THE UNIVERSITY OF ALBERTA

HYPERSTABLE, MODEL REFERENCE ADAPTIVE CONTROL SYSTEMS

by

© RICHARD McGREGOR JOHNSTONE

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF MASTER OF SCIENCE

IN

CHEMICAL ENGINEERING

(PROCESS CONTROL)

DEPARTMENT OF CHEMICAL ENGINEERING

EDMONTON, ALBERTA

FALL, 1978

To My Wife, Barbara

ABSTRACT

In recent years there has been a considerable effort expended towards the refinement of adaptive control techniques. A large number of articles dealing with the general area of adaptive control and stability were reviewed as part of this thesis project. It was concluded that the generalized model reference approach to adaptive control was the most useful and that this could be further subdivided into two classes. The first, termed "direct" adaptive control, refers to any adaptive system in which the adaptation mechanism directly updates the controller parameters or control action. The second class, termed "indirect" adaptive control, denotes those systems in which the adaptive strategy includes an explicit identification step. The control calculation is then based on the identification model parameters.

There are presently three distinct methods for the design of model reference adaptive systems. The first is based on gradient search procedures but provides no guarantee of stability. The others, based on Liapunov stability theory and on Popov's hyperstability theorem, guarantee asymptotic stability in the large. It has been shown that both of these latter techniques offer the same potential for solving the design problem when the starting

point is a state space description of the plant and reference model. However, it was concluded that for adaptive systems based on input-output models, the hyperstability approach was more systematic and productive.

The primary contribution of this thesis is an "indirect", hyperstable, model reference adaptive control formulation that requires only input-output measurements from the process to be controlled. The proposed approach is based on Popov's hyperstability theory as used by Landau for "direct" adaptive state space systems. This "indirect" adaptive method guarantees asymptotic convergence of the outputs of the identified model and the unknown plant, plus convergence of the output of the identified plant to that of the reference model. Among other advantages, this approach provides a high degree of filtering to the adaptive control algorithm.

The approach can also be modified to maintain a system property other than stability. It is shown, for example, that a multivariable dynamic precompensator can be adapted to maintain the non-interacting (decoupled) properties of an unknown plant.

Martin-Sanchez also proposed an adaptive control scheme based on an input-output description of the plant. However, it is shown that his approach can also be derived from

Landau's work and is not as general or powerful as the proposed method.

Simulation studies based on single-input, single-output and multi-input, multi-output process sytems have shown that the proposed "indirect", hyperstable, model reference, adaptive control approach is physically realizable and will provide the desired output convergence even in the presence of moderate noise.

ACKNOWLEDGEMENTS

I would like to take this opportunity to thank the following people for their co-operation and help during my period as a student, here, at the University of Alberta:

-- My supervisor, Dr. D. Grant Fisher for his guidance and enthusiasm,

-- Dr. S. Shah for his many helpful suggestions during the thesis production,

-- The DACS centre personnel for their assistance in the use of the University's computing facilities,

-- Dr. D. E. Seborg, who provided partial funding through a grant from Imperial Oil,

-- The National Research Council who also provided financial assistance,

-- All my fellow graduate students whom it has been my pleasure to know,

-- Finally, for love, understanding and material help in preparing the manuscript, I will be indebted always to my wife, Barbara.

TABLE OF CONTENTS

TABLE OF CONTENTS continued

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

<u>Alphabetic</u>

$\underline{\underline{A}}$	state or output coefficient matrix
$\underline{\underline{B}}$	control coefficient matrix
$\underline{\underline{C}}$	output coefficient matrix
$\underline{\underline{D}}$	disturbance coefficient matrix
$\underline{\underline{H}}$	matrix with known eigenvalues
$\underline{\underline{I}}$	identity matrix
$\underline{\underline{J}}$	control coefficient matrix
$\underline{\underline{K}}$	controller matrix
$\underline{\underline{L}}$	sequence of positive definite matrices
$\underline{\underline{P}}$	solution to the Liapunov eqn. or a sequence of positive definite matrices
$\underline{\underline{Q}}$	arbitrary symmetric positive definite matrix
$\underline{\underline{R}}$	controller matrix
$\underline{\underline{S}}$	controller matrix
$\underline{\underline{T}}$	controller matrix
$\underline{\underline{W}}$	invertible sequence of positive definite matrices
$\underline{p}$	column of matrix $\underline{\underline{P}}$
$\underline{d}$	identification model output vector
$\underline{e}$	identification error vector
$\underline{g}$	identification model output vector
$\underline{r}$	setpoint vector

$\underline{s}$	equivalent system state vector
$\underline{u}$	plant input or control vector
$\underline{v}$	equivalent system state vector
$\underline{w}$	nonlinear element output vector
$\underline{x}$	state vector
$\underline{y}$	output vector
J	performance index
K	controller gain
T	period
V	Liapunov function
a	adaptive gain
b	adaptive gain
c	finite constant
d	adaptive gain
e	performance error
j	complex $\sqrt{-1}$
s	Laplace variable
t	time
x	real variable
y	output
z	z - transform variable
$\underline{\underline{C}}(\ )$	compensator transfer matrix
$\underline{\underline{D}}(\ )$	compensator transfer matrix
$\underline{\underline{G}}(\ )$	linear transfer matrix
f( )	function
g( )	function

h( )	function
.(k)	value at the k^{th} instant
s.d.	standard deviation
sup	supremum

<u>Greek</u>

$\underline{\underline{\Gamma}}$	controller matrix
$\underline{\underline{\Delta}}$	nonlinear functional matrix
$\underline{\underline{\Theta}}$	nonlinear functional matrix
$\underline{\underline{\Phi}}$	nonlinear functional matrix
$\underline{\chi}$	nonlinear functional matrix
$\underline{\underline{\Omega}}$	controller matrix
$\underline{\xi}$	disturbance vector
$\underline{\eta}$	equivalent system state vector
$\underline{\nu}$	output vector
Δ	incremental quantity
λ	finite constant
Π	function of x^2
Σ	summation operator
α	adaptive gain
β	adaptive gain
γ	finite constant
∂	adaptive gain
ε	control or identification error
η	inequality function
μ	inequality function

τ	time

<u>Subscripts</u>

I	integral control mode
L	load transfer quantity
d	desired
m	model reference
o	dependent on initial conditions
p	process
OL	open-loop
CL	closed-loop
FB	feedback
FF	feedforward
SP	setpoint
diag	diagonal
est	estimated
pre	precompensator

<u>Superscripts</u>

I	integral adaptation
T	transpose
o	estimated
•	differentiation
'	differentiation
~	identification model quantity
-1	inversion

$\hat{}$	identification model quantity

Special Symbols

$\underline{\underline{\mathcal{A}}}$	state coefficient difference matrix
$\underline{\underline{\mathcal{D}}}$	disturbance coefficient difference matrix
Re	real part of complex variable
∂	partial differentiation
$\forall$	for all
$\mathcal{F}$	nonlinear functional
$\lvert\bullet\rvert$	modulus
$\lVert\bullet\rVert$	norm
$*$	Kronecker product

CHAPTER ONE

Introduction

Although the so-called modern control theory is now a sophisticated and mathematically acceptable tool for the design of automatic control systems, there has not yet, been the expected rush to implement these techniques. In fact, as several authors have pointed out [1 - 5], there has not even been mild interest expressed. True, a lack of trained manpower has attributed to this gap between technology and practice, but perhaps a more philosophical reason exists.

It is the contention of this author, that the ill-defined problems which are more often that not encountered in the industrial world, are completely intractable to most of the modern control methodology which has been developed over the last two decades. It is an unavoidable conclusion that, the more sophisticated the apriori design technique is, the more complicated and accurate the process model must be. Further, it is in this aspect of the design that a good deal of the cost must be borne. Even in cases where adequate knowledge is available for the development of the dynamic model, there is doubt that, due to changing process conditions, instrument

defects, etc., it is of practical value anyway.

Philosophically therefore, the requirement is for a self-adapting control system which can be designed using as little apriori information as possible; which can restructure itself, or learn, to handle changes in process configurations; which is simple to understand and implement and, most importantly, guarantees some measure of stability for the entire control system. Whilst such a scheme is technically not feasible at the present time, the field of adaptive control systems is attempting to meet this need.

Regrettably, there has been a certain reluctance to look seriously at these techniques, probably due to their mathematical complexity and the sheer volume of theoretical contributions that are being made. It is this preponderance of literature that has obscured the essential features of what an adaptive algorithm is and what it can do.

This thesis proposes to discuss adaptive control schemes and, in particular, the model reference adaptive control (MRAC) approach from both a heuristic and a mathematical viewpoint. The references, which are included should provide a reasonable start to the uninitiated; however, the list is by no means definitive and it is proper

to state that the structuring of the field has necessitated the omission of some detail. Further, since this work is of a theoretical nature there has been no concious attempt to compare the various methods, although it is difficult not to rationalize that the new developments are more productive and therefore should provide the basis for future research in the area.

It is surprising that relatively little work has been attempted, in this direction, within this Department, given the interest in a wide range of techniques [6 - 10]. Indeed, this is only the third work which has appeared on adaptive control systems and the first to consider, theoretically, all the major contributions in the area.

1.1 Objectives of the Thesis

The work contained herein has primarily been directed towards examining the input-output formulation for adaptive control developed originally by Martin-Sanchez [11]. In doing so, however, it has been necessary to investigate the chronological development of model reference adaptive control and, because of the "dual" nature of these systems, model reference identification.

Thus, whilst the emphasis is clearly towards the

generalization of the model reference technique, a major contribution lies in the structural analysis of the field.

1.2 Structure of the Thesis

The theoretical results of the thesis have been presented in a more or less chronological sequence, commencing initially with an overview of available adaptive control techniques. A literature survey is included to provide a stepping-stone from which more detailed research may be initiated. Further, a broad outline of the field is considered from a design point of view.

Chapter Three deals with model reference type systems, providing a sequential increase in sophistication from gradient techniques through to the Liapunov stability method and, finally, to the method based on Popov's hyperstability criterion. All of these are, however, still state-space formulations.

In Chapter Four a major break is made, with the discussion of the input-output formulation considered by Martin-Sanchez. The generalization of this model reference technique is discussed in Chapter Five, with a particular example being developed to illustrate the applicability of the concept.

Recourse has been taken to simulation to illustrate certain points of the theoretical approach. The results of these simulations are discussed in Chapter Six. Further, it has been noted that the concept is not just applicable to control or identification schemes per se. There are many instances in which a recursive on-line identification scheme would be of manifest applicability. A case in point is presented in Chapter Seven. Finally, in Chapter Eight, the overall conclusions of the work and several areas of desired future study are summarized.

The amount of material which may be omitted obviously depends on the prior exposure of the reader to the mathematical techniques expressed in the relevant sections (a firm background in nonlinear control theory is felt to be desirable)[1]. It has been the intent of the author to try not to present, in too much detail, work which is already available in the literature, unless there is a clear compulsion to do so.

1

A particularly good reference in this field is the work by Siljak [12].

Generally, it would be expected that some familiarity of the objectives and mechanics of adaptive systems could be assumed. Moreover, if a reasonable idea of the work that Landau has done is allowed, then Chapter Three may be omitted in its entirety. The main theoretical contributions of this thesis have been confined to Chapters Four and Five, with a conceptual summary appearing in Chapter Eight. It would be suggested that readers familiar with hyperstable adaptive system design restrict their attention to these chapters for the first reading.

CHAPTER TWO

Adaptive System Methodology and Literature Review

2.1 Introduction

It has been said that the theory of adaptation and learning "...is one of the most fundamental of modern science and engineering..." [1]. Undoubtedly the concepts of adaptation are widely acceptable in their basic forms, for they can be found at work, in some form or another, from medicine to spaceflight.

It comes as some surprise, therefore, to learn that the science of adaptation is relatively new . To a certain extent it derived out of classical control theory, when it was found that there were some practical situations in which a conventional controller (ie. that based on a fixed control strategy) simply could not supply an acceptable performance. For example, a controller might be required to compensate for such things as:

a) process transfer function variations, either in order or in parameter values, with changes in environment. This situation is manifest in aerospace applications since the input-output relationships which describe an aircraft's flight

characteristics, can change with air speed, height, air pressure, etc.;

b) changes in the system itself. For instance, many systems can change quite dramatically over time through such varied causes as mechanical wear, or because of operational changes (ie. equipment malfunction), etc.;

c) changes in the nature of the inputs and disturbances to which the plant is subjected. This can be especially prevalent in industries which may be termed "marginally economic" and hence are operated much closer to the constraints demanded by changing market trends.

The motives for using adaptive control are not industry dependent and examples may be found in paper manufacture applications as well as in aircraft control. Thus, in a paper mill, the inertia of the winding reel changes as the paper is wound on, and the motor torque, required to maintain constant tension in the web, changes with reel diameter. In a missile, the mass and centre-of-gravity change as fuel is consumed and in a supersonic aircraft, the aerodynamic parameters vary widely as the plane climbs from sea-level to cruising altitude. Accordingly then, under

conditions such as these, the performance of a conventional feedback control system often degrades to beyond acceptable limits.

The obvious solution is to employ some type of controller which accounts for these changes and may, in fact, use them actively in the pursual of the primary control task. Such a system must therefore measure the characteristics of the outputs of the control system and of the process under control and, on the basis of these measurements, adjust the overall system towards some previously defined optimum condition. This type of control system would typically be of the form shown in Figure 2.1.

It should be noted that this description contains, as a subclass, any practical conventional control system, for it can be argued, that in these cases the performance index is measured directly and any major variations in the system parameters can be compensated for by tuning the control loop. Thus in essence, the adaptive loop is closed, though manually.

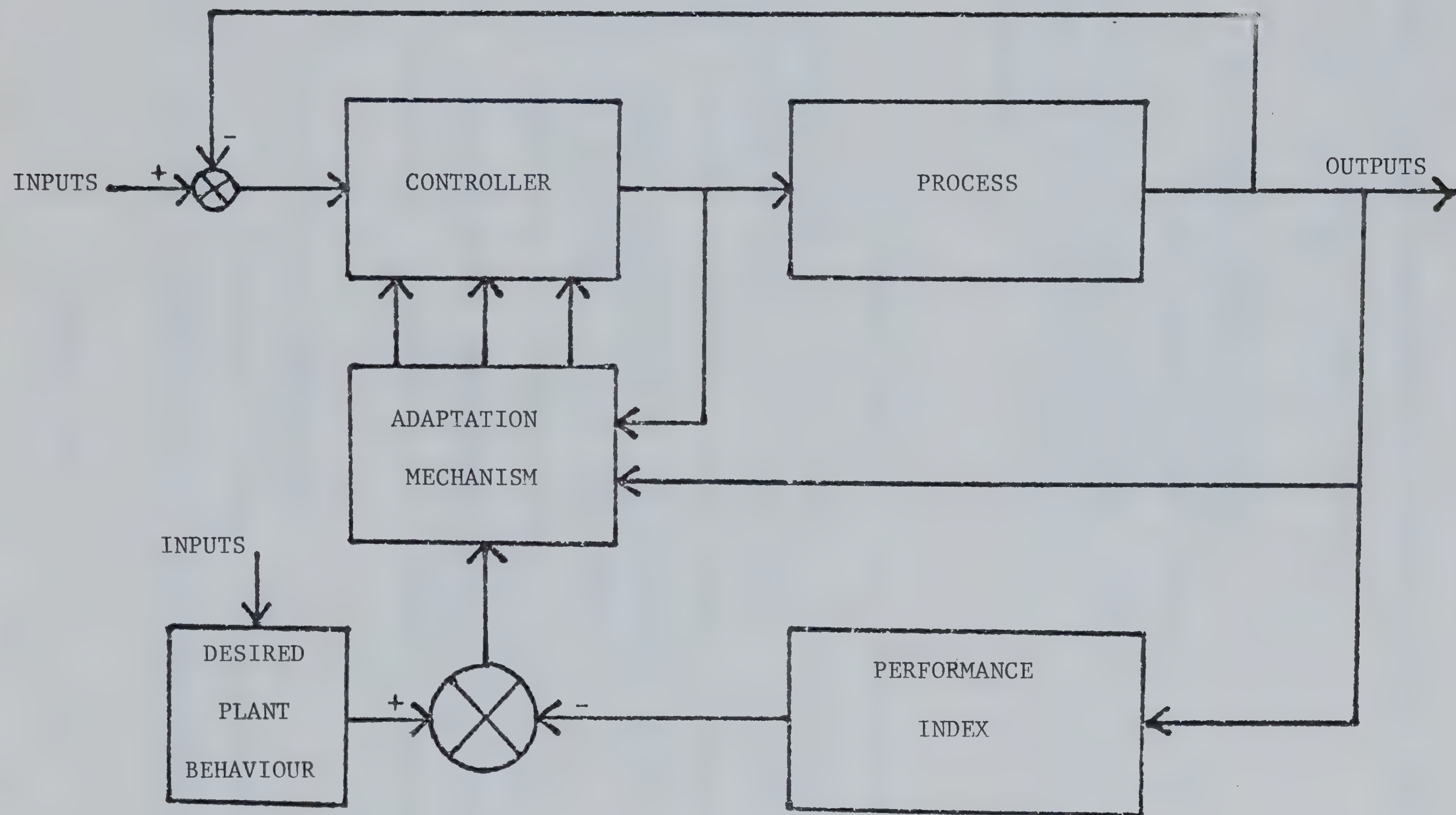


FIGURE 2.1 A TYPICAL ADAPTIVE CONTROL SYSTEM

2.2 Classification of Adaptive Control Systems

Davies [2] has identified six essentially distinct types of basic adaptive control system:

1) Passive adaptive systems:

These static systems are designed to give adequate performance for a large range of parameter variations and environmental disturbances, using a fixed control strategy.

In essence this is a most unsatisfactory type of approach since it can be argued that the design phase is based on a "worst case" philosophy. Not only is it difficult to define a realistic "worst case", it effectively means that "tight" control about a prespecified point becomes almost impossible. This last problem may be alleviated somewhat by a technique known as multi-mode switching; a method which essentially specifies several types of controller, dependent on the system performance with time.

2) Input Signal Adaptation:

In this type of system the adaptation mechanism is purely a function of the characteristics of the

input signal. The adaptive system is, thence, essentially open-loop.

3) Plant Adaptive Control Systems:

This type of control system adjusts its own parameters to compensate for process parameter variations in the controlled plant. A subclass of this type of configuration contains the so-called model reference adaptive systems (MRAS), more of which will be said later.

4) Parameter Adaptive Systems:

Adaptation is achieved in these systems by directly adjusting the process parameters. Applicable mainly to electrical systems, it is exceedingly rare to find this type of adaptive strategy used in chemical process situations since the open-loop process parameters are rarely available for direct adjustment.

5) Input Signal Shaping Adaptation:

This technique appears to be potentially very powerful since the adaptive mechanism can be used to calculate an input signal to "force" the system to respond in some pre-determined way. A particular type of model reference adaptive control approach (the signal synthesis

technique) operates in just this manner, as will be discussed later.

6) Extremum or Optimum Adaptation

Here the basic control scheme is adjusted so that some dependent variable is maintained at a maximum (or a minimum) value. This apparently is a very common approach nowadays, although the method does not appear to offer nearly as many possibilities as those listed above.

2.3 Nature of an Adaptive System

All adaptive schemes presented to date, can be considered as being designed with three, though not always distinct, phases in mind [3]:

1) Identification:

In the context used here, this refers to the definition, at any time, of a desired system behaviour and a performance index. Such a specification may take many forms, varying from simple classical performance measures, such as rise-time, overshoot, etc., through to more sophisticated model reference approaches which specify a desired dynamic model. In its most general form, such a

model could consider long range economic goals as well as short-term control objectives [2].

2) Decision:

This part of the mechanism decides how system performance relates to the desired response. Corrective adjustments are then made according to the algorithm used. Of the three phases of design to be considered in adaptive system configuration, this undoubtedly, is the most well-defined of all.

There are essentially two basic types of algorithmic approach that dominate the literature at the present time:

a) gradient techniques and,

b) stability methods

Of late, the stability approaches have virtually taken over the field, especially for the model reference configuration [4 - 9].

3) Modification:

This process entails the physical action of

"driving" the system performance index towards the desired value. The choice of how this might be done can be narrowed to a certain number of alternatives, although this is a field of active research:

(i) The process parameters, themselves, may be modified. This technique gave rise to the parameter adaptive systems;
(ii) The controller parameters may be adjusted. Quite popular, this technique has been classified as a plant adaptive system;
(iii) An input signal may be generated in such a way that the process approaches the optimum position;
(iv) A simple mode switch may be employed enabling the system to utilize a completely new algorithm, if a certain condition, dependent on the process, is met. Or finally,
(v) Some type of logic-directed controller [10,11] may be adopted.

All of these phases are naturally, inter-related, but

it is true to say that, at some time, a designer must give consideration to each in a more or less separate context.

At this stage it is proposed to discuss each of the above, in detail, as it is apparent that the individual schemes put forward tend to give greater cognizance to one, or at most two, particular aspects of the design.

2.3.1 Identification

The decisions made during this phase often pre-determine the approach to be taken in subsequent system specification, for it is at this stage that the index of performance and desired plant response must be decided upon. Indeed, the desired plant response specification is not a trivial matter since it is obvious that, by definition, no system can perform better than prescribed by its chosen optimum conditions.

This decision must then take into account as many factors as the designer can foresee at the time, including perhaps even some forecast of economic trends and environmental variations.

The choice of both the index of performance and the desired plant response are obviously affected by the measurements which physically can be carried out. For

instance, in most chemical process plants the states of the plant are not all available. Thus, a control scheme based on a function of the error between the state and some desired state is not very practical, unless some sort of state estimation scheme is employed.

Given the inherent difficulties involved in these choices, it is perhaps best to define some properties which would be desirable to include in a performance index. Firstly, the chosen index needs to be readily measurable "on-line" and moreover, it is essential that this may be done as accurately as possible. Secondly, the optimum region defined by the desired process behaviour should be as well-defined as possible so that there is nc ambiguity involved in the maintenance of this positicn. Thirdly, the chosen performance index should provide a good indication of relative operating quality over as large a process parameter space as is feasible, and lastly, it should be physically meaningful.

Unfortunately, many performance indices in use do not have all the above properties. Few are measurable and even fewer really have a physical significance. The mean square error and integral square error indices fall into this category. These can easily lead to unstable or physically

unrealizable system designs [12].

The most common performance indices in use [13 - 23], can be described by the equation:

$$J = \int_0^\infty f_1(t)\, f_2(e)\, dt \qquad (1)$$

where e=e(t) is the system error. Unfortunately, this type of error criterion often fails to yield a unique optimum operating condition.

When the input is not deterministic, but stochastic, equation (1) can be modified to become:

$$J = \lim_{T\to\infty} 1/2T \int_{-T}^{T} f_1(t)\, f_2(e)\, dt \qquad (2)$$

where $f_1(t)$ is generally, a strictly positive value.

Finally, it is noted that any index of performance is by necessity process-dependent, for a desirable index for one system may lead to a completely erroneous result for another. Thus, in some situations, it would be "optimal" to have an extremely fast response to an input function, whereas in other plants the reverse might be true. In keeping with other authors [2,24], it is recommended that well-tried measures of system performance only, be utilized.

As has been implied already, the performance index, in

some ways, specifies the desired plant behaviour, although this may be chosen by purely subjective analyses of product quality and the corresponding system performance. This approach is especially prevalent when the product specifications are only vaguely related to the actual operating conditions. Thus, iron ore pellet quality in an indurator, is expressed in terms of a "tumbling index", which is related to the fracture properties of the pellets. The difficulty here is that this figure is dependent on numerous conditions and, as such, does not define a unique operating pcint. In practice these problems are over-shadowed by purely prosaic considerations such as operator experience and biases.

Most of the schemes used today configure the desired plant behaviour block in such a way as to correspond directly with some measure of the plant performance, eg. the desired steady-state operation is an obvious candidate for such a choice. These then, can all be said to include a conceptual "model reference" approach. The model of the plant which supplies the desired behaviour is, in fact, any dynamic or steady-state system which the designer feels properly designates the objectives of the plant under consideration (given the assumption of physical

realizability). The designer thus, can take into account, ideally, as many factors as is wished including the performance index.

2.3.2 Decision

This section considers the algorithmic approaches which can be utilized in adaptive configurations. Originally most of the procedures suggested were based on optimization theory [25 - 27] using surface-slope measurements as indices of performance. Eveleigh [25] has given a very good presentation of these earlier schemes, including those based on steepest descent approaches [28,29], periodic perturbation methods [29 - 34], peak holding systems [35 - 39], and signal synthesis [40]. Also in this work, there appears a summary of applications using these approaches [41 - 53].

It is now known that these techniques do not guarantee closed-loop stability [7,12] and this has lead to the development of other techniques based on Liapunov's direct method and Popov's hyperstability [54 - 65].

This choice is perhaps the simplest of all, since it is influenced primarily by the state of the art, which is clearly in favcur of the hyperstability concept at

present [6].

2.3.3 Modification

As has been noted above, it is only in extremely rare cases that the parameters of the open-loop process are directly adjustable. This then normally necessitates modification of some supplementary control block.

There are two common approaches to the design of adaptive control systems:

(i) controller parameter adaptation or,
(ii) signal synthesis adaptation.

The first category includes any scheme which has as its main aim, the adjustment of the parameters and/or structure of a compensator which subsequently determines the control action. A system such as that described by Figure 2.2 is typical.

The second approach categorizes those schemes which depend on the implementation of an input signal to achieve the desired index of performance figure. Figure 2.3 represents a general scheme.

Examples of both systems may be found in the

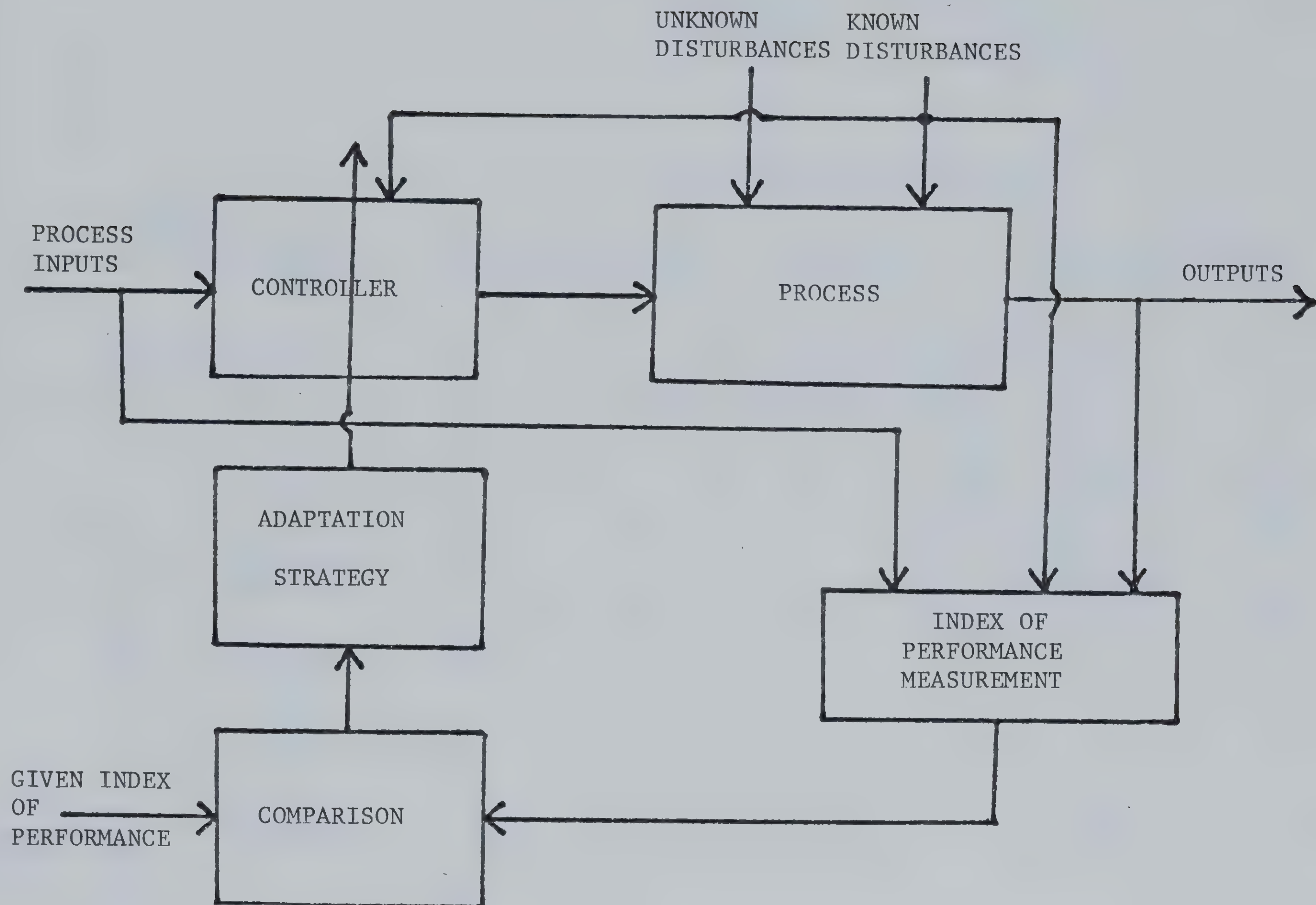


FIGURE 2.2 BASIC CONFIGURATION OF A PLANT ADAPTIVE CONTROL SYSTEM

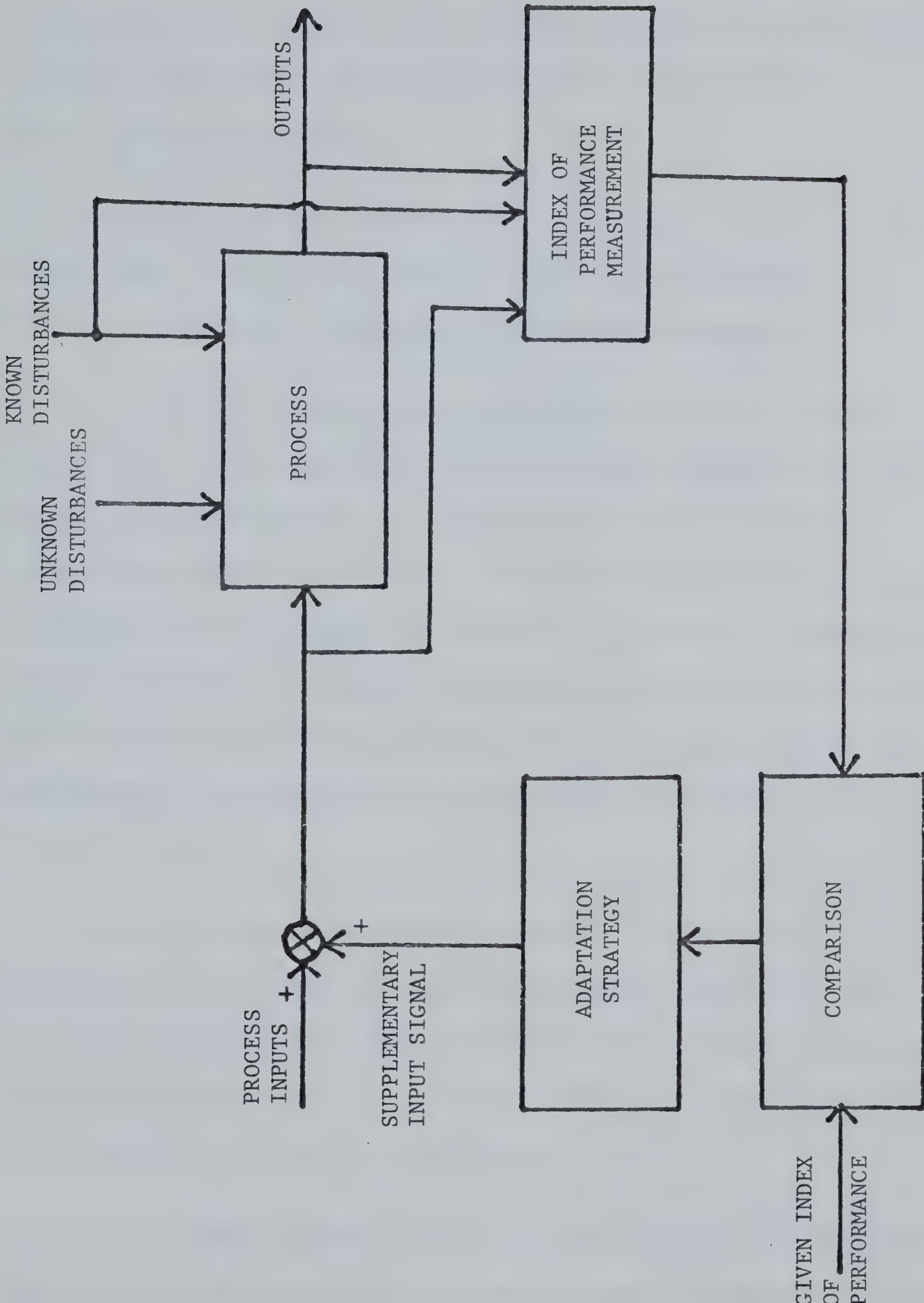


FIGURE 2.3 BASIC CONFIGURATION OF A SIGNAL SYNTHESIS ADAPTATION SYSTEM

literature, as well as those which feature certain aspects of both approaches simultaneously [1 - 9,25,66].

2.4 Other Approaches

Finally, other conceptual approaches are mentioned, since they are of considerable interest at present.

2.4.1 Stochastic Methods and Learning Systems

All of the techniques previously referred to have contained, as an integral part of their formulation, a deterministic nature, ie. noise characteristics are considered where they occur to be destructive and are either filtered out or simply put up with. The use of stochastic models however, includes the randomness as an active element in the pursual of the control task. In the control and estimation literature such algorithms are widely distributed [6,67 - 68].

Another field of interest which is becoming particularly active, is that of learning systems. This type of system learns the unknown information during operations. The learned information is then, in turn, used as experience for future decisions or controls.

A very good presentation of the concepts of learning systems is contained in a booklet issued following the 1973

IFAC conference [69]. This booklet provides an overview of progress in learning systems with emphasis on their application to control problems. Reinforcement learning models, which originated from studies of the psychology of learning, are discussed. Learning algorithms, based on:

(i) Bayesian estimation,
(ii) stochastic automation models,
(iii) stochastic approximation, and
(iv) random search,

are described and compared. The advantages and disadvantages of each learning algorithm, when applied to control problems, and the role of learning techniques in robot and manipulator systems are reviewed. Finally, a bibliography of learning control is presented.

Another useful source is the work by Tsypkin [1] in which is presented a unified approach to the techniques of adaptation and learning . Both he and Lerner [70] have also included bibliographies on Soviet research in the area.

2.5 Conclusions

The general adaptive control system concepts discussed in this chapter provide a background for the more specific material which follows. The references cited here should

give the reader a sufficient start in the various fields of adaptation and learning, although it is beyond the means of any single authcr to cover in depth, an area as broad and detailed as this work has become.

In view of the interest in model reference control systems [4 - 9] of late, the following chapters are devoted to these procedures. The general conceptual approach of model reference systems adheres to the design philosophies referred to above.

Chapter Two

CHAPTER THREE

Model Reference Adaptive Control Techniques

3.1 Introduction

An adaptive system calculates an index of performance based on the inputs and the outputs of the adjustable system. From the comparison of the calculated index of performance values and a set of given ones, the adaptation mechanism modifies the parameters of the adjustable system or generates an auxiliary input, in order to maintain the index of performance values close to the desired ones.

Of the myriad of possible performance criteria that one could imagine, it is certain now, that the so-called model reference techniques offer the most general approach [1 - 7]. Indeed the concept is well-based from a practical point of view, since the desired behaviour of the controlled system can be "forced" to take into account many dynamic elements that would be encountered, under normal operating conditions. These could include noisy measurements, disturbances, setpoint changes, time-varying behaviour and perhaps, even some classes of non-linearity.

The earliest concerted efforts in the design of model reference adaptive control (MRAC) systems came during the

late 1950's and early 1960's. This was principally in the aircraft industry, for autopilot design. The incentive here was clear, since, under some flight conditions a human pilot simply cannot maintain control by purely manual means. Nor for that matter could a fixed control scheme offer satisfactory behaviour, given the rapidly varying conditions. An obvious case in point is the approach phase in the landing of shipboard fighters. Human pilots simply could not cope with such variable inputs as flight deck pitch, air turbulence, and stall indication systems, all of which may possibly be present under combat conditions [8].

The problems of the chemical process industries can certainly be compared directly with those experienced in an aerospace environment. There is an enormous incentive for the development of adaptive control schemes (primarily due to cost factors). Indeed, since chemical plants are known to exhibit nonlinear and some time-varying behaviour it would appear a practical necessity. Moreover, MRAC offers an unique opportunity to look at the cost function since the reference model can be, theoretically, arbitrarily chosen.

The MRAC systems suggested to date, may be broadly classified into three distinct approaches:

(i) gradient techniques;

(ii) Liapunov stability methods, and

(iii) those based on hyperstability.

In the following sections of this chapter, these three classificaticns will be discussed.

As a necessary part of the chronological development a brief literature survey will be presented. For additional details the reader is referred to the several excellent surveys which already exist [1 - 7].

3.2 Gradient Techniques

The techniques of the 50's and early 60's used methods based on gradient searches with the aim of minimizing a function of the difference (error) between the outputs (or states) of the reference model, and the actual process [9]. Thus, in the best known of these, the "MIT Technique" [10,11] (shown in Figure 3.1) the design objective is to minimize an integral of the error squared, $\int \varepsilon^2 \, dt$.

For this scheme, the parameter adjustment law is written as:

$$\dot{K}_c = b \varepsilon \, \partial y_p / \partial K_c \quad \text{.......................................} (1)$$

where K_c is an adjustable controller parameter of the

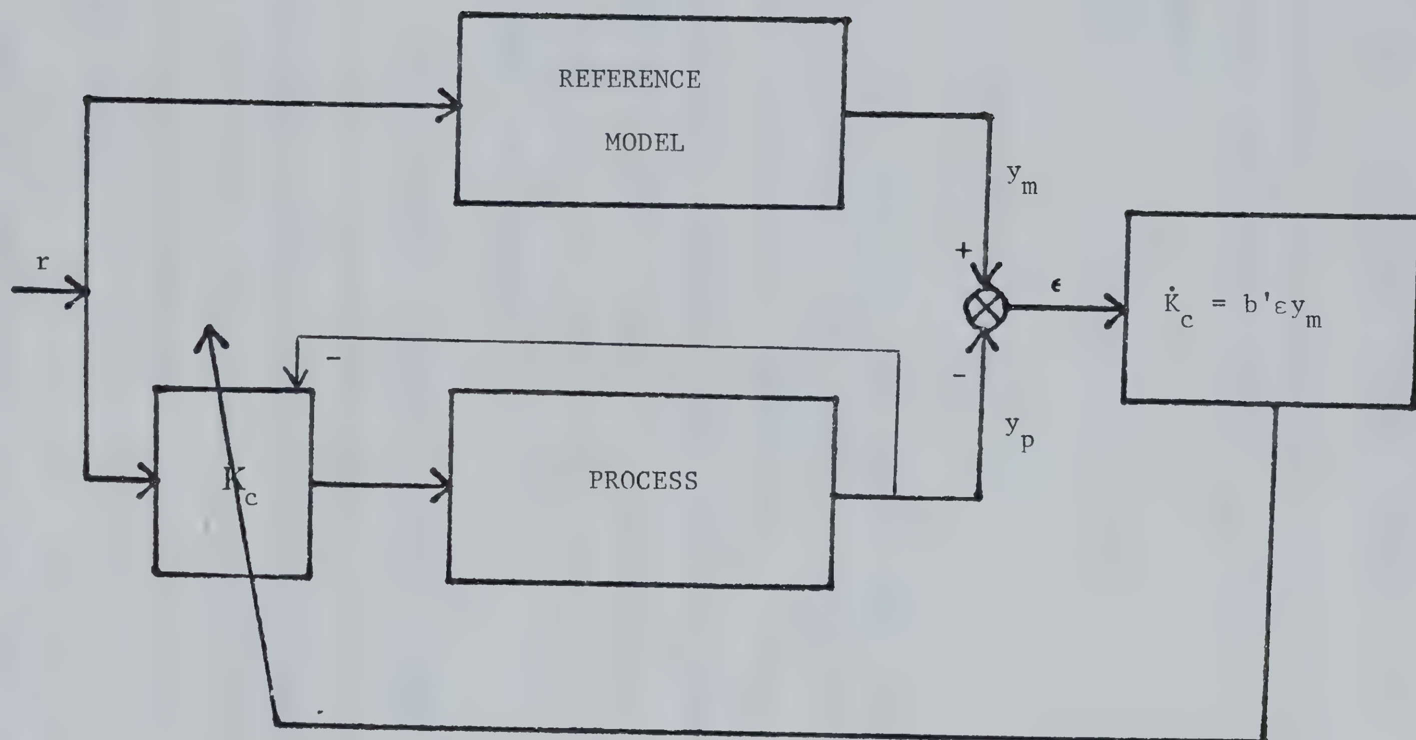


FIGURE 3.1 A PICTORIAL REPRESENTATION OF WHITAKER'S "MIT DESIGN RULE" ADAPTIVE CONTROL SYSTEM

system; y_p is the output of the plant; b is a positive constant and ε is the performance error.

In this case, the sensitivity function, $\partial y_p/\partial k_c$, is proportional to the model output, y_m, and hence, equation (1) becomes:

$$\dot{k}_c = b'\varepsilon\, y_m \quad\quad (2)$$

where b' is the adaptive gain.

This method has been very popular due to its simplicity, although it may require a number of sensitivity filters for multiparameter adjustments. Several improvements (with respect to the speed of response) have been suggested. Among these, the techniques of Donalson [12], Dressler [13], Price [14], Winsor [15] and Monopoli [16], are notable.

All of the gradient methods however, suffer from the disadvantage that they are not globally stable [4,17] and hence, the adaptive gain, which governs the speed of response, is limited. Extensive simulations during the design stage are thus necessary to establish the region of stable operations.

The stability problem was first demonstrated by

Parks [17] and lead, in part, to development of adaptive schemes based on Liapunov's second or direct method.

3.3 Liapunov Design

The technique, using a quadratic Liapunov function, was first suggested by Butchart and Shackcloth [18] although the implementation was later carried out by Parks [17] in the redesign of adaptive systems formerly obtained using the MIT gradient method. The use of a more general Liapunov function by Phillipson [19] and Gilbart et al. [20] has resulted in the introduction of feedforward loops that improve the damping of the adaptive response.

The synthesis has also been generalized to multivariable systems, using a state-space formulation, by Winsor and Roy [21] and Porter and Tatnall [22] and the latter have also extended the scheme to time-varying systems [23]

The "dual" problem of system identification using the Liapunov method has also been considered by several authors [24 - 29] and later work has extended the overall scheme to discrete systems [30,31].

Oliver [32,33] has presented a thorough experimental evaluation of the MRAC technique using a double-effect

evaporator, and has extended the control scheme to include setpoint and integral action.

3.3.1 Theory

The development is relatively straightforward starting with a differential (state-space) representation of the plant:

$$\dot{\underline{x}}_p = \underline{A}\,\underline{x}_p + \underline{B}\,\underline{u} + \underline{D}\,\underline{\xi} \quad \ldots\ldots(3)$$

where $\underline{x}_p$ represents the nx1 state vector

$\underline{u}$ represents the mx1 input or control vector and

$\underline{\xi}$ represents the px1 disturbance vector.

$\underline{A}$, $\underline{B}$ and $\underline{D}$ are process parameter matrices of appropriate order.

Assuming multivariable feedback-feedforward control one can write that:

$$\underline{u} = \underline{K}_{FB}\,\underline{x}_p + \underline{K}_{FF}\,\underline{\xi} \quad \ldots\ldots(4)$$

or, upon substitution of equation (4) in equation (3):

$$\dot{\underline{x}}_p = (\underline{A} + \underline{B}\,\underline{K}_{FB})\underline{x}_p + (\underline{D} + \underline{B}\,\underline{K}_{FF})\underline{\xi} \quad \ldots\ldots(5)$$

which can be written in closed-loop form as:

$$\dot{\underline{x}}_p = \underline{A}_p\,\underline{x}_p + \underline{D}_p\,\underline{\xi} \quad \ldots\ldots(6)$$

where $\underline{\underline{A}}_p$ and $\underline{\underline{B}}_p$ denote, respectively, nxn and nxp closed-loop parameter matrices.

A desired closed-loop reference model is now chosen as:

$$\dot{\underline{x}}_m = \underline{\underline{A}}_m \underline{x}_m + \underline{\underline{D}}_m \underline{\xi} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

where $\underline{x}_m$ is the nx1 model state vector.

A measure of the system performance, for the scheme, is supplied by defining an error vector, $\underline{\varepsilon}$, such that:

$$\underline{\varepsilon} = \underline{x}_m - \underline{x}_p \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Subtracting equation (5) from equation (7) results in:

$$\dot{\underline{\varepsilon}} = \underline{\underline{A}}_m \underline{\varepsilon} + (\underline{\underline{A}}_m - \underline{\underline{A}}_p)\underline{x}_p + (\underline{\underline{D}}_m - \underline{\underline{D}}_p)\underline{\xi} \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

and if:

$$\underline{\underline{A}}_m - \underline{\underline{A}}_p = \underline{\underline{\alpha}}$$

$$\underline{\underline{D}}_m - \underline{\underline{D}}_p = \underline{\underline{\delta}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

then a Liapunov function:

$$V = \underline{\varepsilon}^T \underline{\underline{P}}\, \underline{\varepsilon} + \sum_{i=1}^{n} \sum_{j=1}^{n} 1/a_{ij}\, \alpha_{ij}^2 + \sum_{i=1}^{n} \sum_{j=1}^{p} 1/d_{ij}\, \delta_{ij}^2 \quad \ldots\ldots\ldots (11)$$

can be written.

Chapter Three

$\underline{\underline{P}}$ is the positive definite matrix solution of the Liapunov equation:

$$\underline{\underline{A}}_m^T \underline{\underline{P}} + \underline{\underline{P}}\,\underline{\underline{A}}_m = -\underline{\underline{Q}} \quad \ldots\ldots(12)$$

and $\underline{\underline{Q}}$ is an arbitrary, symmetric positive-definite matrix. $a_{ij}>0$ and $d_{ij}>0$ are adaptive loop gains and α_{ij} $(i=1 \rightarrow n, j=1 \rightarrow n)$ and δ_{ij} $(i=1 \rightarrow n, j=1 \rightarrow p)$ are, respectively, the i,j^{th} elements of $\underline{\underline{A}}$ and $\underline{\underline{D}}$.

Differentiating equation (11) with respect to time and using equations (9) and (12), with $\underline{\underline{Q}} = \underline{\underline{I}}$, one obtains:

$$\dot{V} = -\underline{\varepsilon}^T\underline{\varepsilon} + 2\sum_{i=1}^{n}\sum_{j=1}^{n}(1/a_{ij}\,\dot{\alpha}_{ij} + x_{pj}\,\underline{\varepsilon}^T\,\underline{P}_i)\alpha_{ij}$$

$$+ 2\sum_{i=1}^{n}\sum_{j=1}^{p}(1/d_{ij}\,\dot{\delta}_{ij} + \xi_j\,\underline{\varepsilon}^T\,\underline{P}_i)\delta_{ij} \quad \ldots\ldots(13)$$

where $\underline{P}_i$ denotes the i^{th} column of the matrix $\underline{\underline{P}}$.

In order that $\dot{V}$ be at least negative semi-definite, as required for stability, it is sufficient that $\alpha_{ij}=0$ and $\delta_{ij}=0$ (ie. the model and the process are identical) or that:

$$1/a_{ij}\,\dot{\alpha}_{ij} + x_{pj}\,\underline{\varepsilon}^T\,\underline{P}_i = 0$$

$$i = 1 \rightarrow n, \; j = 1 \rightarrow n \quad \ldots\ldots(14)$$

$$1/d_{ij} \; \dot{\delta}_{ij} + \xi_j \underline{\varepsilon}^T \underline{P}_i = 0$$

$i = 1 \to n \;,\; j = 1 \to p$ ....(15)

These equations may be rearranged to give:

$$\dot{\alpha}_{ij} = -x_{pj} \; \underline{\varepsilon}^T \underline{P}_i \; a_{ij}$$

$i = 1 \to n \;,\; j = 1 \to n$ ....(16)

$$\dot{\delta}_{ij} = - \xi_j \underline{\varepsilon}^T \underline{P}_i \; d_{ij}$$

$i = 1 \to n \;,\; j = 1 \to p$ ....(17)

As Oliver [32] notes, V will decrease monotonically with time until either V equals zero or some positive constant. In the latter case this suggests that $\underline{\varepsilon}=0$ but also that $(\underline{\underline{A}}_m - \underline{\underline{A}}_p) \neq 0$ and/or $(\underline{\underline{D}}_m - \underline{\underline{D}}_p) \neq 0$ and, consequently, the closed-loop model and the process parameter matrices need not be equal in order that the error between their states be zero.

The next development is to use the adaptation laws given by equations (16) and (17) to obtain an adaptation

mechanism that can be physically realized.

Define:

$$\underline{\Gamma} = \underline{B}\ \underline{K}_{FB}$$

and:

$$\underline{\Omega} = \underline{B}\ \underline{K}_{FF} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

Now, assuming that the open-loop plant and closed-loop model parameter matrices are time-invariant:

$$\dot{\underline{A}} = \dot{\underline{A}}_m - \dot{\underline{A}}_p = -\dot{\underline{A}}_p = -(\dot{\underline{A}} + \dot{\underline{\Gamma}}) = -\dot{\underline{\Gamma}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

and:

$$\dot{\underline{\Phi}} = \dot{\underline{D}}_m - \dot{\underline{D}}_p = -\dot{\underline{D}}_p = -(\dot{\underline{D}} + \dot{\underline{\Omega}}) = -\dot{\underline{\Omega}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

Using equations (16) and (17) in conjunction with the relationships, derived as equations (19) and (20), one finally obtains the adaptation laws:

$$\dot{\gamma}_{ij} = x_{pj}\ \underline{\varepsilon}^T\ \underline{P}_i\ a_{ij}$$

$$i = 1 \rightarrow n\ ,\ j = 1 \rightarrow n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

$$\dot{\omega}_{ij} = \xi_j \underline{\varepsilon}^T \underline{P}_i d_{ij}$$

$$i = 1 \rightarrow n \ , \ j = 1 \rightarrow p \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

This leads to a configuration depicted as Figure 3.2.

The original theory is then modified, by Oliver [32], to include adaptive integral and setpoint control modes such that:

$$\underline{u}(t) = \underline{\underline{K}}_{FB} \ \underline{x}_p(t) + \underline{\underline{K}}_I \int_0^t \underline{y}(t)\, dt$$

$$+ \underline{\underline{K}}_{FF} \ \underline{\xi}(t) + \underline{\underline{K}}_{SP} \ \underline{y}_{SP}(t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

where $\underline{\underline{K}}_I$ and $\underline{\underline{K}}_{SP}$ are, respectively, integral and setpoint control mode matrices.

$\underline{y}(t)$ represents a qx1 vector of the so-called "integral states" [34] and:

$$\underline{y}(t) = \underline{\underline{C}} \ \underline{x}_p(t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

ie. the "integral states" are nothing more than those original states in which offset has been deemed undesirable.

$\underline{y}_{SP}$ represents the, at most, rx1 [34,35] desired values of the "setpoint states".

Both these extensions are handled by reformulating the

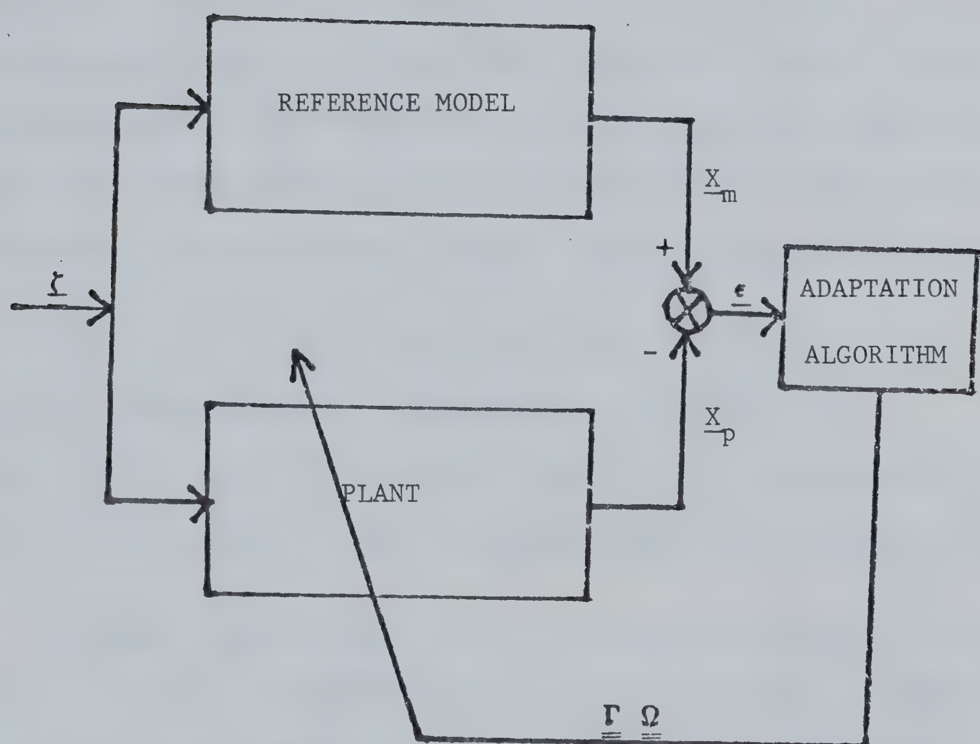


FIGURE 3.2 LIAPUNOV DESIGN OF A MODEL REFERENCE ADAPTIVE CONTROL SYSTEM

problem such that equations of the form of (6) and (7) are obtained [32].

The main disadvantage of the Liapunov method is that the entire state vector must be available for measurement, which is not often possible. Extensive effort has been applied to this area, mainly in the addition of state-estimation to the original adaptive scheme, using Kalman filters[1] [36] and the so-called adaptive observers [37 - 40] which simultaneously estimate the states and parameters of an unknown linear, time-invariant system.

There is also at least one study which uses the Liapunov technique, as proposed by Sutherlin and Boland [41], and a full-order observer to reconstruct the entire state vector from measured inputs and outputs [42].

Another disadvantage of the Liapunov design rule is that it may not be applicable to those classes of plant in which the plant parameters are not directly adjustable.

1

For a very good work on Kalman filtering, the reader is referred to a booklet which has been produced under the auspices of the Guidance and Control Panel of NATO-AGARD [80] (North American Treaty Organization - Advisory Group for Aerospace Research and Development).

Such a case was mentioned by Winsor and Roy [21] and a solution, though quite complex, was offered by Gilbart et al. [20]. Hang [43] has shown that this general adaptive rule is restricted to the class of plants in which all the controllable parameters appear explicitly as individual elements of the plant and control matrices. He has further presented a design which alleviates this problem.

3.4 Hyperstability Design

Of the two methods, based on absolute stability, that derived from Popov's results in the field of hyperstable systems [44 - 46] appears to yield the most general, practical and systematic approach. This is in spite of the fact that, from a theoretical point of view, the approach via Liapunov's second method and via hyperstability theory have the same potential for solving the design problem.

Landau [47], in 1969, was the first to present a detailed analysis showing how a nonlinear hyperstable system could be designed to carry out the MRAC task; although Anderson [48] had looked at the hyperstability problem from a positivity point of view in 1968.

The theorems associated with these developments include all the results obtained by Butchart and Shackcloth [18],

Parks [17] and Winsor and Roy [21] using Liapunov's second method.

The hyperstability work has been extended to include discrete systems [49,50] and series-parallel, parallel, and series configurations have been investigated [1,2,51]. Use has been made of the positivity lemma [52] and positive definite kernels [52,53] to provide an alternate proof of stability and several control schemes have been proposed [54 - 59].

A lucid description of the results of hyperstable system analysis can be found in [60]. In addition, hyperstability concepts have been used in several other design methods [61 - 66] and for system identification [67 - 74] based on the "dual" nature of the MRAC technique.

Although most of the work done to date by Landau and others has dealt with state-space systems, the results are readily extended to include input-output formulations as shown by Martin-Sanchez [75 - 77]. This, as noted in Chapter Four, has certain distinct advantages allowing, for example, the inclusion of explicit time-delays.

Chapter Three

3.4.1 Theory

An outline of the general approach, as detailed by Landau, will be presented for the continuous state-space formulation. The discrete formulation of Martin-Sanchez appears in Chapter Four as a distinct subset of the hyperstability theory.

As with the Liapunov approach the starting point is the dynamic equation describing the plant.

Suppose:

$$\dot{\underline{x}}_p = \underline{A}(t)\,\underline{x}_p + \underline{B}(t)\,\underline{u} + \underline{D}(t)\,\underline{\xi} \quad \ldots\ldots\ldots\ldots(25)$$

and:

$$\underline{y}_p = \underline{C}\,\underline{x}_p \quad \ldots\ldots\ldots\ldots(25a)$$

where the $\underline{\underline{A}}(t)$, $\underline{\underline{B}}(t)$ and $\underline{\underline{D}}(t)$ denote time-varying adjustable open-loop parameter matrices and $\underline{y}_p$ is an rx1 output vector.

A reference model equation is then chosen such that:

$$\dot{\underline{x}}_m = \underline{A}_m\,\underline{x}_m + \underline{B}_m\,\underline{u} + \underline{D}_m\,\underline{\xi} \quad \ldots\ldots\ldots\ldots(26)$$

and:

$$\underline{y}_m = \underline{C}\,\underline{x}_m \quad \ldots\ldots\ldots\ldots(26a)$$

The plant and model equations are understood to be of the same order (ie. the dimension of $\underline{x}_m$ is the same as that of $\underline{x}_p$ etc.). Defining an error vector, $\underline{\varepsilon}$, such that:

$$\underline{\varepsilon} = \underline{x}_m - \underline{x}_p \quad \text{...(27)}$$

it is clear that an equation:

$$\dot{\underline{\varepsilon}} = \underline{\underline{A}}_m \underline{\varepsilon} + (\underline{\underline{A}}_m - \underline{\underline{A}}(t))\underline{x}_p + (\underline{\underline{B}}_m - \underline{\underline{B}}(t))\underline{u}$$

$$+ (\underline{\underline{D}}_m - \underline{\underline{D}}(t))\underline{\xi} \quad \text{...(28)}$$

can be written.

The next development is to define a linear compensator, $\underline{\underline{D}}$, such that the output $\underline{v}(t)$ is given by:

$$\underline{v} = \underline{\underline{D}}\,\underline{\varepsilon} \quad \text{...(29)}$$

and assume that the parameter adjustment laws are of the following type:

$$\dot{\underline{\underline{A}}}(t) = \underline{\Phi}(\underline{v}(\tau),t)$$

$$\dot{\underline{\underline{B}}}(t) = \underline{\chi}(\underline{v}(\tau),t)$$

$$\dot{\underline{\underline{D}}}(t) = \underline{\Delta}(\underline{v}(\tau),t) \quad \text{...(30)}$$

$$\tau \leq t$$

where $\underline{\underline{\Phi}}$, $\underline{\underline{X}}$ and $\underline{\underline{\Delta}}$ denote nonlinear matrix functions of the compensator output, $\underline{v}(t)$.

Finally, only those systems which satisfy the following integral inequality are considered [48]:

$$\int_0^t \underline{v}^T(\tau)\, \underline{w}(\tau)\, d\tau \geq -\lambda_0^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

where λ_o is a finite constant which depends on the initial state of the system and eventually on $\sup_{o<\tau<t} \|\underline{x}(\tau)\|$ but not on t. $\underline{v}(t)$ is the output of the linear compensator block and $\underline{w}(t)$ is the output of a nonlinear block to be defined.

The Popovian nonlinearity condition [44 - 46] constitutes a subset of this approach. Suppose that the output of the nonlinearity $\underline{w}(t)$ satisfies the following:

$$0 \leq \underline{w}(t) \leq k\, \underline{v}(t) < \infty$$

where k is a finite constant such that:

$$0 \leq k < \infty$$

The inequality (31) then becomes:

$$k \int_0^t \underline{v}^T(\tau)\, \underline{v}(\tau)\, d\tau \geq 0 \geq -\lambda\, \hat{\delta} \qquad \forall t$$

At this stage a theorem may be stated which determines the global stability of the entire system.

Theorem 3.1

Sufficient conditions such that those model reference systems described by equations (25) - (30) and inequality (31) be asymptotic hyperstable are:

1) the transfer matrix:

$$\underline{\underline{G}}(s) = \underline{\underline{D}}\,(s\underline{\underline{I}} - \underline{\underline{A}}_m)^{-1}$$

be strictly positive real;

2) the vectors $(\underline{\underline{A}}_m - \underline{\underline{A}}(t))\underline{x}$, $(\underline{\underline{B}}_m - \underline{\underline{B}}(t))\underline{u}$, $(\underline{\underline{D}}_m - \underline{\underline{D}}(t))\underline{\xi}$ and $\underline{v}(t)$ be of the same dimension and,

3) the computing block of the matrices $\dot{\underline{\underline{A}}}(t)$, $\dot{\underline{\underline{B}}}(t)$ and $\dot{\underline{\underline{D}}}(t)$ must introduce functions with the following form:

$$\underline{\underline{\Phi}}\,(t) = [\,\phi_{ij}\,(t)] = [\alpha_{ij}\;\; v_i\;\; x_{pj}\,]$$

$$i = 1 \rightarrow n\ , \ j = 1 \rightarrow n$$

$$\underline{\chi}(t) = [\eta_{ij}(t)] = [\beta_{ij} v_i u_j]$$

$$i = 1 \rightarrow n, \quad j = 1 \rightarrow m$$

and:

$$\underline{\Theta}(t) = [\theta_{ij}(t)] = [\delta_{ij} v_i \xi_j]$$

$$i = 1 \rightarrow n, \quad j = 1 \rightarrow p$$

α_{ij}, β_{ij} and δ_{ij} are finite positive constants.

<u>Proof</u>

Consider equation (25). If a vector, $\underline{w}$, is defined such that:

$$\underline{w}(t) = (\underline{A}_m - \underline{A}(t))\underline{x}_p + (\underline{B}_m - \underline{B}(t))\underline{u}$$

$$+ (\underline{D}_m - \underline{D}(t))\underline{\xi} \quad \text{.......................................}(32)$$

then from equation (28):

$$\dot{\underline{\varepsilon}} = \underline{A}_m \underline{\varepsilon} + \underline{I}\,\underline{w} \quad \text{.......................................}(33)$$

If equation (33) is considered with the integral

inequality (31) and equation (29), an equivalent system may be described such as that depicted in Figure 3.3. $\underline{\underline{G}}(s)=\underline{\underline{D}}(s\underline{\underline{I}} - \underline{\underline{A}}_m)^{-1}$ is the transfer matrix of the linear block. For asymptotic hyperstability the strict positive realness of $\underline{\underline{G}}(s)$ is required [44 - 47]. Condition 1 of Theorem 3.1 is thus necessary.

The second condition of the Theorem follows from the definitions of $\underline{w}$ and the inequality which the nonlinearity must satisfy.

Finally, the third condition can be shown to be sufficient in order that the inequality is satisfied.

From the scalar representation of equation (32) it may be shown that a sufficient condition for the inequality to be verified is that integrals of the form:

$$\int_0^t -(a_{mij} - a_{ij}(\tau))x_{pj}(\tau)v_i(\tau)d\tau \geq -\lambda^2_{aij}$$

$$i = 1 \rightarrow n\,,\quad j = 1 \rightarrow n$$

$$\int_0^t -(b_{mij} - b_{ij}(\tau))u_j(\tau)v_i(\tau)d\tau \geq -\lambda^2_{bij}$$

$$i = 1 \rightarrow n\,,\quad j = 1 \rightarrow m$$

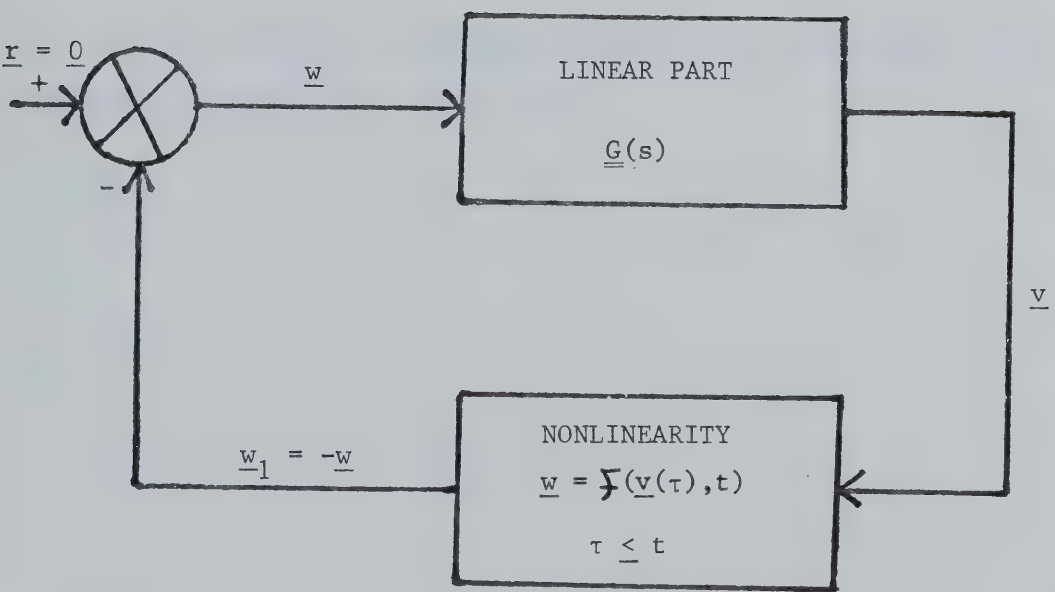


FIGURE 3.3 EQUIVALENT NONLINEAR MULTIVARIABLE CONTROL SYSTEM

$$\int_0^t -(d_{mij} - d_{ij}(\tau))\xi_j(\tau)v_i(\tau)d\tau \geq -\lambda_{dij}^2$$

$i = 1 \to n\ ,\ j = 1 \to p$ ....(34)

are applicable.

λ_{aij}, λ_{bij} and λ_{dij} are finite constants which depend on the initial state of the system but not on time, t.

If the model is assumed to be time-invariant then the parameter adaptation laws give:

$$-(\dot{a}_{mij} - \dot{a}_{ij}(\tau)) = \dot{a}_{ij}(\tau) = \alpha_{ij}\ v_i(\tau)\ x_{pj}(\tau)$$

$i = 1 \to n\ ,\ j = 1 \to n$

$$-(\dot{b}_{mij} - \dot{b}_{ij}(\tau)) = \dot{b}_{ij}(\tau) = \beta_{ij}\ v_i(\tau)\ u_j(\tau)$$

$i = 1 \to n\ ,\ j = 1 \to m$

$$-(\dot{d}_{mij} - \dot{d}_{ij}(\tau)) = \dot{d}_{ij}(\tau) = \delta_{ij}\ v_i(\tau)\ \xi_j(\tau)$$

$i = 1 \to n,\ j = 1 \to p$ ....(35)

The inequalities (34) can then be written as:

$$\int_0^t x_{pj}(\tau)\, v_i(\tau)\, [\int_0^\tau \alpha_{ij}\, v_i(\tau')\, x_{pj}(\tau')\, d\tau' + f_{ij}(0)]d\tau$$

$$= \alpha_{ij}/2 \{ [\int_0^t v_i(\tau')\, x_{pj}(\tau')\, d\tau' + f_{ij}(0)]^2 - f_{ij}^2(0)\}$$

$$\sum -\alpha_{ij}/2\ f_{ij}^2(0) = -\lambda^2_{aij}$$

$$i = 1 \to n\ ,\ j = 1 \to n$$

$$\int_0^t u_j(\tau)\, v_i(\tau)\, [\ \int_0^\tau \beta_{ij}\, v_i(\tau')\, u_j(\tau')d\tau' + g_{ij}(0)]d\tau$$

$$= \beta_{ij}/2 \{ [\int_0^t v_i(\tau')\, u_j(\tau')d\tau' + g_{ij}(0)]^2 - g_{ij}^2(0)\}$$

$$\sum -\beta_{ij}/2\ g_{ij}^2(0) = -\lambda^2_{bij}$$

$$i = 1 \to n\ ,\ j = 1 \to m$$

$$\int_0^t \xi_j(\tau)\, v_i(\tau)\, [\ \int_0^\tau \delta_{ij}\, v_i(\tau')\, \xi_j(\tau')d\tau' + h_{ij}(0)]d\tau$$

$$= \delta_{ij}/2 \{[\int_0^t v_i(\tau')\, \xi_j(\tau')d\tau' + h_{ij}(0)]^2 - h_{ij}^2(0)\}$$

$$\sum -\delta_{ij}/2\ h_{ij}^2(0) = -\lambda^2_{dij}$$

$$i = 1 \to n\ ,\ j = 1 \to p \quad\quad (36)$$

where $f_{ij}(0)$, $g_{ij}(0)$ and $h_{ij}(0)$ represent the initial conditions of the corresponding integrals.

Chapter Three

Theorem 3.1 is thus proved.

Theorem 3.1 determines the way in which the parameters of the open-loop plant may be adjusted in order that the adaptive system be globally hyperstable. However, it is only in very rare instances that the plant may be directly adjustable, in this manner. The design of a physically realizable algorithm is, hence, not satisfied completely at this point.

Of the practical schemes that have been suggested it is apparent that the approach taken by Bethoux and Courtiol [59] is closest to the "classical" configuration implemented by Oliver [32]. There are two differences in that Bethoux and Courtiol have made the assumption that the model and the plant parameter matrices verify Erzberger's [78] perfect model following conditions and, also, a proportional adaptive mode is introduced --- this is claimed to aid the speed of response of the system.

In the approach to be outlined here, it is desired to point out the parallels existing between the two stability methods of MRAC design and, thus, the additional assumptions of Bethoux and Courtiol's work are not imposed.

The configuration of the adaptive system is as depicted

in Figure 3.4 and described by the following equations:

$$\dot{\underline{x}}_p = \underline{A}\,\underline{x}_p(t) + \underline{B}\,\underline{u}(t) + \underline{D}\,\underline{\xi}(t)$$

and:

$$\underline{y} = \underline{C}\,\underline{x}_p \quad \ldots\ldots(37)$$

where the symbols are as for equation (3).

Assuming multivariable feedback - feedforward control, one can write:

$$\underline{u}(t) = \underline{K}_{FB}(t)\,\underline{x}_p + \underline{K}_{FF}(t)\,\underline{\xi} \quad \ldots\ldots(38)$$

or upon substitution in equation (37):

$$\dot{\underline{x}}_p(t) = (\underline{A} + \underline{B}\,\underline{K}_{FB}(t))\,\underline{x}_p + (\underline{D} + \underline{B}\,\underline{K}_{FF}(t))\,\underline{\xi} \quad \ldots\ldots(39)$$

which can be written in closed-loop form as:

$$\dot{\underline{x}}_p(t) = \underline{A}_p(t)\,\underline{x}_p + \underline{D}_p(t)\,\underline{\xi} \quad \ldots\ldots(40)$$

The desired closed-loop reference model is chosen as:

$$\dot{\underline{x}}_m(t) = \underline{A}_m\,\underline{x}_m + \underline{D}_m\,\underline{\xi}$$

$$\underline{y}_m = \underline{C}\,\underline{x}_m \quad \ldots\ldots(41)$$

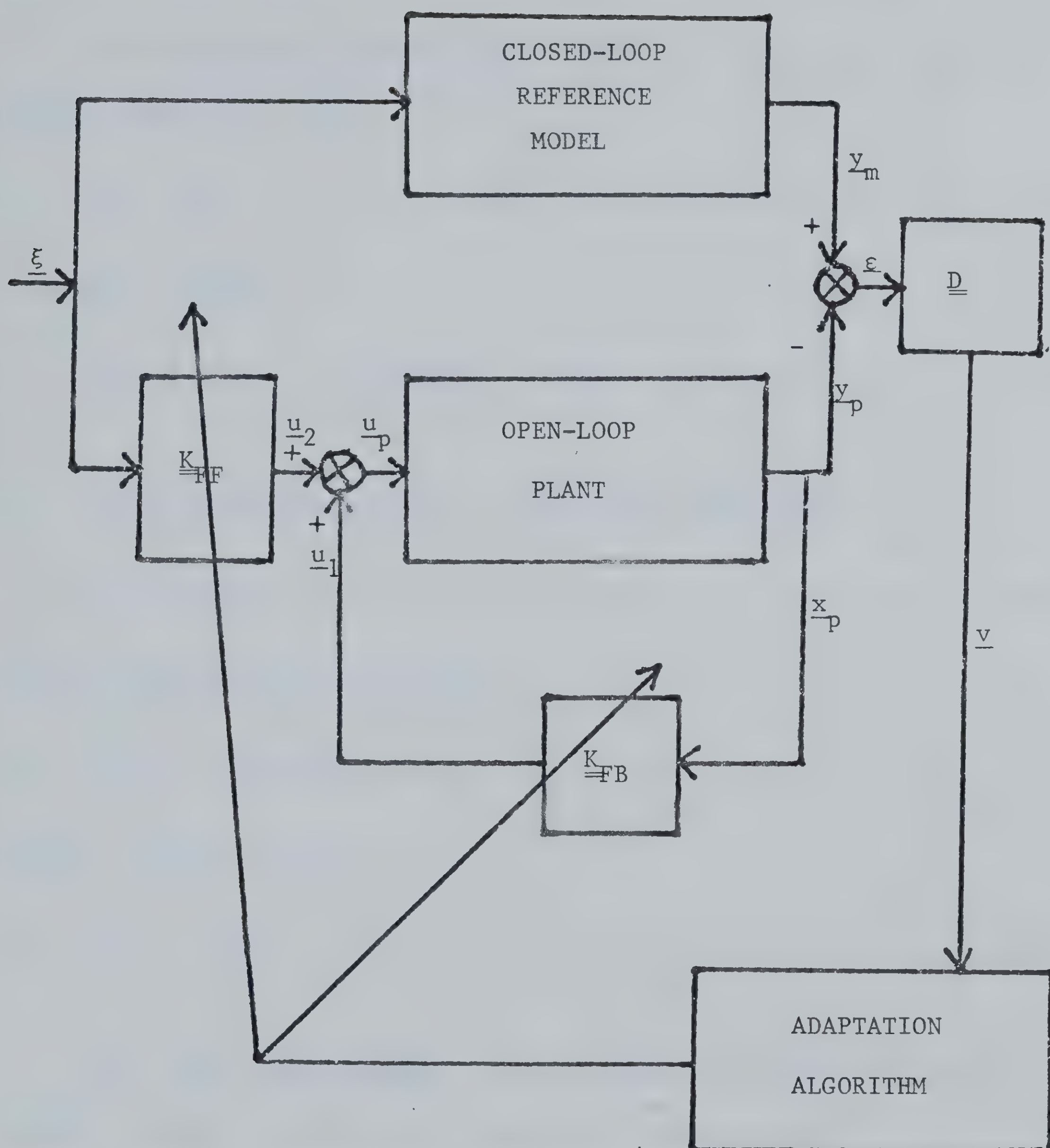


FIGURE 3.4 "CLASSICAL" MULTIVARIABLE FEEDBACK - FEEDFORWARD SCHEME FOR MODEL REFERENCE ADAPTIVE CONTROL

Here, as in the Liapunov stability example, the model and the plant are assumed to be of the same order.

Subtracting equation (40) from (41) and defining an error vector, $\underline{\varepsilon}$, as:

$$\underline{\varepsilon} = \underline{x}_m - \underline{x}_p \quad \ldots\ldots (42)$$

one can obtain:

$$\dot{\underline{\varepsilon}} = \underline{\underline{A}}_m \underline{\varepsilon} + (\underline{\underline{A}}_m - \underline{\underline{A}}_p(t))\,\underline{x}_p + (\underline{\underline{D}}_m - \underline{\underline{D}}_p(t))\,\underline{\xi} \quad \ldots\ldots (43)$$

If a compensator, $\underline{\underline{D}}$, is defined such that:

$$\underline{v} = \underline{\underline{D}}\,\underline{\varepsilon} \quad \ldots\ldots (44)$$

and a vector, $\underline{w}$, such that:

$$\underline{w} = (\underline{\underline{A}}_m - \underline{\underline{A}}_p(t))\,\underline{x}_p + (\underline{\underline{D}}_m - \underline{\underline{D}}_p(t))\,\underline{\xi} \quad \ldots\ldots (45)$$

then, one may write:

$$\dot{\underline{\varepsilon}} = \underline{\underline{A}}_m \underline{\varepsilon} + \underline{\underline{I}}\,\underline{w} \quad \ldots\ldots (46)$$

The next development is to assume that the nonlinear block, whose output is $\underline{w}$, obeys the inequality:

$$\int_0^t \underline{v}^T(\tau)\,\underline{w}(\tau)\,d\tau \geq -\lambda_0^2 \quad \ldots\ldots (47)$$

where λ_o is a finite constant depending only on the initial state of the system.

Further, require that:

$$\dot{\underline{A}}_p(t) = \underline{\Phi}\,(\underline{v}(\tau),t) \qquad \tau \leq t$$

and:

$$\dot{\underline{D}}_p(t) = \underline{\Delta}\,(\underline{v}(\tau),t) \qquad \tau \leq t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (48)$$

Equations (44) - (46), (48) and inequality (47) define an equivalent, autonomous, nonlinear feedback system such as that depicted in Figure 3.3. It is, therefore, by analogy with the results presented as Theorem 3.1, possible to state necessary and sufficient conditions such that the derived equivalent system (and therefore the original system) will be asymptotic hyperstable.

<u>Theorem 3.2</u>

Necessary and sufficient conditions such that the model reference adaptive system described by equations (44) - (46), (48) and inequality (47) be asymptotic hyperstable are that:

1) the transfer matrix

$$\underline{G}(s) = \underline{D}(s\underline{I} - \underline{A}_m)^{-1}$$

be strictly positive real;

2) the vectors $(\underline{\underline{A}}_m - \underline{\underline{A}}_p(t))\underline{x}_p$, $(\underline{\underline{D}}_m - \underline{\underline{D}}_p(t))\underline{\xi}$ and $\underline{v}$ be of the same dimension and,

3) the adaptation laws for the matrices $\dot{\underline{\underline{A}}}_p(t)$ and $\dot{\underline{\underline{D}}}_p(t)$ must introduce functions of the following form:

$$\underline{\Phi}(t) = [\phi_{ij}(t)] = [\alpha_{ij} v_i x_{pj}]$$

$$i = 1 \rightarrow n, \quad j = 1 \rightarrow n$$

and:

$$\underline{\Theta}(t) = [\theta_{ij}(t)] = [\delta_{ij} v_i \xi_j]$$

$$i = 1 \rightarrow n, \quad j = 1 \rightarrow p$$

where α_{ij} and δ_{ij} are finite positive constants.

The proof is analogous to that given for Theorem 3.1.

Theorem 3.2 defines how the elements of the matrices $\dot{\underline{\underline{A}}}_p(t)$ and $\dot{\underline{\underline{D}}}_p(t)$ should adapt. However the elements which can be physically manipulated are $\underline{\underline{K}}_{FB}$ and $\underline{\underline{K}}_{FF}$.

For the purposes of the following, define:

$$\underline{\underline{\Gamma}} = \underline{\underline{B}}\,\underline{\underline{K}}_{FB}$$

and:

$$\underline{\underline{\Omega}} = \underline{\underline{B}}\,\underline{\underline{K}}_{FF} \qquad (49)$$

Supposing that the plant is time-invariant it is possible to obtain corresponding equations for the adaptation of $\underline{\underline{K}}_{FB}$ and $\underline{\underline{K}}_{FF}$.

$$\dot{\underline{\underline{A}}}_p(t) = (\dot{\underline{\underline{A}}} + \dot{\underline{\underline{\Gamma}}}) = \dot{\underline{\underline{\Gamma}}} = \underline{\underline{B}}\,\dot{\underline{\underline{K}}}_{FB} = [\alpha_{ij}\ v_i\ x_{pj}]$$

and:

$$\dot{\underline{\underline{D}}}_p(t) = (\dot{\underline{\underline{D}}} + \dot{\underline{\underline{\Omega}}}) = \dot{\underline{\underline{\Omega}}} = \underline{\underline{B}}\,\dot{\underline{\underline{K}}}_{FF} = [\ \delta_{ij}\ v_i\ \ \xi_j\ ] \qquad (50)$$

The equivalence between the hyperstability approach and the Liapunov development is now readily established. To obtain asymptotic hyperstability it is required to design a compensator matrix, $\underline{\underline{D}}$, such that the transfer matrix of the linear part of the equivalent system is strictly positive real. Using the Popov - Yakubovich - Kalman lemma [1,79] one obtains that $\underline{\underline{D}}=\underline{\underline{P}}$, where $\underline{\underline{P}}$ is the matrix solution to the Liapunov equation (12). Further,

$$v_i(t) = \underline{D}_i\ \underline{\varepsilon} = \underline{\varepsilon}^T\ \underline{D}_i^T \qquad (51)$$

where $\underline{D}_i$ is the i^{th} row of the compensator matrix $\underline{\underline{D}}$ (or $\underline{\underline{P}}$).

Thus if $\underline{\underline{C}}=\underline{\underline{I}}$ (ie. all the states are available for measurement) then the adaptation laws (50) reduce to those obtained for the Liapunov approach (equations (21) and (22)).

The equations may be further extended, as Oliver has done, to include adaptive integral and setpoint control [32].

The development that appears in the preceding pages has shown the equivalence between the two stability approaches, for the design of MRAC systems. In general it must be concluded that hyperstability theory offers more encompassing results.

Extensions to include unmeasurable disturbances and certain classes of nonlinear plant are also possible, since the difficulty of specifying a specific Liapunov function is avoided. The development of these are left until Chapter Five in order that they may be included in a still more general approach, utilizing input-output (ie. transfer function type) formulations.

3.5 Conclusions

The presentation of this chapter has centered on three approaches to the design of MRAC systems. This general outline represents a chronological development in the theory over the last twenty years or more. The chapter has dealt with state-space formulations primarily because, until recently, this has been the predominant trend.

As will be shown in the following chapters, not only is this approach unnecessary, it appears to have been limiting. It should be noted here, also, that although only continuous systems were discussed in detail all these results are readily extended to discrete systems using the analogous discrete version of the hyperstability theorem [50].

From the point of view of the methods investigated so far, several observations may be offered. Firstly, the two schemes which guarantee stability are obviously more attractive from a practical and theoretical standpoint than optimization/gradient methods. Secondly, the hyperstability approach can lead to more general results than the design using Liapunov's direct method. In particular, for the continuous example discussed, the hyperstable scheme becomes identical to that outlined by Oliver, when all the states

are measureable.

Chapter Three

CHAPTER FOUR

An Input-Output Formulation of the Model Reference Adaptive Control Problem

4.1 Introduction

Up until 1976, nearly all of the major contributions in the area of research in model reference techniques, had been firmly planted in the state-space. This was in spite of the fact that formulations, using transfer function type notation, had already been suggested [1 - 3]. This is by no means the case today.

Despite the successes of the earlier schemes there are disadvantages inherent in the state-space development, namely, those of state-inaccessibility, time-delays, and general plant implementation problems, such as the definition of a desirable trajectory. Various adaptive techniques [7 - 16], or add-on approaches, such as coupled state-estimation [4], or state-variable filters [5 - 6, 17 - 18], and the like, have solved the problems in some cases. But for a truly comprehensive theory to be utilized in the manner that Popov has foreshadowed, it is necessary to take a step "backwards".

The scheme presented by Martin-Sanchez [19 - 21] has

done precisely this -- reformulating the problem so that the starting point is an input-output description of the plant. The hyperstability theorem may then be used in a somewhat similar manner to that described in Chapter Three.

This chapter will dwell on the theoretical aspects of Martin-Sanchez's approach. Moreover, even though only a discrete synthesis technique will be detailed, it should be noted, at the outset, that Martin-Sanchez's method does represent a true conceptual generalization of the state-space formulation for the hyperstability theory.

4.2 Theory

The general structure of the adaptive control scheme, utilized by Martin-Sanchez, can be depicted as in Figure 4.1. In the diagram $\underline{y}_d(k+1)$, represents the "desired output" vector. As with the familiar model reference concept, the driver block determines the dynamic behaviour of the desired output and thence, the plant. This block, in fact, can be designed in such a way as to dynamically compensate for such practical phenomena as noisy measurements, and technically logical decisions may also be incorporated.

The starting point for the development is what is

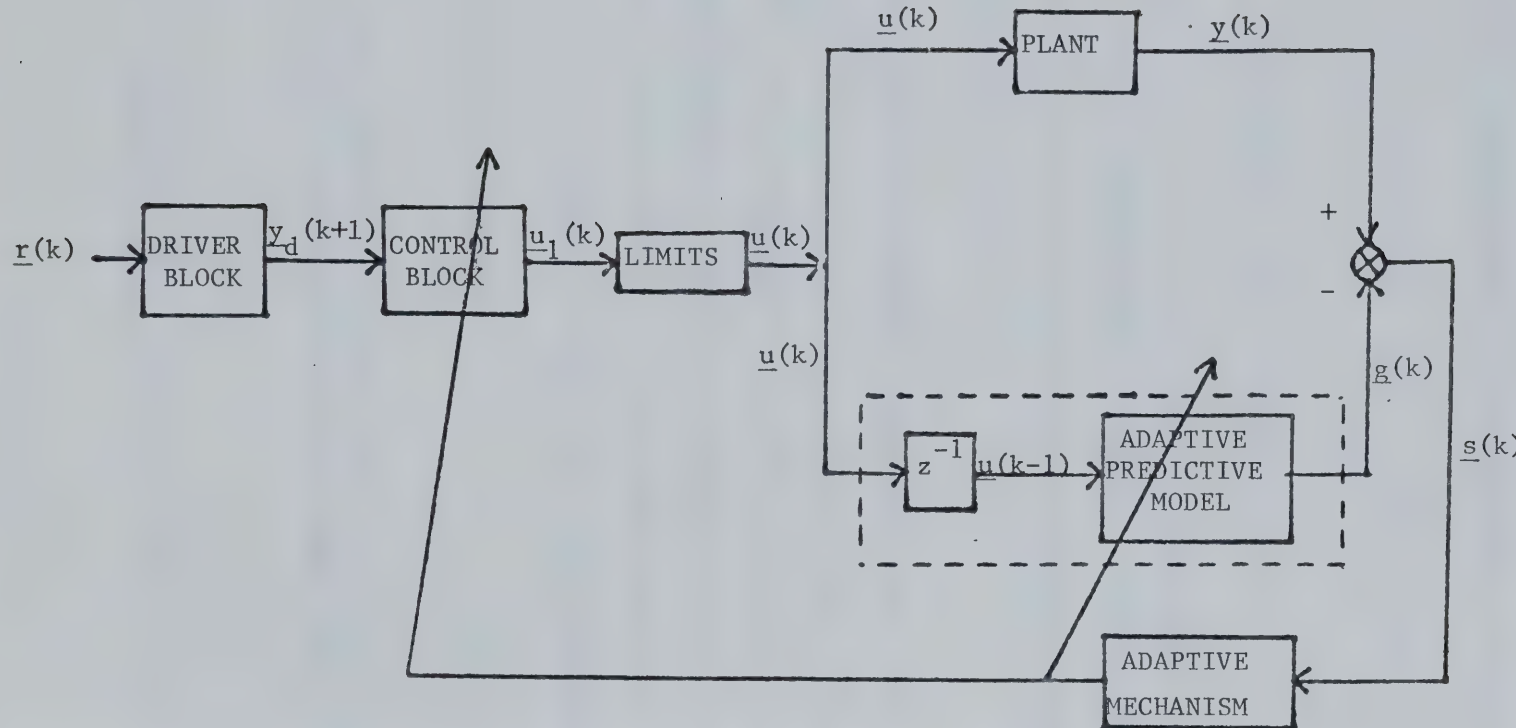


FIGURE 4.1 MARTIN-SANCHEZ'S ADAPTIVE CONTROL SCHEME

equivalent to a transfer function description of the input-output behaviour of the open-loop plant:

$$\underline{y}(k) = \sum_{i=1}^{h} \underline{\underline{A}}_i(k)\, \underline{y}(k-i) + \sum_{j=1}^{f} \underline{\underline{B}}_j(k)\, \underline{u}(k-j)$$

$$+ \sum_{l=1}^{g} \underline{\underline{D}}_l(k)\, \underline{\xi}(k-l) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where $\underline{y}(k-i)$, $\underline{u}(k-j)$, and $\underline{\xi}(k-l)$ represent nx1 output, nx1 input, and px1 disturbance vectors, respectively.[1] $\underline{\underline{A}}_i(k)$, $\underline{\underline{B}}_j(k)$ and $\underline{\underline{D}}_l(k)$ are time-varying process parameter matrices of appropriate order.[2]

The starting equation employed here, does not include the time-delays explicitly as does [21]. Instead, equation (1) is assumed to include elements corresponding to the time-delays as separate entities, i.e. a change of variable can be assumed to have been made s.t. $i = i' + r$ and $j = j' + t$, where r and t are the time-delays. This

---

[1] If the number of inputs does not equal the number of outputs, then supplementary conditions need to be invoked [21].

[2] It is assumed that these matrices admit only a finite number of bounded changes as $k \to \infty$.

approach has been taken to emphasize the generality of the solution. However, if the time-delays are approximately known, they may be included in the formulation so that an equation such as [I.1] [21] is obtained. This has the effect of reducing the order of the identification problem, and in practice, would be considered desirable.

An identification model can now be defined as:

$$\underline{d}(k) = \sum_{i=1}^{h_1} \tilde{\underline{A}}_i(k-1)\,\underline{y}(k-i) + \sum_{j=1}^{f_1} \tilde{\underline{B}}_j(k-1)\,\underline{u}(k-j)$$

$$+ \sum_{l=1}^{g_1} \tilde{\underline{D}}_l(k-1)\,\underline{\xi}(k-l) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

and, an identification error as:

$$\underline{e}(k) = \underline{y}(k) - \underline{d}(k) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

The identification model is assumed to be of the same order as the plant equation (1), ie. $h_1=h$, $f_1=f$ and $g_1=g$. This assumption is formally required mathematically, but there are many practical applications where it is difficult to satisfy this assumption, eg. distributed systems where

the theoretical process order is infinite[1].

If we now introduce the vector, $\underline{g}(k)$, given by:

$$\underline{g}(k) = \sum_{i=1}^{h} \tilde{\underline{A}}_i(k)\,\underline{y}(k-i) + \sum_{j=1}^{f} \tilde{\underline{B}}_j(k)\,\underline{u}(k-j)$$

$$+ \sum_{l=1}^{g} \tilde{\underline{D}}_l(k)\,\underline{\xi}(k-l) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

a linear transformation may be defined such that:

$$\underline{s}(k) = \underline{y}(k) - \underline{g}(k) =$$

$$\sum_{i=1}^{h} (\underline{A}_i(k) - \tilde{\underline{A}}_i(k))\,\underline{y}(k-i) + \sum_{j=1}^{f} (\underline{B}_j(k) - \tilde{\underline{B}}_j(k))\,\underline{u}(k-j) +$$

$$\sum_{l=1}^{g} (\underline{D}_l(k) - \tilde{\underline{D}}_l(k))\,\underline{\xi}(k-l) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

This transformation defines an equivalent mapping which reconfigures the system, defined by equations (1), (4) and (5) as an autonomous, nonlinear feedback system (Figure 4.2).

1

The overall technique has been shown tc handle the case in which structural differences are present, under simulation conditions [19]. In any case, it can be argued that the plant order is infinite so that during application the problem of structural differences always arises.

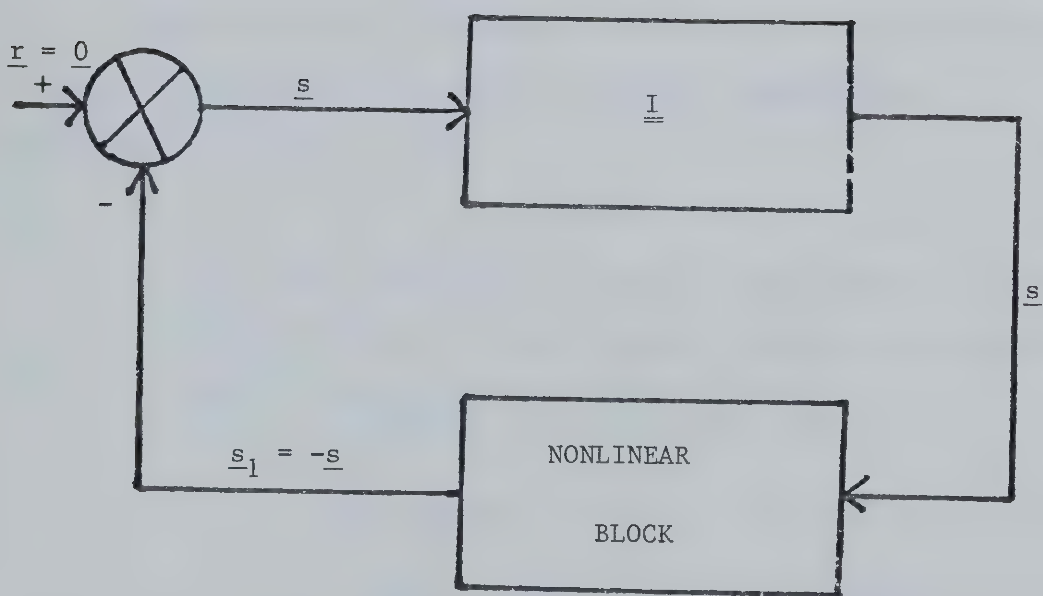


FIGURE 4.2 EQUIVALENT AUTONOMOUS, NONLINEAR FEEDBACK SYSTEM

The discrete hyperstability theorem [22] gives necessary and sufficient conditions such that a system, configured as depicted in Figure 4.2, will be asymptotic hyperstable (which necessarily implies asymptotic stability in the large).

<u>Theorem 4.1</u>

A necessary and sufficient condition in order that the system, depicted as Figure 4.2, be an asymptotic, hyperstable system is that:

(i) the nonlinear element's input-output behaviour should belong to the family whose characteristics may be described by the inequality:[1]

$$\eta(k_0,k_1) = \sum_{k=k_0}^{k_1} \underline{s}^T(k)\, \underline{s}_1(k) \geq -\lambda_0^2 \qquad \forall k_1 \geq k_0 \quad ....(6)$$

λ_0 is a finite constant perhaps dependent on the initial system state but not on time.

---

1 This condition defines a "weakly" hyperstable system [23].

Proof

The proof of this theorem follows directly from that presented by Landau [22].

The system considered for this general theorem involves a nonlinear, autonomous feedback system, as depicted in Figure 4.3. The nonlinear element is assumed to satisfy the condition:

$$\eta(k_0,k_1) = \sum_{k=k_0}^{k_1} \underline{v}^T(k)\, \underline{w}_1(k) \geq -\lambda_0^2 \qquad \forall k_1 \geq k_0$$

where λ_o depends only on the initial state of the system.

A necessary and sufficient condition for the system, depicted in Figure 4.3, to be asymptotic hyperstable is that $\underline{\underline{G}}(z)$ be strictly positive real which implies that:

1) $\underline{\underline{G}}(z)$ is positive real $\forall |z|>1$;

2) the poles of $\underline{\underline{G}}(z)$ lie in the circular domain $|z|<1$ and;

3) $\underline{\underline{G}}(z) + \underline{\underline{G}}^T(z^*)$ should be positive definite Hermitian $\forall |z|=1$.

It can be shown that $\underline{\underline{G}}(z)=\underline{\underline{I}}$ satisfies all of these conditions. Moreover, setting $\underline{s}_1=\underline{w}_1$ and $\underline{s}=\underline{v}$, it is obvious that the inequality condition, (6), is sufficient. At the

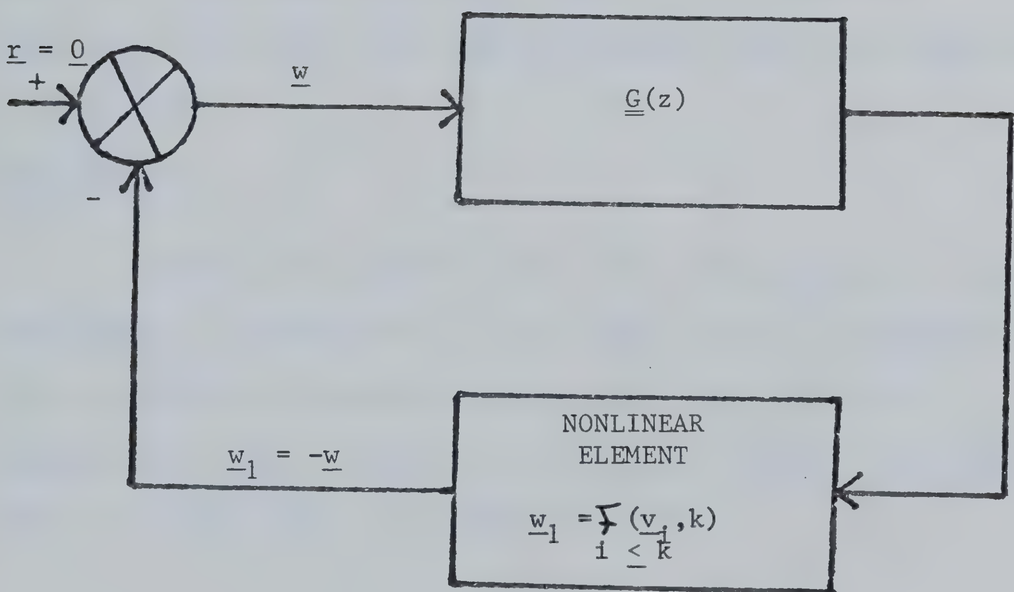


FIGURE 4.3 GENERAL AUTONOMOUS NONLINEAR FEEDBACK SYSTEM

same time it is necessary, since any nonlinear system satisfying inequality (6), is "weakly hyperstable", by definition [22]. Combined in a feedback manner with the hyperstable identity block, one obtains a hyperstable configuration. For proof of this last assertion, see [23].

Theorem 4.2 leads, finally, to sufficient conditions such that the identification system described by equations (1), (4), (5) and inequality (6), is asymptotic hyperstable.

<u>Theorem 4.2</u>

A sufficient condition for the identification system described by equations (1), (4) and (5) and inequality (6) to be asymptotic hyperstable is that the nonlinear adaptation laws for the matrices $\tilde{\underline{\underline{A}}}_i(k)$, $\tilde{\underline{\underline{B}}}_j(k)$ and $\tilde{\underline{\underline{D}}}_1(k)$ admit functions of the following form:

$$\tilde{\underline{A}}_i(k) = \underline{\Phi}_i(\underline{s}_i, k) \qquad i \leq k$$

$$i = 1 \rightarrow h$$

$$\tilde{\underline{B}}_j(k) = \underline{X}_j(\underline{s}_j, k) \qquad j \leq k$$

$$j = 1 \rightarrow f$$

$$\tilde{\underline{D}}_l(k) = \underline{\Theta}_l(\underline{s}_l, k) \qquad l \leq k$$

$$l = 1 \rightarrow g \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

where:

$$\underline{\Phi}_i(\underline{s}_i, k) = [\phi_{itq}] = [\alpha_{itq} \, s_t(k) \, y_q(k - i) + \tilde{a}_{itq}(k - 1)]$$

$$i = 1 \rightarrow h, \; t = 1 \rightarrow n, \; q = 1 \rightarrow n$$

$$\underline{X}_j(\underline{s}_j, k) = [\eta_{jtq}] = [\beta_{jtq} \, s_t(k) \, u_q(k - j) + \tilde{b}_{jtq}(k - 1)]$$

$$j = 1 \rightarrow f, \; t = 1 \rightarrow n, \; q = 1 \rightarrow n$$

$$\underline{\Theta}_l(\underline{s}_l, k) = [\theta_{ltq}] = [\delta_{ltq} \, s_t(k) \, \xi_q(k - l) + \tilde{d}_{ltq}(k - 1)]$$

$$l = 1 \rightarrow g, \; t = 1 \rightarrow n, \; q = 1 \rightarrow p \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

α_{itq}, β_{jtq} and δ_{ltq} are strictly positive constant coefficients.

Proof

The scalar representation of equation (3) is used in conjunction with Theorem 4.1.

For inequality (6) to be satisfied, it is sufficient that:

$$- \sum_{k=k_0}^{k_1} s_t^2(k) \geq - \lambda_t^2 \qquad \forall k_1 \geq k_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(9)$$

where λ_t, $(t=1 \rightarrow n)$ are finite constants only dependent on the initial state of the system.

Using the scalar representation of equation (3), inequality (9) becomes:

$$\sum_{k=k_0}^{k_1} \Big\{ \sum_{i=1}^{h} \sum_{q=1}^{n} [\tilde{a}_{itq}(k) - a_{itq}(k)]\, s_t(k)\, y_q(k-i)$$
$$+ \sum_{j=1}^{f} \sum_{q=1}^{n} [\tilde{b}_{jtq}(k) - b_{jtq}(k)]\, s_t(k)\, u_q(k-j)$$
$$+ \sum_{l=1}^{g} \sum_{q=1}^{p} [\tilde{d}_{ltq}(k) - d_{ltq}(k)]\, s_t(k)\, \xi_q(k-l) \Big\} \geq - \lambda_t^2$$

$t = 1 \rightarrow n$ ....(10)

In turn, sufficient conditions such that inequality (10) is satisfied are that:

$$\sum_{k=k_0}^{k_1} [\tilde{a}_{itq}(k) - a_{itq}(k)]\, s_t(k)\, y_q(k-i) \geq - \lambda_{aitq}^2$$

$i = 1 \rightarrow h$, $t = 1 \rightarrow n$, $q = 1 \rightarrow n$

$$\sum_{k=k_0}^{k_1} [\tilde{b}_{jtq}(k) - b_{jtq}(k)]\, s_t(k)\, u_q(k-j) \geq - \lambda_{bjtq}^2$$

$j = 1 \rightarrow f$, $t = 1 \rightarrow n$, $q = 1 \rightarrow n$

$$\sum_{k=k_0}^{k_1} [\tilde{d}_{ltq}(k) - d_{ltq}(k)]\, s_t(k)\, \xi_q(k-l) \geq -\lambda^2_{dltq}$$

$l = 1 \to g$, $t = 1 \to n$, $q = 1 \to p$ ....(11)

where λ_{aitq}, λ_{bjtq} and λ_{dltq} are finite constants not dependent on time[1].

The adaptation laws of the theorem are now invoked so that the inequalities (11) become:

$$\sum_{k=k_0}^{k_1} [\sum_{h=k_0}^{k} \alpha_{itq}\, s_t(h)\, y_q(h-i) - a_{itq}(k)]\, s_t(k)\, y_q(k-i)$$

$\geq -\lambda^2_{aitq}$ ....(a)

$i = 1 \to h$, $t = 1 \to n$, $q = 1 \to n$

$$\sum_{k=k_0}^{k_1} [\sum_{h=k_0}^{k} \beta_{jtq}\, s_t(h)\, u_q(h-j) - b_{jtq}(k)]\, s_t(k)\, u_q(k-j)$$

$\geq -\lambda^2_{bjtq}$ ....(b)

$j = 1 \to f$, $t = 1 \to n$, $q = 1 \to n$

---

1 Note that a_{itq}, b_{jtq} and d_{ltq} are assumed to admit only a finite number of bounded changes as $k \to \infty$.

Chapter Four

$$\sum_{k=k_0}^{k_1} \left[\sum_{h=k_0}^{k} \delta_{ltq}\, s_t(h)\, \xi_q(h - l) - d_{ltq}(k) \right] s_t(k)\, \xi_q(k - l)$$

$$\geq - \lambda^2_{dltq} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (c)$$

$$l = 1 \rightarrow g\ ,\ t = 1 \rightarrow n\ ,\ q = 1 \rightarrow p \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

$$\forall\, k_1 \geq k_0$$

α_{itq}, β_{jtq} and δ_{ltq} are strictly positive constants.

The inequalities (12) have the following form:

$$\sum_{k=k_0}^{k_1} x(k) \left\{ \sum_{l=k_0}^{k} \beta x(l) + c \right\} = 1/2\beta \left(\sum_{k=k_0}^{k_1} x(k) + c/\beta \right)^2 +$$

$$1/2\beta \sum_{k=k_0}^{k_1} x^2(k) - c^2/2\beta \geq - c^2/2\beta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

where c is a constant and β is a strictly positive value.

For example, consider inequality (12a) above with:

$$x(k) \equiv s_t(k) y_q(k - i)$$

$$\beta \equiv \alpha_{itq}$$

$$c \equiv -a_{itq}$$

$$l \equiv h$$

Theorem 4.2 is thus proved.

The adaptation rules, given by the Theorem, are not directly useful in their present form since it is the identification error which is directly measurable and not $\underline{s}(k)$. It is possible, however, to relate $\underline{s}(k)$ to $\underline{e}(k)$ by means of a variable, diagonal gain matrix, $\underline{\underline{K}}(k)$.

Theorem 4.3

The vector, $\underline{s}(k)$, is related to the identification error vector, $\underline{e}(k)$, by a variable, diagonal gain matrix, $\underline{\underline{K}}(k)$, defined by:

$$\underline{\underline{K}}(k) = [k_{tt}(k)]_{diag} = [1/1 + \sum_{i=1}^{h}\sum_{q=1}^{n} \alpha_{itq}\, y_q^2(k - i) + \sum_{j=1}^{f}\sum_{q=1}^{n} \beta_{jtq}\, u_q^2(k - j) + \sum_{l=1}^{g}\sum_{q=1}^{p} \delta_{ltq}\, \xi_q^2(k - l)]$$

$$t = 1 \rightarrow n \quad \ldots\ldots(14)$$

Proof

The theorem defines a relationship:

$$\underline{s}(k) = \underline{\underline{K}}(k)_{diag}\, \underline{e}(k) \quad \ldots\ldots(15)$$

From the scalar representation of equation (5) and the adaptation laws introduced in Theorem 4.2, one obtains:

$$s_t(k) = \sum_{i=1}^{h} \sum_{q=1}^{n} [a_{itq}(k) - \tilde{a}_{itq}(k-1) - \alpha_{itq} s_t(k)$$

$$y_q(k-i)] \, y_q(k-i) + \sum_{j=1}^{f} \sum_{q=1}^{n} [b_{jtq}(k) - \tilde{b}_{jtq}(k-1) -$$

$$\beta_{jtq} \, s_t(k) \, u_q(k-j)] \, u_q(k-j) + \sum_{l=1}^{g} \sum_{q=1}^{p} [d_{ltq}(k) -$$

$$\tilde{d}_{ltq}(k-1) - \delta_{ltq} \, s_t(k) \, \xi_q(k-l)] \, \xi_q(k-l)$$

$$t = 1 \rightarrow n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

Using the scalar forms of equations (2), (3), and (16):

$$s_t(k) = e_t(k) - s_t(k)[\sum_{i=1}^{h} \sum_{q=1}^{n} \alpha_{itq} \, y_q^2(k-i) +$$

$$\sum_{j=1}^{f} \sum_{q=1}^{n} \beta_{jtq} \, u_q^2(k-j) + \sum_{l=1}^{g} \sum_{q=1}^{p} \delta_{ltq} \, \xi_q^2(k-l)] \ldots (17)$$

Equation (17) can now be rearranged to give:

$$s_t(k) = k_{tt}(k) \, e_t(k)$$

$$t = 1 \rightarrow n$$

where:

$$k_{tt}(k) = 1/[1 + \sum_{i=1}^{h} \sum_{q=1}^{n} \alpha_{itq} \, y_q^2(k-i) +$$

$$\sum_{j=1}^{f} \sum_{q=1}^{n} \beta_{jtq} \; u_q^2\,(k-j) + \sum_{l=1}^{g} \sum_{q=1}^{p} \delta_{ltq} \; \xi_q^2\,(k-l)]$$

or in matrix form:

$$\underline{s}(k) = \underline{K}(k)_{diag} \; \underline{e}(k)$$

Theorems 4.1, 4.2 and 4.3 define the asymptotic hyperstable identification system, depicted in Figure 4.4.

The identification system just outlined, is similar to that described by Landau [24,25]. However, as Landau [25] has noted, stable parallel model reference adaptive systems (which correspond to the "output error" method in recursive identification), always are affected by bias in the presence of noise obscured measurements. Although this bias may be reduced, significantly, if the adaptation gains are low, the obvious disadvantage would be the low speed of convergence. This observation lead to another scheme with decreasing adaptation gains [25], although more on this will be said in the next chapter.

Now that an explicit identification of the open-loop plant is available, at all times, a control system may be derived. The essential step is to assume that the output of the driver block, $\underline{y}_d(k+1)$, is equal to the identification

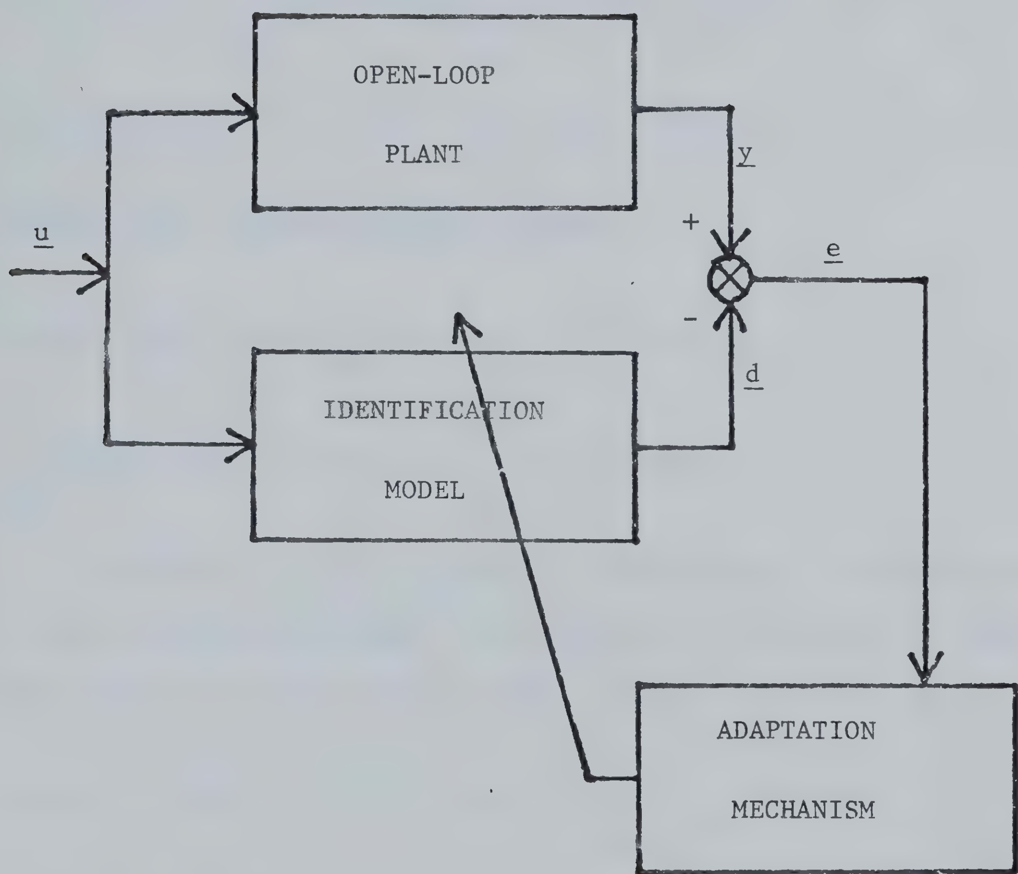


FIGURE 4.4 A RECURSIVE ASYMPTOTIC HYPERSTABLE IDENTIFICATION SCHEME

model output at the $(k+1)^{th}$ interval[1], ie.:

$$\underline{y}_d(k+1) = \underline{d}(k+1) \quad \text{.......................................(18)}$$

From equation (2), then, one obtains:

$$\underline{d}(k+1) = \sum_{i=1}^{h} \tilde{\underline{A}}_i(k)\, \underline{y}(k-i+1) + \sum_{j=1}^{f} \tilde{\underline{B}}_j(k)\, \underline{u}(k-j+1) + \sum_{l=1}^{g} \tilde{\underline{D}}_l(k)\, \underline{\xi}(k-l+1) \quad \text{.......................................(19)}$$

which, upon rearrangement, gives:

$$\underline{u}(k) = \tilde{\underline{B}}_1^{-1}(k)\{\underline{y}_d(k+1) - \sum_{i=1}^{h} \tilde{\underline{A}}_i(k)\, \underline{y}(k-i+1) - \sum_{j=2}^{f} \tilde{\underline{B}}_j(k)\, \underline{u}(k-j+1) - \sum_{l=1}^{g} \tilde{\underline{D}}_l(k)\, \underline{\xi}(k-l+1)\} \quad \text{.......(20)}$$

A control error may now be defined as the difference, at any instant, between the values of the process output and the output of the driver block. So:

$$\underline{e}_1(k) = \underline{y}(k) - \underline{y}_d(k) \quad \text{.......................................(21)}$$

1

This procedure seems to have been inspired by work on inverse control schemes by Godbole and Smith, and others [26 - 31].

It is now possible to show that the asymptotic hyperstability of the identification system also implies the asymptoticity of the entire control scheme.

Theorem 4.4

The asymptotic hyperstability of the identification scheme, defined by equations (1), (2), (3), (14), (15) and inequality, (6), implies that the control system, described by equations (1), (2), (3), (14), (15), (20), (21) and inequality, (6), is asymptotic hyperstable.

Proof

The proof of this theorem is dependent on three assumptions:

1) that the input to the driver block and the disturbances are always bounded;

2) the driver block is described by a stable dynamical system, and,

3) $\tilde{\underline{\underline{B}}}_1(k)$ is non-singular $\forall k$.

From equation (5) and the adaptation laws of Theorem 4.2, the following equation is obtained:

$$\underline{s}(k) = \sum_{i=1}^{h} (\underline{\Delta}_i(k) - \Delta\tilde{\underline{\Delta}}_i(k) - \tilde{\underline{\Delta}}_i(k-1))\, \underline{y}(k-i)$$

$$+ \sum_{j=1}^{f} (\underline{B}_j(k) - \Delta\tilde{\underline{B}}_j(k) - \tilde{\underline{B}}_j(k-1))\, \underline{u}(k-j)$$

$$+ \sum_{l=1}^{g} (\underline{D}_l(k) - \Delta\tilde{\underline{D}}_l(k) - \tilde{\underline{D}}_l(k-1))\, \underline{\xi}(k-l) \quad \text{..........(22)}$$

where:

$$\Delta\tilde{\underline{A}}_i(k) = [\Delta\tilde{a}_{itq}(k)] = [\alpha_{itq} \; s_t(k)\, y_q(k-i)]$$

$$i = 1 \rightarrow h, \; t = 1 \rightarrow n, \; q = 1 \rightarrow n$$

$$\Delta\tilde{\underline{B}}_j(k) = [\Delta\tilde{b}_{jtq}(k)] = [\beta_{jtq} \; s_t(k)\, u_q(k-j)]$$

$$j = 1 \rightarrow f, \; t = 1 \rightarrow n, \; q = 1 \rightarrow n$$

$$\Delta\tilde{\underline{D}}_l(k) = [\Delta\tilde{d}_{ltq}(k)] = [\delta_{ltq} \; s_t(k)\, \xi_q(k-l)]$$

$$l = 1 \rightarrow g, \; t = 1 \rightarrow n, \; q = 1 \rightarrow p \quad \text{....................(23)}$$

Equation (19) may now be rewritten as:

$$\underline{y}_d(k) = \sum_{i=1}^{h} \tilde{\underline{A}}_i(k-1)\, \underline{y}(k-i) + \sum_{j=1}^{f} \tilde{\underline{B}}_j(k-1)\, \underline{u}(k-j) +$$

$$\sum_{l=1}^{g} \tilde{\underline{D}}_l(k-1)\, \underline{\xi}(k-l) \quad \text{.................................(24)}$$

and, so, the control error vector, $\underline{e}_1(k)$, may be written:

$$\underline{e}_1(k) = \sum_{i=1}^{h} (\underline{A}_i(k) - \tilde{\underline{A}}_i(k-1))\, \underline{y}(k-i) + \sum_{j=1}^{f} (\underline{B}_j(k) - \tilde{\underline{B}}_j(k-1))\, \underline{u}(k-j) + \sum_{l=1}^{g} (\underline{D}_l(k) - \tilde{\underline{D}}_l(k-1))\, \underline{\xi}(k-l) \quad (25)$$

Equation (5) can be rewritten as:

$$\underline{\varepsilon}(k-1) = \sum_{i=1}^{h} (\underline{A}_i(k-1) - \tilde{\underline{A}}_i(k-1))\, \underline{y}(k-i-1) + \sum_{j=1}^{f} (\underline{B}_j(k-1) - \tilde{\underline{B}}_j(k-1))\, \underline{u}(k-j-1) + \sum_{l=1}^{g} (\underline{D}_l(k-1) - \tilde{\underline{D}}_l(k-1))\, \underline{\xi}(k-l-1) \;..............(26)$$

So from equations (25) and (26), one obtains:

$$\underline{e}_1(k) = \underline{\varepsilon}(k-1) + \sum_{i=1}^{h} (\underline{A}_i(k-1) - \tilde{\underline{A}}_i(k-1))\, [\underline{y}(k-i) - \underline{y}(k-i-1)] + \sum_{j=1}^{f} (\underline{B}_j(k-1) - \tilde{\underline{B}}_j(k-1))\, [\underline{u}(k-j) - \underline{u}(k-j-1)] + \sum_{l=1}^{g} (\underline{D}_l(k-1) - \tilde{\underline{D}}_l(k-1))\, [\underline{\xi}(k-l) - \underline{\xi}(k-l-1)] + \sum_{i=1}^{h} [\underline{A}_i(k) - \underline{A}_i(k-1)]\, \underline{y}(k-i) + \sum_{j=1}^{f} [\underline{B}_j(k) - \underline{B}_j(k-1)]\, \underline{u}(k-j) + \sum_{l=1}^{g} [\underline{D}_l(k) - \underline{D}_l(k-1)]\, \underline{\xi}(k-l) \;..(27)$$

The last three terms of the right hand side of equation (27) will be equal to zero $\forall k$ s.t. $k \geq k_c$, where k_c is some instant. This follows from the condition that the process parameters may admit only a finite number of bounded changes as $k \to \infty$.

Further, it can be shown that the assumption of bounded disturbances, coupled with the convergence properties of the identification scheme [19], implies that the terms related to the disturbance vector always remain bounded.

It now only remains to prove that the input and output vectors always remain bounded. Consider the hypothesis that at least one of $y_q(k-i-1)$ or $u_q(k-i-1)$, $(i=1 \to n) \to \infty$ as $k \to \infty$.

From the asymptotic hyperstability properties of the identification scheme $\underline{s}(k-1) \to 0$ as $k \to \infty$. Therefore, considering the scalar form of equation (26), it is obvious that $(a_{itq}(k-1) - \tilde{a}_{itq}(k-1))$ and $(b_{jtq}(k-1) - \tilde{b}_{jtq}(k-1))$ must approach zero faster than $y_q(k-i-1)$ or $u_q(k-i-1)$ approaches infinity.

Furthermore, from equations (1) and (20), it is manifest that $(\underline{y}(k-i) - \underline{y}(k-i-1))$ and $(\underline{u}(k-j) - \underline{u}(k-j-1))$

cannot possibly tend to infinity faster than $\underline{y}(k-i-1)$ and $\underline{u}(k-j-1)$. But this means that equation (27) implies that $\underline{e}_1(k)$ will always remain bounded. Thus, from equation (21) and assumptions 1 and 2, it can be seen that the process output, and thus the input, always remain bounded. This result is at variance with the original hypothesis. The argument proceeds, then, that the input and output vectors are bounded.

Finally, as the identification system approaches equilibrium, $\underline{s}(k) \to 0$ or from equations (23),

$$\Delta\tilde{\underline{A}}_i(k) \to 0, \quad \Delta\tilde{\underline{B}}_j(k) \to 0, \quad \Delta\tilde{\underline{D}}_l(k) \to 0$$

$$i = 1 \to h, \; j = 1 \to f, \; l = 1 \to g$$

and so $\underline{e}_1(k) \to 0$, from equations (22) and (25).

This proves Theorem 4.4.

<u>Note:</u>

Theorems 4.2 through 4.4 inherently have assumed two further conditions which have not been explicitly stated in the proofs given:

1. The driver block must specify a desired output

such that the generated control action is physically realizable, ie. some type of projection technique which ensures that the identified plant does not contain right-half plane zeroes, must be employed. In particular, the final argument contained in the proof of Theorem 4.4 is invalid if such a strategy is not used, for $\underline{u} \to \infty$ if an unstable controller is stipulated and hence, from equation (27) $\underline{e}_1(k) \to \infty$.

2. The plant must be strictly open-loop stable. This condition is enforced to ensure a stable identification sequence. From Theorem 4.3:

$$s_t(k) = k_{tt}(k)\, e_t(k)$$

$$t = 1 \to n$$

where:

$$k_{tt}(k) = 1/(1 + \sum_{i=1}^{h} \sum_{q=1}^{n} \alpha_{itq}\, y_q^2(k-i) + \sum_{j=1}^{f} \sum_{q=1}^{n} \beta_{jtq}\, u_q^2(k-j) + \sum_{l=1}^{g} \sum_{q=1}^{p} \delta_{ltq}\, \xi_q^2(k-l))$$

So that if $\underline{y} \to \infty$ and $\underline{u}$ is bounded, it is possible for $\underline{e}(k) \to \infty$ as $\underline{s}(k) \to 0$. Once again the final argument contained in the proof of Theorem 4.4

will be invalid in such a situation as $y \to \infty$.

4.3 Discussion of the Adaptive-Inverse Development

It is essential to note that the theoretical results given above, have all been arrived at by Martin-Sanchez and are presented in an, as yet, unpublished paper [21]. The important ideas, however, have all appeared in an earlier work [19].

Whilst the general approach of using an adaptive inverse appears to be quite sound (in fact Godbole and Smith [26] suggested just such an approach in 1972) there are several objections to the methods and proofs that Martin-Sanchez has outlined. Firstly, as Johnson and Larimore [32] point out[1], this type of approach is strictly only applicable for minimum phase model descriptions, since open right-half plane zeroes can lead to an unstable inverse controller which would then refute the proof of Theorem 4.4. Martin-Sanchez [33], in his reply, notes that this problem may be alleviated, somewhat, by insisting that the driver block never generate an output which will render necessary

---

1

This same conclusion had been noted much earlier by Godbole and Smith [26] and Forney [30].

such a situation. This approach necessitates more information about the plant operation than is desirable.

It is true to say, thus, from a theoretical point of view, that a plant inverse method such as that described in this chapter, is strictly applicable to minimum phase models only.

Further, it should be noted that it is also desirable to only consider, essentially, time-invariant plants, as the proof relies on the fact that at some instant the plant can be assumed to be stationary.[1]

Another objection relates to the nonlinear adaptive relationship between the identification error, given by equation (3), and the vector $\underline{s}$. It is quite possible, especially in a noisy environment, for the identification error to go to infinity as $\underline{s} \rightarrow 0$. An unstable plant would clearly lead to this situation -- the problem may, however, occur for other circumstances as well unless some type of projection technique is employed, which will ensure that the parameter estimates remain in a stable region [34].

1

I am indebted to Dr. M.N. Karim for this interpretation of finite, bounded changes in the process parameters.

Finally, the concept of the driver block appears to have been a central issue in this approach and yet, the design of this block from the point of view of output realizability, is not elaborated upon apart from noting that noise characteristics may be taken into account [19], ie. noise filters may be incorporated in the driver block.

4.4 Conclusions

In this chapter, the theoretical design of a practical adaptive control system using the inverse system approach, has been considered. This approach was presented by Martin-Sanchez in 1976, although previous work had been published in Spanish by the same author. Further, he has applied this scheme to a pilot distillation column, at the University of Alberta, with very encouraging results [21].

It has been mentioned that there are several aberrations with this technique that have not been regarded as such. The claims that Martin-Sanchez makes for his procedure must be viewed in this light.

In the next chapter, there will be presented a more generalized approach than that reported here, using the original theoretical contributions of Landau. Martin-Sanchez's scheme will be considered as a subset of

this technique, although for the purposes of tractability, the adaptive inverse system methodology is subsequently retained.

Chapter Four

CHAPTER FIVE

On a General Self-Adaptive Control Scheme

5.1 Introduction

Of the various designs, which can be classified under the general heading of adaptive control, the model reference technique has proved the most popular. In particular, the two general approaches which guarantee closed-loop stability, hyperstability and Liapunov theory, have received a great deal of attention in recent years [1 - 5].

The general feeling today, is that Popov's hyperstability theory [6,7] offers the more widely applicable results, in the field. Indeed schemes based on this methodology have been developed to include input-output formulations (see Chapter Four) which are probably the most practical system descriptions, given the inherent problems associated with state-space approaches [8 - 13].

This chapter will introduce a general self-adaptive control scheme based on an asymptotic hyperstable identification method developed by Landau [14]. The view has been taken that any adaptive control technique must be able to be integrated into actual operating plants with as little change of operating philosophy as possible.

5.2 General Methodology

The general input-output formulation of the control problem allows a very practical method of implementing the self-corrective type of control scheme. In particular, by considering the outputs per se, it avoids the problems that most plague the state-space approaches:

(i) explicit time-delays,

(ii) finding a readily appreciable performance index and, finally,

(iii) state inaccessibility.

Of the various techniques that could be imagined for the implementation of a closed-loop model reference adaptive control scheme, two broad classifications can be identified:

1) Direct adaptation of the parameters of a controller/precompensator. Such a scheme is depicted in Figure 5.1.

2) Open-loop identification followed by parameter adaptation.

An explicit open-loop identification of the plant is available at each instant. The

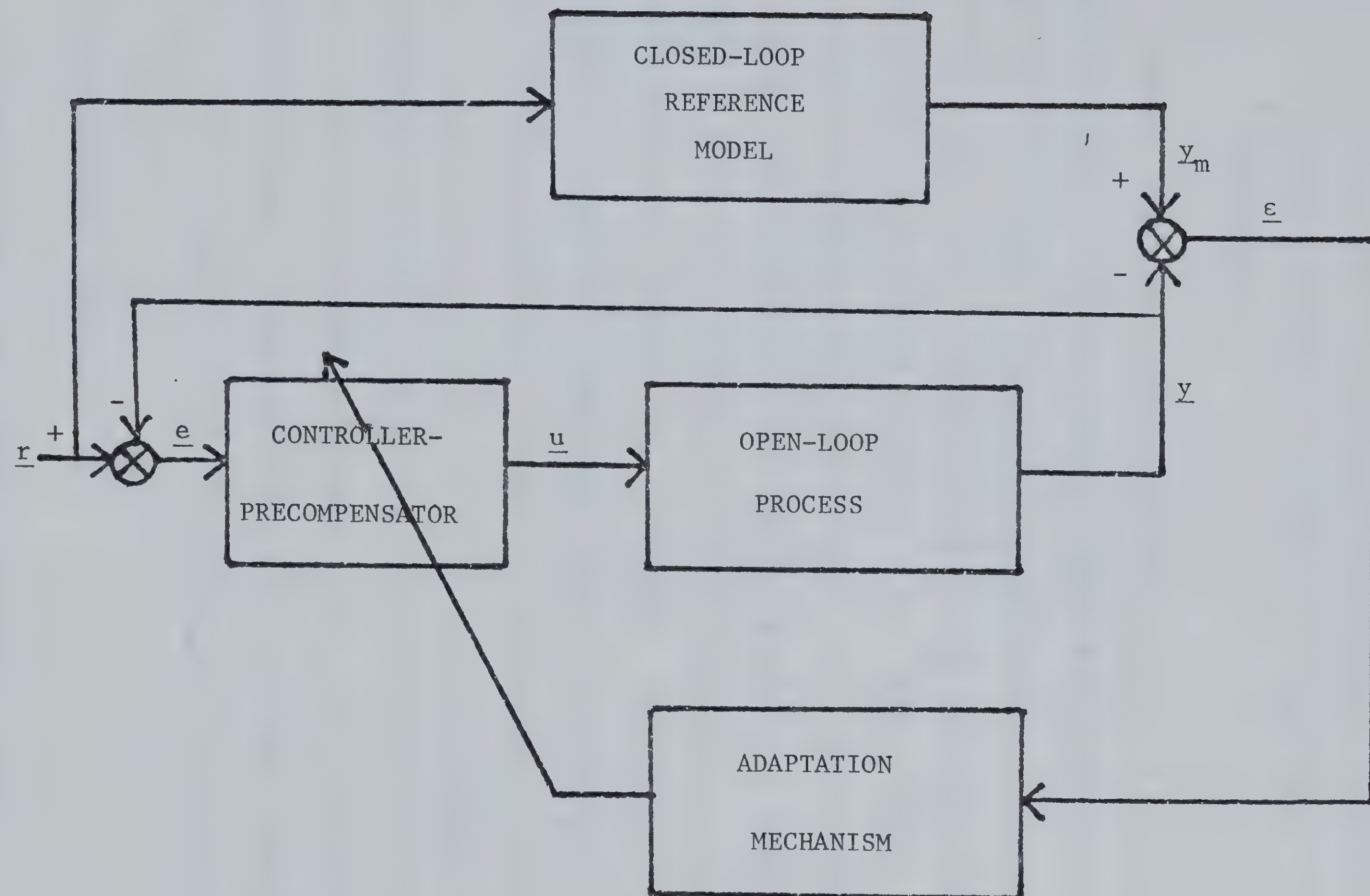


FIGURE 5.1 A MODEL REFERENCE ADAPTIVE CONTROL METHOD BASED ON ADAPTATION OF THE CONTROL BLOCK PARAMETERS

parameters of the controller can then be updated or a control signal calculated, to yield the required output. Figure 5.2 represents this technique.

Whilst the first approach would appear to be the most straightforward for purposes of implementation, there are some disadvantages. Firstly, an approximate knowledge of the open-loop plant (over and above the necessary structural information) is usually required [15 - 16]. Also, some of the plant parameters are unreachable (ie. physically not available for adaptation) in the more general case. To alleviate this last problem, a more restrictive starting formulation must normally be accepted. The second method, meanwhile, suggests a much more fruitful procedure; an explicit identification is used in conjunction with a primary control block. The control and identification modes are separated and, as such, the control block can be designed in any convenient manner [17]. In particular, this technique offers the possibility of a genuine two-level control scheme such as that described by Nikiforuk et al. [18 - 19].

5.3 A Particular Example

To demonstrate the design philosophy briefly alluded to

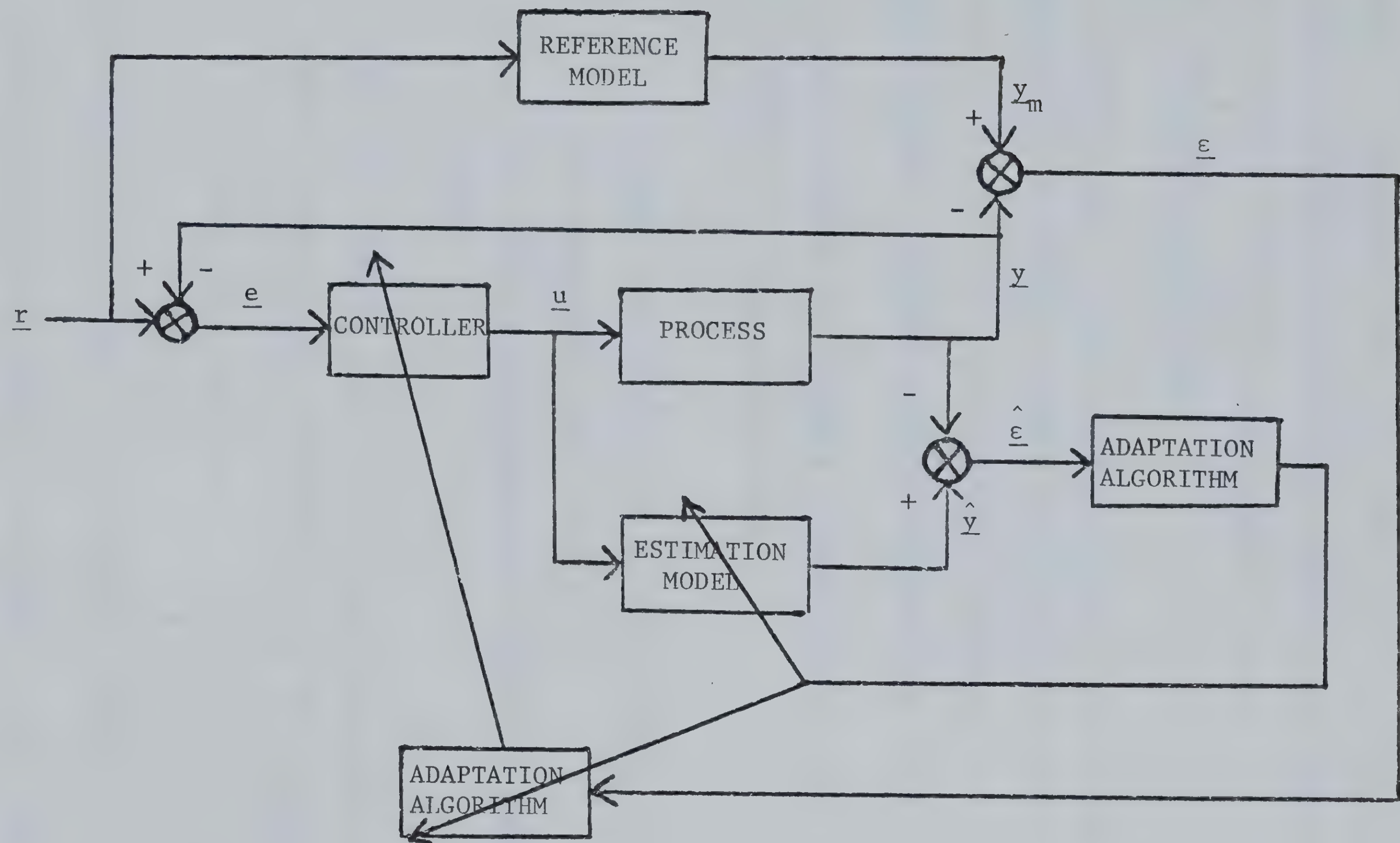


FIGURE 5.2 PRIOR OPEN-LOOP IDENTIFICATION ADAPTIVE CONTROL METHOD

above, a technique will be developed based on the adaptive inverse procedure suggested by Godbole and Smith [20] and later implemented by Martin-Sanchez [21].

5.3.1 Problem Formulation

Unlike most of the previous work, the present design approach is directed towards the adaptation of the parameters of the precompensator/controller of a standard multivariable feedback system depicted as Figure 5.3.

For the sake of clarity disturbances are ignored, for the present, in this development.

$\underline{\underline{G}}_{OL}(z)$ represents the pulse transfer matrix for the open-loop plant and,

$\underline{\underline{K}}(z)$ is a precompensator/controller transfer matrix.

The input-output data of the loop, depicted in Figure 5.3, can be conveniently represented by the formulation:

$$\underline{y}(z) = \underline{\underline{G}}_{CL}(z)\,\underline{r}(z) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where $\underline{\underline{G}}_{CL}(z)$ is a closed-loop transfer matrix.

Equation (1) may also be written, in the time-domain, as:

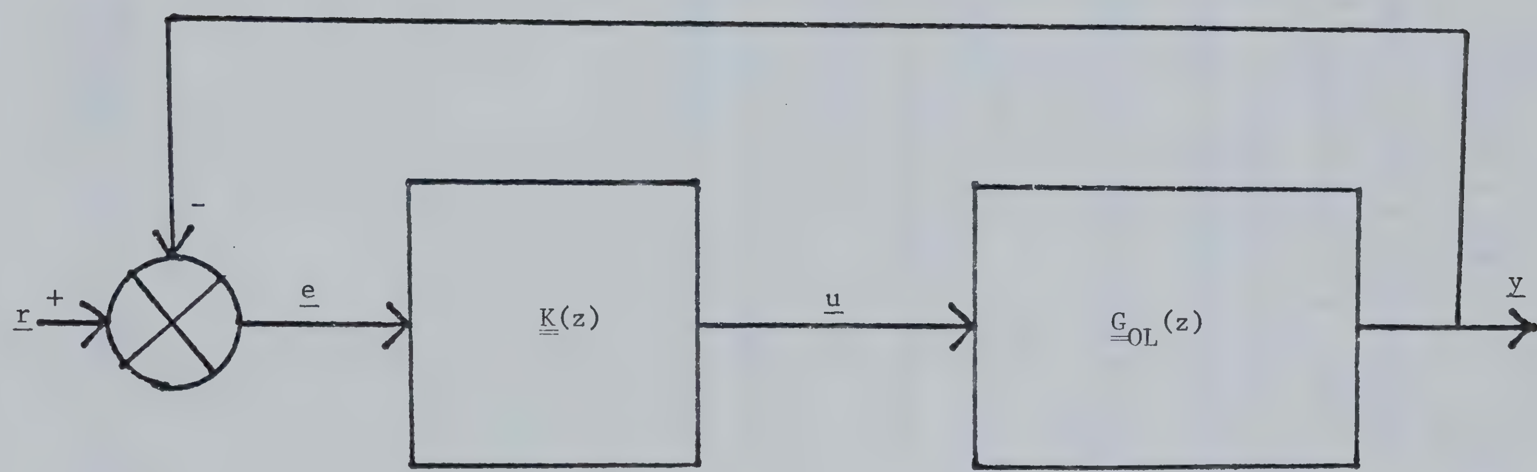


FIGURE 5.3 MULTIVARIABLE FEEDBACK CONTROL SYSTEM

$$\underline{y}(k) = \sum_{i=1}^{h} \underline{\underline{A}}_{ip}(k)\, \underline{y}(k-i) + \sum_{j=1}^{f} \underline{\underline{B}}_{jp}(k)\, \underline{r}(k-j) \quad \ldots\ldots.(2)$$

where $\underline{y}$ and $\underline{r}$ denote n dimensional output and setpoint vectors, respectively. $\underline{\underline{A}}_{ip}$ and $\underline{\underline{B}}_{jp}$ are process parameter matrices, of appropriate order. These matrices may admit a finite number of bounded changes as $k \to \infty$.

A reference model is now chosen as:

$$\underline{y}_m(k) = \sum_{i=1}^{h_1} \underline{\underline{A}}_{im}\, \underline{y}_m(k-i) + \sum_{j=1}^{f_1} \underline{\underline{B}}_{jm}\, \underline{r}(k-j) \quad \ldots\ldots\ldots\ldots(3)$$

where the m subscripted variables denote time-invariant model parameters.[1]

This model need not have the same structure as the closed-loop plant, provided that:

$h_1 \leq h$

and:

$f_1 \leq f$ ....(4)

---

[1]

$\underline{\underline{A}}_{im}$ and $\underline{\underline{B}}_{jm}$ can, in fact, admit a finite number of bounded changes, as $k \to \infty$.

If an error vector is defined as:

$$\underline{\varepsilon}(k) = \underline{y}_m(k) - \underline{y}(k) \quad \text{.........................} (5)$$

then from equations (2) - (4), it can be seen that:

$$\underline{\varepsilon}(k) = \sum_{i=1}^{h_1} \underline{\underline{A}}_{im}\, \underline{\varepsilon}(k-i) + \sum_{i=1}^{h_1} (\underline{\underline{A}}_{im} - \underline{\underline{A}}_{ip}(k))\, \underline{y}(k-i) -$$

$$\sum_{i=h_1+1}^{h} \underline{\underline{A}}_{ip}(k)\, \underline{y}(k-i) + \sum_{j=1}^{f_1} (\underline{\underline{B}}_{jm} - \underline{\underline{B}}_{jp}(k))\, \underline{r}(k-j) -$$

$$\sum_{j=f_1+1}^{f} \underline{\underline{B}}_{jp}(k)\, \underline{r}(k-j) \quad \text{.........................} (6)$$

This error vector then provides a direct measure of the current system performance[1] for the entire control system.

Define:

$$\underline{v}(k) = \underline{\varepsilon}(k) + \sum_{i=1}^{p_1} \underline{\underline{D}}_i\, \underline{\varepsilon}(k-i) \quad \text{.........................} (7)$$

where $\underline{\underline{D}}_i$, $(i=1 \rightarrow p_1)$ are the coefficient matrices of a linear filter, $\underline{\underline{D}}(z)$.

---

[1]

Note that "measure of current system performance" has a different interpretation from the "index of performance", employed in Chapter Two. Here the term merely signifies a measure of achievement of the control objective.

Now let:

$$\underline{w}(k) = \sum_{i=1}^{h_1} (\underline{A}_{im} - \underline{A}_{ip}(k))\, \underline{v}(k - i) - \sum_{i=h_1+1}^{h} \underline{A}_{ip}(k)\, \underline{v}(k - i)$$

$$+ \sum_{j=1}^{f_1} (\underline{B}_{jm} - \underline{B}_{jp}(k))\, \underline{r}(k - j) - \sum_{j=f_1+1}^{f} \underline{B}_{jp}(k)\, \underline{r}(k - j) \quad (8)$$

and:

$$\underline{w}_1(k) = -\underline{w}(k) \quad \ldots\ldots(9)$$

where equation (8) is understood to represent a nonlinear function of $\underline{v}$ and time.

Incorporating equation (8) into (6), one obtains:

$$\underline{\varepsilon}(k) = \sum_{i=1}^{h_1} \underline{A}_{im}\, \underline{\varepsilon}(k - i) + \underline{I}\, \underline{w}(k) \quad \ldots\ldots(10)$$

or, in z-transform notation:

$$\underline{\varepsilon}(z) = (\underline{I} - \sum_{i=1}^{h_1} \underline{A}_{im}\, z^{-i})^{-1}\, \underline{w}(z) \quad \ldots\ldots(11)$$

whereupon, from equation (7):

$$\underline{v}(z) = \underline{D}(z)\, (\underline{I} - \sum_{i=1}^{h_1} \underline{A}_{im}\, z^{-i})^{-1}\, \underline{w}(z) \quad \ldots\ldots(12)$$

or:

$$\underline{v}(z) = \underline{G}(z)\, \underline{w}(z) \quad \ldots\ldots(12a)$$

In addition, it is assumed that only those pairs $(\underline{v}, \underline{w}_1)$

which satisfy the following inequality condition are considered[1]:

$$\eta(k_0,k_1) = \sum_{k=k_0}^{k_1} \underline{v}^T(k)\, \underline{w}_1(k) \geq -\lambda_0^2 \qquad \forall k_1 \geq k_0 \quad .....(13)$$

where λ_0 is a finite constant perhaps dependent on the initial state of the system, but not on time.

5.3.2 Theory

Equations (7) - (9), (12) and inequality (13) define a system which can be depicted as in Figure 5.4.

As a basis for later discussion it is noted that the nonlinear control system, defined by Figure 5.4, is equivalent to the scheme represented by Figure 5.5.

Note that the approach illustrated by Figure 5.5 implies adaption of the closed-loop plant parameters. This is easily done in simulation studies, but in experimental applications the actual plant parameters may not be adjustable. An alternative approach is presented later (c.f. Fig. 5.6) which alleviates this practical diffculty.

1

This condition has been used to define a "weakly" hyperstable system [22].

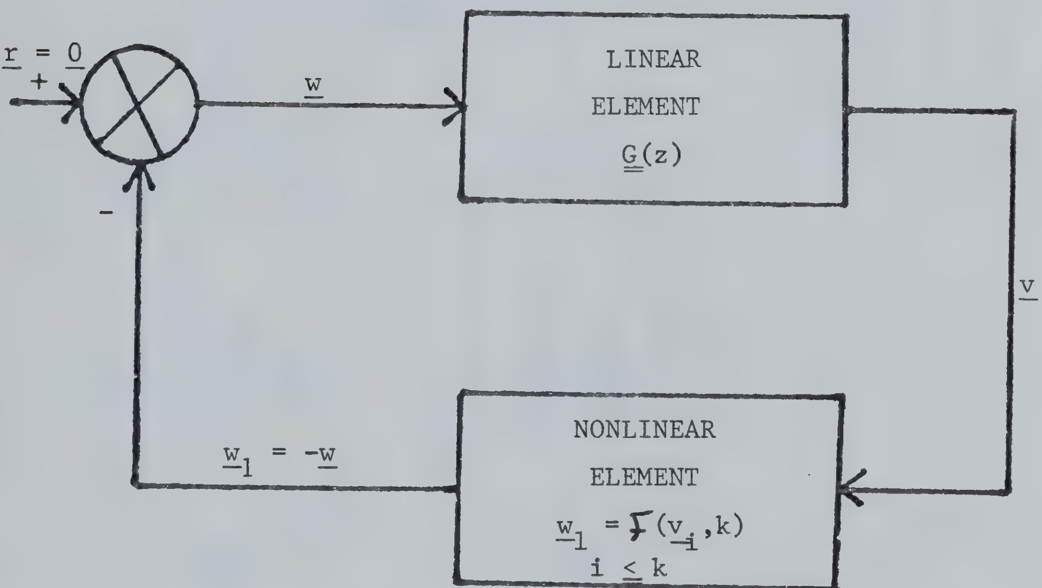


FIGURE 5.4 AUTONOMOUS, NONLINEAR FEEDBACK CONTROL SYSTEM

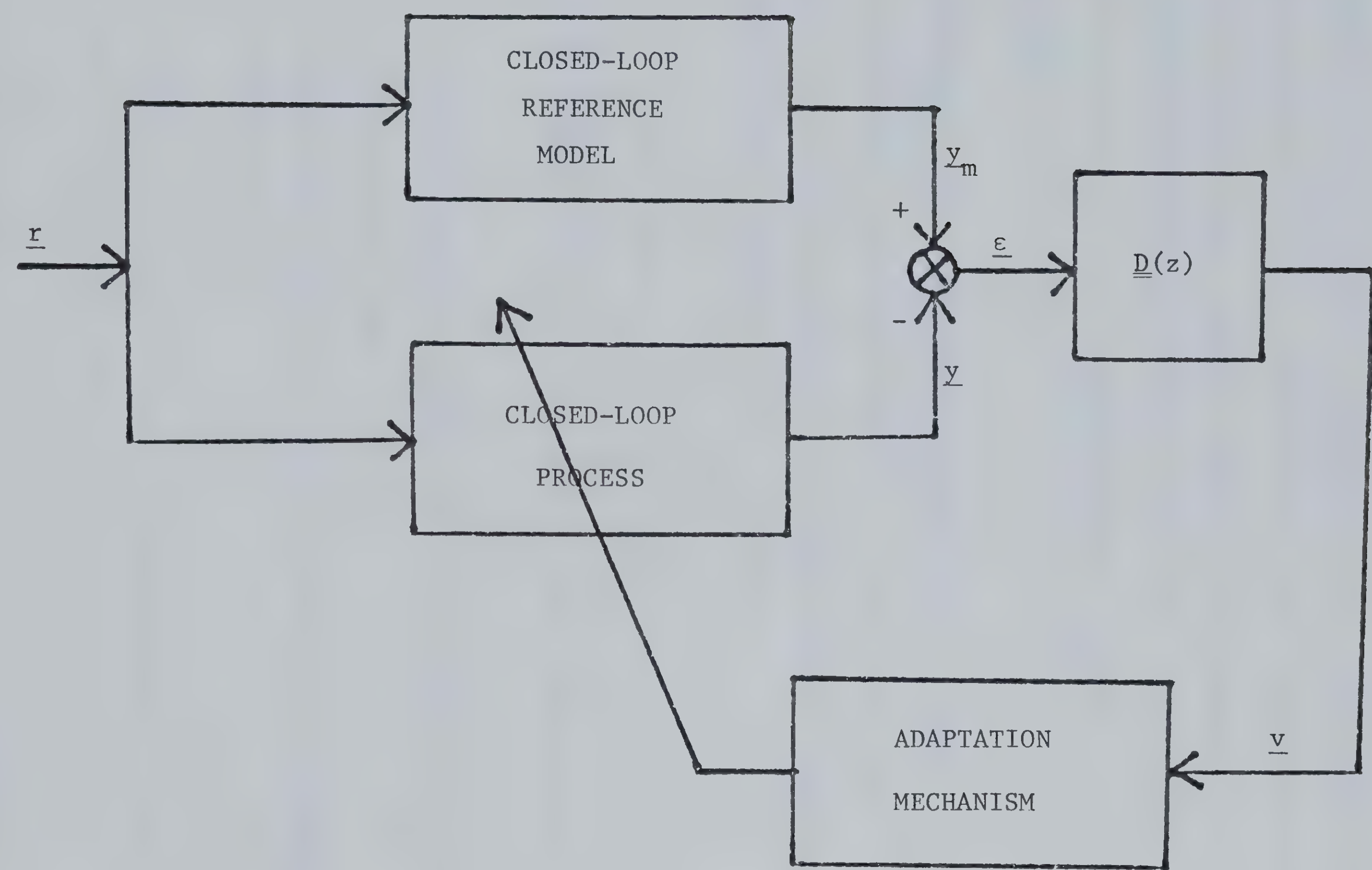


FIGURE 5.5 MODEL REFERENCE ADAPTIVE CONTROL SCHEME

Landau has presented a theorem [23 - 24] which gives necessary and sufficient conditions such that the discrete nonlinear and/or time-varying system, represented in Figure 5.4, and described by equations (7) - (9), (12) and inequality (13), will be asymptotically hyperstable.

Theorem 5.1

Necessary and sufficient conditions in order that the system, described by equations (7) - (9), (12) and inequality (13), be an asymptotic hyperstable system are that the transfer matrix, $\underline{\underline{G}}(z)$, given by:

$$\underline{\underline{G}}(z) = \underline{\underline{D}}(z)\,(\underline{\underline{I}} - \sum_{i=1}^{h_1} \underline{\underline{A}}_{im}\, z^{-i})^{-1}$$

is strictly positive real discrete in the sense that:

(i) the poles of $\underline{\underline{G}}(z)$ lie within the circular domain $|z|<1$ and,

(ii) $\underline{\underline{G}}(z) + \underline{\underline{G}}^T(z^*)$ should be positive definite Hermitian.

The proof may be considered as analogous to that for the continuous hyperstability theorem [25].

Using this theorem and the equivalence established between the systems of Figure 5.4 and Figure 5.5, it is

possible to state conditions such that the proposed adaptive control system is asymptotically hyperstable.

Theorem 5.2

Sufficient conditions such that the discrete, adaptive control system, shown in Figure 5.5, is asymptotic hyperstable are the following:

(i) the transfer matrix:

$$\underline{G}(z) = \underline{D}(z)\,(\underline{I} - \sum_{i=1}^{h_1} \underline{A}_{im}\, z^{-i})^{-1}$$

must be strictly positive real discrete;

(ii)

$$(\underline{A}_{im} - \underline{A}_{ip}(k))\,\underline{v}(k-i) \qquad i = 1 \rightarrow h_1$$

$$\underline{A}_{ip}(k)\,\underline{v}(k-i) \qquad i = h_1 + 1 \rightarrow h$$

$$(\underline{B}_{jm} - \underline{B}_{jp}(k))\,\underline{r}(k-j) \qquad j = 1 \rightarrow f_1$$

$$\underline{B}_{jp}(k)\,\underline{r}(k-j) \qquad j = f_1 + 1 \rightarrow f$$

and $\underline{v}$ must all have the same dimension and,

(iii) the adaptation laws for $\underline{\underline{A}}_{ip}(k)$ and $\underline{\underline{B}}_{jp}(k)$ must admit the following nonlinear matrix

functions:

$$\underline{\Phi}_i(k) = [\phi_{itq}(k)] = [\alpha_{itq}\ v_t(k)\ y_q(k-i)]$$

$$i = 1 \to h\ ,\ t = 1 \to n\ ,\ q = 1 \to n$$

$$\underline{X}_j(k) = [\eta_{jtq}(k)] = [\beta_{jtq}\ v_t(k)\ r_q(k-j)]$$

$$j = 1 \to f\ ,\ t = 1 \to n\ ,\ q = 1 \to n$$

α_{itq} and β_{jtq} are strictly positive coefficients.[1]

Proof

Equations (7) - (9), (12) and inequality (13) define a discrete, nonlinear feedback system, such as is depicted in Figure 5.4, where:

$$\underline{G}(z) = \underline{D}(z)\,(\underline{I} - \sum_{i=1}^{h_1} \underline{\Delta}_{im}\, z^{-i})^{-1}$$

and:

$$\underline{w}_1(k) = \mathcal{F}(\underline{v}_i, k) \qquad i \leq k$$

1

If the reference model matrices contain zeroes in any elements it is possible to set the corresponding gains, of the adaptive closed-loop plant, to zero.

$\mathcal{F}$ denotes a nonlinear functional dependence.

In order that the system of Figure 5.4, or equivalently Figure 5.5, be an asymptotic hyperstable system, it is sufficient (from Theorem 5.1) that:

1) $\underline{\underline{G}}(z)$ be positive real discrete, and

2)
$$\eta(k_0,k_1) = \sum_{k=k_0}^{k_1} \underline{v}^T(k)\, \underline{w}_1(k) \geq -\lambda_0^2 \qquad \forall k_1 \geq k_0$$

where λ_o is a finite constant dependent only on the initial system state.

Thus, condition (1) of Theorem 5.2 is proved.

The second condition of Theorem 5.2, follows from the definitions of $\underline{w}$ and $\underline{w}_1$, in that this condition must hold for either to be defined.

The third condition may be proved by using the definition of $\underline{w}_1$:

$$\underline{w}_1(k) = \sum_{i=1}^{h_1} (\underline{A}_{ip}(k) - \underline{A}_{im})\, \underline{y}(k-i) + \sum_{i=h_1+1}^{h} \underline{A}_{ip}(k)\, \underline{y}(k-i)$$
$$+ \sum_{j=1}^{f_1} (\underline{B}_{jp}(k) - \underline{B}_{jm})\, \underline{r}(k-j) + \sum_{j=f_1+1}^{f} \underline{B}_{jp}(k)\, \underline{r}(k-j) \quad (14)$$

Substituting this into $\eta(k_o,k_1)$, one obtains the condition that:

$$\sum_{k=k_o}^{k_1} \underline{v}^T(k) \left\{ \sum_{i=1}^{h_1} (\underline{A}_{ip}(k) - \underline{A}_{im}) \underline{y}(k-i) + \sum_{i=h_1+1}^{h} \underline{A}_{ip}(k) \underline{y}(k-i) + \sum_{j=1}^{f_1} (\underline{B}_{jp}(k) - \underline{B}_{jm}) \underline{r}(k-j) + \sum_{j=f_1+1}^{f} \underline{B}_{jp}(k) \underline{r}(k-j) \right\} \geq -\lambda_o^2$$

$$\forall k_1 \geq k_o \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(15)$$

For this inequality to be satisfied, it is sufficient that the following scalar functions are verified:

$$\sum_{k=k_o}^{k_1} v_t(k) (a_{itq}(k) - a_{itqm}) y_q(k-i) \geq -\lambda^2_{aitq}$$

$$i = 1 \to h_1 \text{ , } t = 1 \to n \text{ , } q = 1 \to n$$

$$\sum_{k=k_o}^{k_1} v_t(k) a_{itq}(k) y_q(k-i) \geq -\lambda^2_{a'itq}$$

$$i = h_1 + 1 \to h \text{ , } t = 1 \to n \text{ , } q = 1 \to n$$

$$\sum_{k=k_o}^{k_1} v_t(k) (b_{jtq}(k) - b_{jtqm}) r_q(k-j) \geq -\lambda^2_{bjtq}$$

$$j = 1 \to f_1 \text{ , } t = 1 \to n \text{ , } q = 1 \to n$$

$$\sum_{k=k_o}^{k_1} v_t(k)\, b_{jtq}(k)\, r_q(k-j) \geq -\lambda^2_{b'jtq}$$

$j = f_1 + 1 \to f$, $t = 1 \to n$, $q = 1 \to n$ ....(16)

λ_{aitq}, $\lambda_{a'itq}$, λ_{bjtq} and $\lambda_{b'jtq}$ are finite constants independent of time.

Introducing the adaptation laws, of Theorem 5.2, inequalities (16) become:

$$\sum_{k=k_o}^{k_1} v_t(k)\, (\alpha_{itq}\; v_t(k)\, y_q(k-i) + a_{itq}(k-1) - a_{itqm})$$

$$y_q(k-i) \geq -\lambda^2_{aitq}$$

$i = 1 \to h_1$, $t = 1 \to n$, $q = 1 \to n$

$$\sum_{k=k_o}^{k_1} v_t(k)\, (\alpha_{itq}\; v_t(k)\, y_q(k-i) + a_{itq}(k-1))\, y_q(k-i)$$

$$\geq -\lambda^2_{a'itq}$$

$i = h_1 + 1 \to h$, $t = 1 \to n$, $q = 1 \to n$

$$\sum_{k=k_o}^{k_1} v_t(k)\, b_{jtq}(k)\, r_q(k-j) \geq -\lambda^2_{b'jtq}$$

$j = f_1 + 1 \rightarrow f\ ,\ t = 1 \rightarrow n\ ,\ q = 1 \rightarrow n$ ....(16)

λ_{aitq}, $\lambda_{a'itq}$, λ_{bjtq} and $\lambda_{b'jtq}$ are finite constants independent of time.

Introducing the adaptation laws, of Theorem 5.2, inequalities (16) become:

$$\sum_{k=k_o}^{k_1} v_t(k)\,(\alpha_{itq}\ v_t(k)\, y_q(k-i) + a_{itq}(k-1) - a_{itqm})$$

$$y_q(k-i) \geq -\lambda^2_{aitq}$$

$i = 1 \rightarrow h_1\ ,\ t = 1 \rightarrow n\ ,\ q = 1 \rightarrow n$

$$\sum_{k=k_o}^{k_1} v_t(k)\,(\alpha_{itq}\ v_t(k)\, y_q(k-i) + a_{itq}(k-1))\, y_q(k-i)$$

$$\geq -\lambda^2_{a'itq}$$

$i = h_1 + 1 \rightarrow h\ ,\ t = 1 \rightarrow n,\ q = 1 \rightarrow n$

$$\sum_{k=k_o}^{k_1} v_t(k)\,(\beta_{jtq}\ \ v_t(k)\, r_q(k - j) + b_{jtq}(k - 1) - b_{jtqm})$$

$$r_q(k - j) \geq - \lambda^2_{bjtq}$$

$$j = 1 \rightarrow f_1 \ , \ t = 1 \rightarrow n \ , \ q = 1 \rightarrow n$$

$$\sum_{k=k_o}^{k_1} v_t(k)\,(\beta_{jtq}\ \ v_t(k)\, r_q(k - j) + b_{jtq}(k - 1))\ r_q(k - j)$$

$$\geq - \lambda^2_{b'jtq}$$

$$j = f_1 + 1 \rightarrow f \ , \ t = 1 \rightarrow n \ , \ q = 1 \rightarrow n \ \ldots\ldots\ldots\ldots\ldots(17)$$

If the adaptation laws are written in recursive form as:

$$a_{itq}(k) = \sum_{l=k_o}^{k} \alpha_{itq}\ \ v_t(l)\ y_q(l - i)$$

$$i = 1 \rightarrow h \ , \ t = 1 \rightarrow n \ , \ q = 1 \rightarrow n$$

$$b_{jtq}(k) = \sum_{l=k_o}^{k} \beta_{jtq}\ \ v_t(l)\ r_q(l - j)$$

$$j = 1 \rightarrow f \ , \ t = 1 \rightarrow n \ , \ q = 1 \rightarrow n$$

it is readily apparent that relations (17) are analogous to:

$$\sum_{k=k_o}^{k_1} x(k)\ (\sum_{l=k_o}^{k} \beta x(l) + c) = 1/2\beta(\sum_{k=k_o}^{k_1} x(k) + c/\beta)^2 +$$

Chapter Five

$$\underline{y}(k - i + 1) + \sum_{j=1}^{f_1} (\underline{B}_{jm} - \underline{B}_{jp}(k + 1))\, \underline{r}(k - j + 1)$$

$$- \sum_{j=f_1+1}^{f} \underline{B}_{jp}(k + 1)\, \underline{r}(k - j + 1) + \sum_{i=1}^{p_1} \underline{D}_i\, \underline{\varepsilon}(k - i + 1)$$

or, in scalar form:

$$v_t(k + 1) = \sum_{i=1}^{h_1} \sum_{q=1}^{n} a_{itqm}\, y_{qm}(k - i + 1) -$$

$$\sum_{i=1}^{h} \sum_{q=1}^{n} a_{itq}(k + 1)\, y_q(k - i + 1) +$$

$$\sum_{j=1}^{f_1} \sum_{q=1}^{n} (b_{jtqm} - b_{jtq}(k + 1))\, r_q(k - j + 1) -$$

$$\sum_{j=f_1+1}^{f} \sum_{q=1}^{n} b_{jtq}(k + 1)\, r_q(k - j + 1) +$$

$$\sum_{i=1}^{p_1} \sum_{q=1}^{n} d_{itq}\, \varepsilon_q(k - i + 1)$$

$$t = 1 \rightarrow n$$

Using condition (iii), of Theorem 5.2, one then obtains:

$$v_t(k + 1) = \sum_{i=1}^{h_1} \sum_{q=1}^{n} a_{itqm}\, y_{qm}(k - i + 1)$$

$$- \sum_{i=1}^{h} \sum_{q=1}^{n} (\alpha_{itq}\, v_t(k + 1)\, y_q(k - i + 1) + a_{itq}(k))$$

$$y_q(k - i + 1) + \sum_{j=1}^{f_1} \sum_{q=1}^{n} (b_{jtqm} - \beta_{jtq}\, v_t(k + 1)$$

$$r_q(k-j+1) - b_{jtq}(k))\, r_q(k-j+1)$$

$$- \sum_{j=f_1+1}^{f} \sum_{q=1}^{n} (\beta_{jtq}\, v_t(k+1)\, r_q(k-j+1) +$$

$$b_{jtq}(k))\, r_q(k-j+1) + \sum_{i=1}^{p_1} \sum_{q=1}^{n} d_{itq}\, \varepsilon_q(k-i+1)$$

$t = 1 \rightarrow n$

So that:

$$v_t(k+1) = \sum_{i=1}^{h_1} \sum_{q=1}^{n} (a_{itqm}\, y_{qm}(k-i+1) - a_{itq}(k)$$

$$y_q(k-i+1)) - \sum_{i=h_1+1}^{h} \sum_{q=1}^{n} a_{itq}(k)\, y_q(k-i+1) +$$

$$\sum_{j=1}^{f_1} \sum_{q=1}^{n} (b_{jtqm} - b_{jtq}(k))\, r_q(k-j+1) - \sum_{j=f_1+1}^{f} \sum_{q=1}^{n} b_{jtq}(k)$$

$$r_q(k-j+1) - v_t(k+1) \left\{ \sum_{i=1}^{h} \sum_{q=1}^{n} \alpha_{itq}\, y_q^2(k-i+1) + \right.$$

$$\left. \sum_{j=1}^{f} \sum_{q=1}^{n} \beta_{jtq}\, r_q^2(k-j+1) \right\} + \sum_{i=1}^{p_1} \sum_{q=1}^{n} d_{itq}\, \varepsilon_q(k-i+1)$$

$t = 1 \rightarrow n$

or:

$$v_t(k+1) = X/Y \quad \ldots\ldots(18)$$

where:

$$X = \sum_{i=1}^{h_1}\sum_{q=1}^{n} (a_{itqm}\; y_{qm}(k-i+1) - a_{itq}(k)\, y_q(k-i+1)) - \sum_{i=h_1+1}^{h}\sum_{q=1}^{n} a_{itq}(k)\, y_q(k-i+1) + \sum_{j=1}^{f_1}\sum_{q=1}^{n} (b_{jtqm} - b_{jtq}(k))\, r_q(k-j+1) - \sum_{j=f_1+1}^{f}\sum_{q=1}^{n} b_{jtq}(k)\, r_q(k-j+1) + \sum_{i=1}^{p_1}\sum_{q=1}^{n} d_{itq}\; \varepsilon_q(k-i+1) \quad \ldots\ldots(19)$$

and:

$$Y = 1 + \sum_{i=1}^{h}\sum_{q=1}^{n} \alpha_{itq}\; y_q^2(k-i+1) + \sum_{j=1}^{f}\sum_{q=1}^{n} \beta_{jtq}\; r_q^2(k-j+1) \quad \ldots\ldots(20)$$

$$t = 1 \rightarrow n$$

Using equation (18), it is therefore possible to estimate the parameter matrices, $\underline{\underline{A}}_{ip}(k+1)$ and $\underline{\underline{B}}_{jp}(k+1)$.

5.3.3 Practical Methods of Implementation

A practical difficulty arises out of the laws of adaptation, in that the closed-loop parameter matrices are,

strictly, not accessible directly for adaptation in a real system. Instead, use is made of a compensator/controller block.

Consider the system shown as Figure 5.6.

The closed-loop transfer matrix may be derived for the inner control-loop as:

$$\underline{\underline{G}}_{CL} = (\underline{\underline{I}} + \underline{\underline{G}}_{OL}\ \underline{\underline{K}})^{-1}\ \underline{\underline{G}}_{OL}\ \underline{\underline{K}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

where:

$\underline{\underline{I}}$ is the nxn identity matrix;

$\underline{\underline{G}}_{OL}$ represents the open-loop plant transfer matrix;

$\underline{\underline{K}}$ is the controller/precompensator transfer matrix and,

$\underline{\underline{G}}_{CL}$ is the actual closed-loop transfer matrix, at any instant.

If $\underline{\underline{G}}_{CL}(z)$ in equation (21) is set equal to the desired closed-loop transfer matrix, then:

$$\underline{\underline{K}}(z) = \underline{\underline{G}}^{-1}_{OL}(z)\ \underline{\underline{G}}_{CL}(z)\ (\underline{\underline{I}} - \underline{\underline{G}}_{CL}(z))^{-1} \quad \ldots\ldots\ldots\ldots\ldots (22)$$

where, from the z-transform form of equation (2):

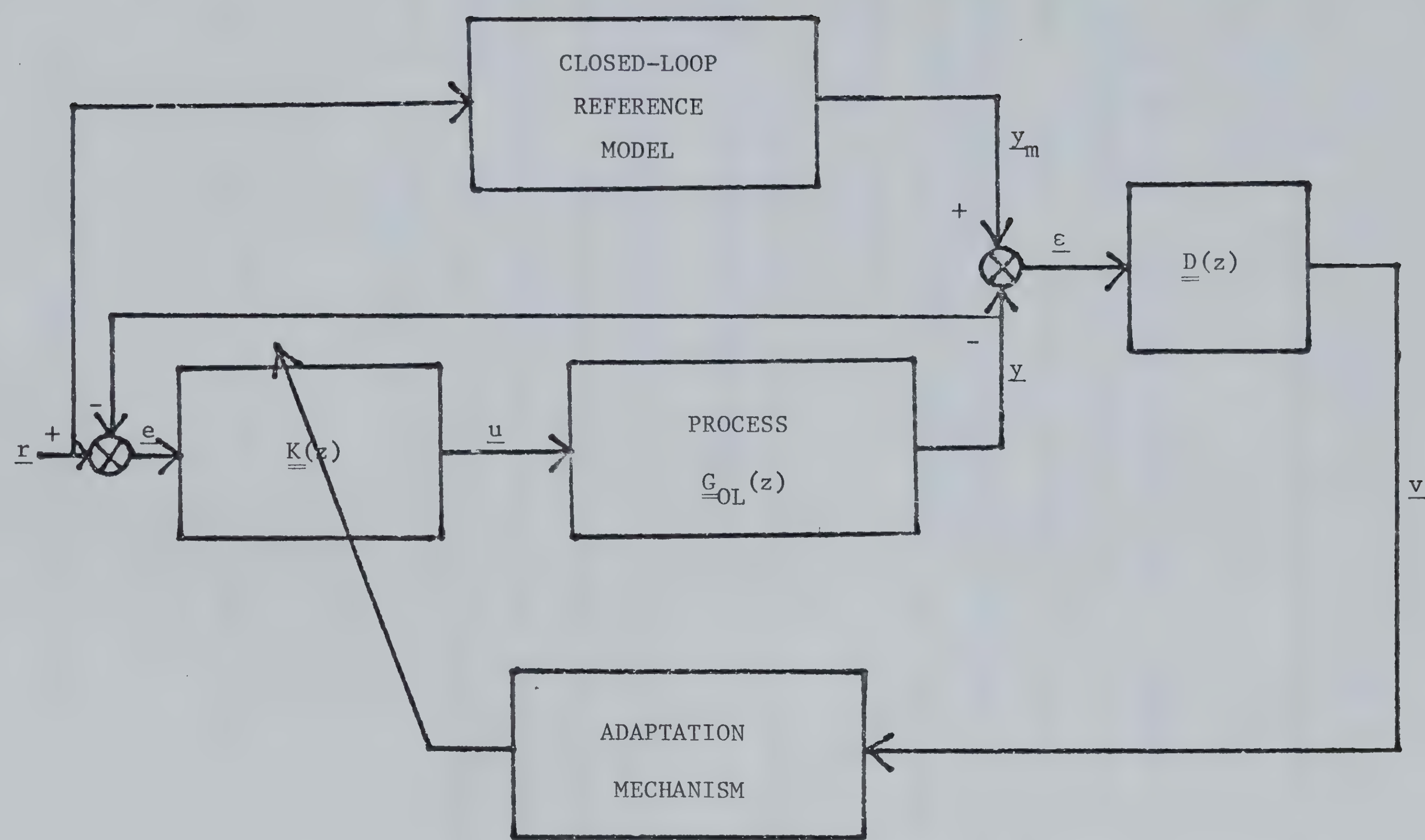


FIGURE 5.6 PRACTICAL MODEL REFERENCE ADAPTIVE CONTROL SYSTEM

$$\underline{\underline{G}}_{CL}(z) = (\underline{\underline{I}} - \sum_{i=1}^{h} \underline{\underline{A}}_{ip} \; z^{-i})^{-1} \; (\sum_{j=1}^{f} \underline{\underline{B}}_{jp} \; z^{-j})$$

and, $\underline{\underline{A}}_{ip}$ and $\underline{\underline{B}}_{jp}$ are obtained from the adaptation laws (Theorem 5.2).

It becomes obvious, however, that a complete knowledge of the open-loop plant transfer matrix is assumed in this scheme. Whilst it is still possible to progress with this design, when limited information is available as to the variation in $\underline{\underline{G}}_{OL}$, there are simpler and more elegant approaches available.

To overcome these difficulties, an augmented output method is considered (see Figure 5.7).

Considering only the blocks enclosed by the dashed boundary, it can be seen that the method is equivalent to that depicted in Figure 5.6, except that the open-loop transfer matrix, $\underline{\underline{G}}_{OL}$, is replaced by $\underline{\underline{G}}_{est}$ and adaptation is based on $\underline{y}$* rather than on $\underline{y}$.

Thus,

$$\underline{\underline{G}}_{CL} = (\underline{\underline{I}} + \underline{\underline{G}}_{est} \; \underline{\underline{K}})^{-1} \; \underline{\underline{G}}_{est} \; \underline{\underline{K}}$$

and:

$$\underline{\underline{K}} = \underline{\underline{G}}_{est}^{-1} \; \underline{\underline{G}}_{CL} \; (\underline{\underline{I}} - \underline{\underline{G}}_{CL})^{-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(23)$$

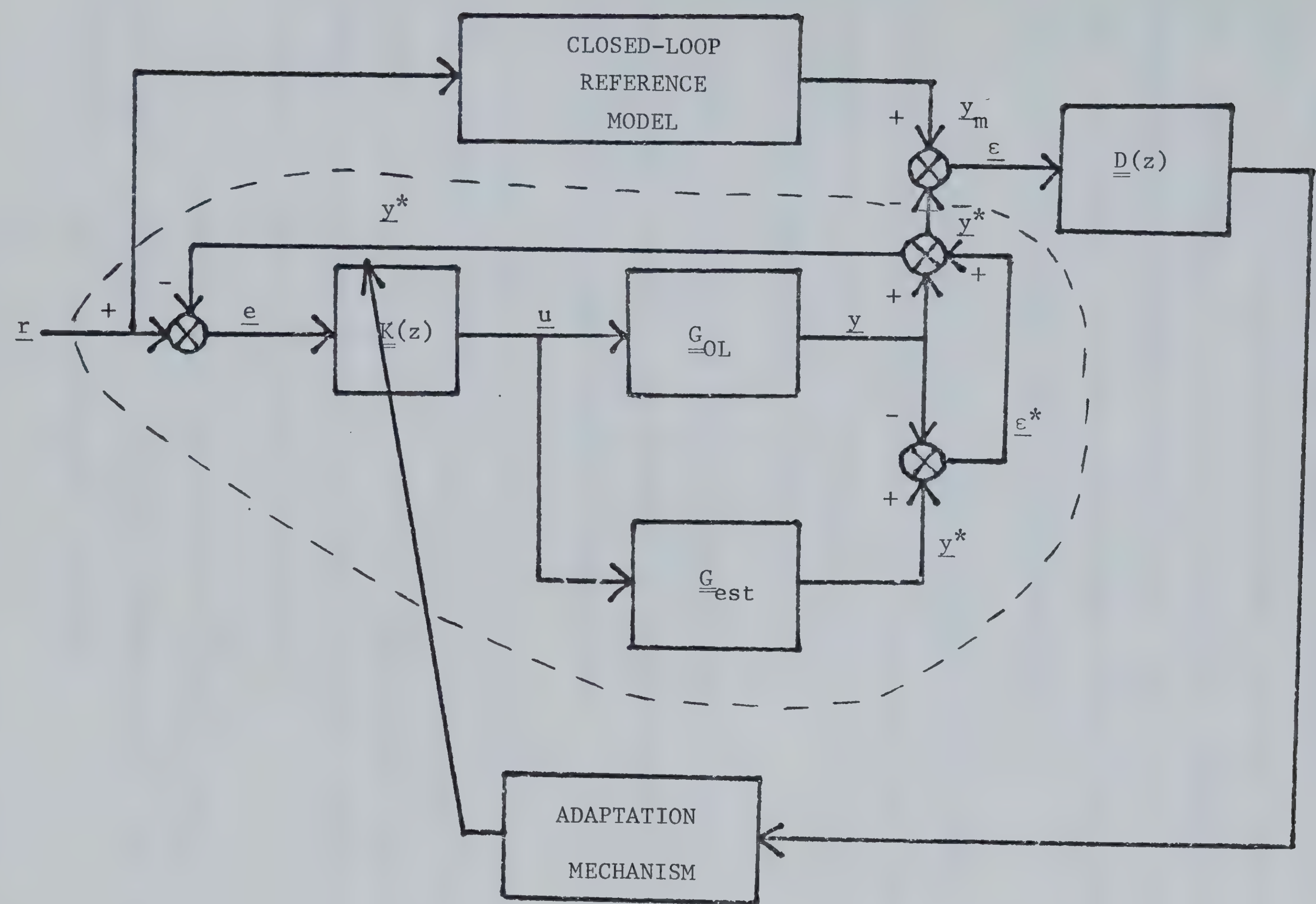


FIGURE 5.7 AUGMENTED OUTPUT METHOD FOR MODEL REFERENCE ADAPTIVE CONTROL

That this system is asymptotic hyperstable can be proved in an analogous fashion to that detailed for Theorem 5.2. Therefore $\underline{y}^* \rightarrow \underline{y}_m$ as $k \rightarrow \infty$ or from the principle of superposition of linear systems $\underline{y} \rightarrow \underline{y}_m - \underline{\varepsilon}^*_{ss}$ as $k \rightarrow \infty$, where $\underline{\varepsilon}^*_{ss}$ represents the steady-state error between the outputs of the actual process and of the known $\underline{\underline{G}}_{est}$.

Unfortunately, there exists a steady-state offset introduced by this technique. Once again this may not be disasterous if certain information, other than the structure of the open-loop plant, is allowed.

There is one more modification, however, which will bring this proposed model reference adaptive control system more in line with the original objectives.

The final step is to consider that $\underline{\underline{G}}_{est} = \hat{\underline{\underline{G}}}_{OL}$, where $\hat{\underline{\underline{G}}}_{OL}$ is an identified transfer matrix of the open-loop plant. Further, it can be considered that a hyperstable recursive scheme can be employed for this task. Thus, a system as depicted in Figure 5.8 would be resolved.

It is apparent that the system is again equivalent to that shown in Figure 5.6, with $\hat{\underline{\underline{G}}}_{OL}$ replacing $\underline{\underline{G}}_{OL}$ and, hence, $\hat{\underline{y}}$ replacing $\underline{y}$ as the controlled variable.

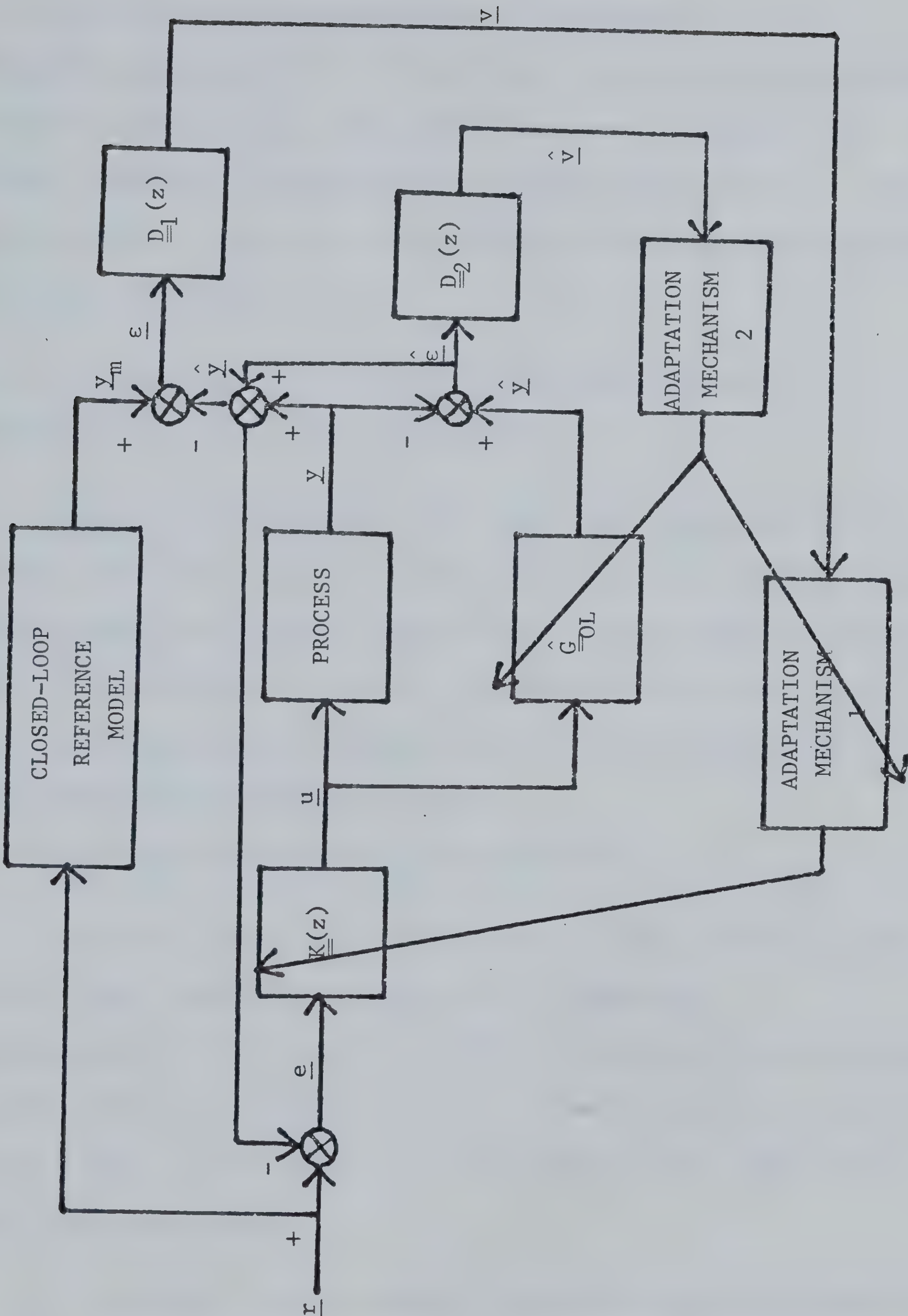


FIGURE 5.8 AUGMENTED OUTPUT MODEL REFERENCE ADAPTIVE CONTROL SYSTEM WITH OPEN-LOOP IDENTIFICATION

Since the adaptation block 2 is assumed to be asymptotic hyperstable $\hat{\underline{\varepsilon}} \to 0$ as $k \to \infty$. Also, adaptation block 1, which effectively encompasses the entire system is, itself, asymptotic hyperstable so that $\hat{\underline{y}} \to \underline{y}_m$ as $k \to \infty$. This implies that $\underline{y} \to \underline{y}_m$ as $k \to \infty$ by the principle of superposition of linear systems.

Some discussion of the identification scheme is an obvious requirement, at this stage.

Whilst it is strictly true that any asymptotically stable identification procedure could be implemented, only those recursive schemes based on the hyperstability concept will be considered in detail. In particular, the scheme recently presented by Landau [14] and investigated by Ljung [26] appears advantageous.

5.4 Hyperstable Recursive Identification

The model reference structure has been shown to supply a very sound conceptual approach to recursive identification [14, 26, 27]. This procedure takes advantage of the so-called "dual" nature of model reference adaptive systems (ie. of the arbitrary way in which the model and plant are specified).

It is noted, at the outset, that the approach below, is

essentially that taken by Martin-Sanchez [21] in the development of his identification scheme. This technique however, is more general and alleviates the problems of biased estimation, in the face of noise obscured measurements, for single-input single-output (SISO) systems.

The system, developed below, will be equivalent in concept to the scheme presented by Landau [14]. Ljung [26] has further investigated the convergence properties of this type of system and obtained criteria such that convergence of the estimates to the true values occurs with probability one.

5.4.1 Theory

Suppose that the open-loop plant can be described by an equation of the form:

$$\underline{y}(k) = \sum_{i=1}^{h} \underline{\underline{A}}_i \, \underline{y}(k-i) + \sum_{j=1}^{f} \underline{\underline{B}}_j \, \underline{u}(k-j) \quad \ldots\ldots\ldots\ldots(24)$$

where:

$\underline{y}(k-i)$ are n dimensional output vectors

$\underline{u}(k-j)$ are m dimensional input vectors

$\underline{\underline{A}}_i$ and $\underline{\underline{B}}_j$ are time-invariant process parameter

matrices of appropriate order[1].

An adjustable estimation model is now chosen such that:

$$\hat{\underline{y}}(k) = \sum_{i=1}^{h} \hat{\underline{\underline{A}}}_i(k)\, \hat{\underline{y}}(k - i) + \sum_{j=1}^{f} \hat{\underline{\underline{B}}}_j(k)\, \underline{u}(k - j) \quad \ldots\ldots\ldots (25)$$

where:

$\hat{\underline{y}}(k-i)$ are n dimensional identification model output vectors

$\hat{\underline{\underline{A}}}_i(k)$ and $\hat{\underline{\underline{B}}}_j(k)$ are estimation model parameter matrices of appropriate dimension.

It is noted, here, that there is no theoretical requirement for the estimation model to have the same structure as the open-loop plant. However, as a rule, it would normally be apparent that to ensure a meaningful identification it would be desired that this be so.

An identification error, $\hat{\underline{\varepsilon}}(k)$, and a vector, $\hat{\underline{v}}(k)$, are defined now, such that:

$$\hat{\underline{\varepsilon}} = \hat{\underline{y}}(k) - \underline{y}(k) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

1

It can be shown that these matrices can admit a finite number of bounded changes as $k \to \infty$.

and:

$$\underline{\hat{v}}(k) = \sum_{i=o}^{p_1} \underline{\underline{D}}_i \, \underline{\hat{\varepsilon}}(k - i) \quad \text{.....(27)}$$

where $\underline{\underline{D}}_o = \underline{\underline{I}}$.

Using equations (24) - (26), one obtains:

$$\underline{\hat{\varepsilon}}(k) = \sum_{i=1}^{h} \underline{\underline{A}}_i \, \underline{\hat{\varepsilon}}(k - i) - \sum_{i=1}^{h} (\underline{\underline{A}}_i - \underline{\underline{\hat{A}}}_i(k)) \, \underline{\hat{y}}(k - i) -$$

$$\sum_{j=1}^{f} (\underline{\underline{B}}_j - \underline{\underline{\hat{B}}}_j(k)) \, \underline{u}(k - j) \quad \text{.....(28)}$$

or, defining vectors $\underline{w}(k)$ and $\underline{w}_1(k)$ as:

$$\underline{w}(k) = \sum_{i=1}^{h} (\underline{\underline{\hat{A}}}_i(k) - \underline{\underline{A}}_i) \, \underline{\hat{y}}(k - i) + \sum_{j=1}^{f} (\underline{\underline{\hat{B}}}_j(k) - \underline{\underline{B}}_j) \, \underline{u}(k - j)$$

$$\text{.....(29)}$$

and:

$$\underline{w}_1(k) = -\underline{w}(k) \quad \text{.....(30)}$$

$$\underline{\hat{\varepsilon}}(k) = \sum_{i=1}^{h} \underline{\underline{A}}_i \, \underline{\hat{\varepsilon}}(k - i) + \underline{\underline{I}} \, \underline{w}(k) \quad \text{.....(31)}$$

Using equation (27) and introducing z-transform notation, equation (31) becomes:

$$\hat{\underline{v}}(z) = \underline{D}(z)\,(\underline{I} - \sum_{i=1}^{h} \underline{A}_i\, z^{-i})^{-1}\, \underline{w}(z) \qquad \ldots\ldots\ldots\ldots\ldots\ldots(32)$$

If it is assumed that the adaptation laws for $\hat{\underline{\underline{A}}}_i(k)$ and $\hat{\underline{\underline{B}}}_j(k)$ admit nonlinear functions of the following form:

$$\hat{\underline{\Phi}}_i(k) = [\ \hat{\phi}_{itq}(\hat{\underline{v}}_i, k)] \qquad i \leq k$$

and:

$$\hat{\underline{\chi}}_j(k) = [\ \hat{\eta}_{jtq}(\hat{\underline{v}}_j, k)] \qquad j \leq k \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(33)$$

then it is apparent that the system described by equations (29) - (32) can be depicted in the manner of Figure 5.4. It is further supposed that only those pairs $(\hat{\underline{v}}, \underline{w}_1)$ which satisfy the following inequality will be considered:

$$\eta(k_0, k_1) = \sum_{k=k_0}^{k_1} \hat{\underline{v}}^T(k)\, \underline{w}_1(k) \geq -\lambda_0^2 \qquad \forall k_1 \geq k_0 \ \ldots.(34)$$

where λ_o is a finite constant not dependent on time.

Thus, it is clear that the discrete hyperstability theorem can be used to define an asymptotic hyperstable, recursive identification scheme.

Theorem 5.3

In order that the system, represented by equations (24) - (27), (33) and inequality (34) be an asymptotic hyperstable system, it is sufficient that:

(i) $\underline{\underline{G}}(z)=\underline{\underline{D}}(z)(\underline{\underline{I}} - \sum_{i=1}^{h} \underline{\underline{A}}_i z^{-i})^{-1} - 1/2\lambda\underline{\underline{I}}$ is a strictly positive real discrete transfer matrix;

(ii) $(\underline{\underline{A}}_i - \hat{\underline{\underline{A}}}_i(k))\underline{y}(k-i)$, $(i=1 \rightarrow h)$, $(\underline{\underline{B}}_j - \hat{\underline{\underline{B}}}_j(k))\underline{u}(k-j)$, $(j=1 \rightarrow f)$ and $\underline{v}$ must all have the same dimension and,

(iii) the adaptation laws for $\hat{\underline{\underline{A}}}_i(k)$ and $\hat{\underline{\underline{B}}}_j(k)$ must admit the following nonlinear matrix functions:

$$\hat{\underline{\underline{\Phi}}}_i(k) = [\ \hat{\phi}_{itq}(k)] = [\alpha_{itq}(k-1)\ \hat{v}_t(k)\ \hat{y}_q(k-i)]$$

$i = 1 \rightarrow h,\ t = 1 \rightarrow n,\ q = 1 \rightarrow n$

$$\hat{\underline{\underline{\chi}}}_j(k) = [\ \hat{\eta}_{jtq}(k)] = [\beta_{jtq}(k-1)\ \hat{v}_t(k)\ u_q(k-j)]$$

$j = 1 \rightarrow f,\ t = 1 \rightarrow n,\ q = 1 \rightarrow m$

where:

$$\alpha_{itq}(k) = \alpha_{itq}(k-1) + 1/\lambda[(\alpha_{itq}(k-1)\ \hat{y}_q(k-i))^2 /$$

$$(1 - 1/\lambda\ \alpha_{itq}(k-1)\ \hat{y}^2_q(k-i))]$$

$i = 1 \rightarrow h,\ t = 1 \rightarrow n,\ q = 1 \rightarrow n$

$$\beta_{jtq}(k) = \beta_{jtq}(k-1) + 1/\lambda[(\beta_{jtq}(k-1)\, u_q(k-j))^2 /$$

$$(1 - 1/\lambda\ \beta_{jtq}(k-1)\, u_q^2(k-j))]$$

$$j = 1 \rightarrow f,\ t = 1 \rightarrow n,\ q = 1 \rightarrow m$$

α_{itq} and β_{jtq} are strictly negative coefficients.

Remarks

1. The first condition, of the theorem, is the analog of the original condition for single-input, single-output identification, first arrived at by Ljung [26] and later mentioned in an addenum by Landau [28].

2. The introduction of the λ term can be seen as a method of affecting the rate of decrease of the magnitude of the adaptation gains.

3. For $\lambda=1$, one obtains the algorithm given in [14] only with decreasing magnitude adaptation gains for which $\underline{\underline{G}}(z)$ becomes:

$$\underline{\underline{G}}(z) = \underline{\underline{D}}(z)\,(\underline{\underline{I}} - \sum_{i=1}^{h} \underline{\underline{A}}_i\, z^{-i})^{-1} - 1/2\ \underline{\underline{I}}$$

Chapter Five

4. For $\lambda \to \infty$, the decreasing magnitude adaptation gains $\alpha_{itq}(k)$ and $\beta_{jtq}(k)$ tend to constant gains α_{itq} and β_{jtq} , and $\underline{\underline{G}}(z)$ can be given by:

$$\underline{\underline{G}}(z) = \underline{\underline{D}}(z)\left(\underline{\underline{I}} - \sum_{i=1}^{h} \underline{\underline{A}}_i\, z^{-i}\right)^{-1}$$

5. According to Ljung [26], condition (1), given by the theorem, assures also that the parameter estimates for a single-input, single-output system converge with probability one to the real ones, in the case of measurements disturbed by noise ($\lambda < \infty$). The proof of this for the multivariable case is not available at this time.

<u>Proof of Theorem 5.3</u>

An auxiliary variable $\underline{v}^*(k)$ is introduced such that:

$$\underline{v}^*(k) = \hat{\underline{v}}(k) + 1/2\lambda\ \underline{w}_1(k) = \hat{\underline{v}}(k) - 1/2\lambda\ \underline{w}(k) \quad \text{.........}(35)$$

From equations (28) - (30), one obtains:

$$\hat{\underline{\varepsilon}}(k) = \sum_{i=1}^{h} \underline{\underline{A}}_i\ \hat{\underline{\varepsilon}}(k-i) + \underline{\underline{I}}\ \underline{w}(k)$$

or from equation (27):

$$\hat{\underline{v}}(z) = \underline{D}(z)\,(\underline{I} - \sum_{i=1}^{h} \underline{\Delta}_i\, z^{-i})^{-1}\, \underline{w}(z)$$

so, from equation (35):

$$\underline{v}^*(z) = \underline{D}(z)\,(\underline{I} - \sum_{i=1}^{h} \underline{\Delta}_i\, z^{-i})^{-1}\, \underline{w}(z) - 1/2\lambda\, \underline{w}(z)$$

$$= [\underline{D}(z)\,(\underline{I} - \sum_{i=1}^{h} \underline{\Delta}_i\, z^{-i})^{-1} - 1/2\lambda\, \underline{I}]\, \underline{w}(z) \quad \ldots\ldots\ldots\ldots(36)$$

Also, consider only those pairs ($\underline{v}^*, \underline{w}_1$) which satisfy the following inequality relation:

$$\sum_{k=k_o}^{k_1} \underline{v}^*(k)\, \underline{w}_1(k) \geq -\lambda_o^2 \qquad \forall k_1 \geq k_o \quad \ldots\ldots\ldots\ldots\ldots(37)$$

where λ_o is a finite constant only dependent on the initial system state.

Equations (24) - (27), (29) - (30) and (35) - (36), and inequality (37) describe an equivalent nonlinear, autonomous feedback system such as that depicted in Figure 5.9.

A necessary and sufficient condition for the system, of Figure 5.9, to be asymptotic hyperstable, is that $\underline{\underline{G}}(z)$ be strictly positive real discrete (from the discrete hyperstability theorem -- see Theorem 5.1).

The first condition of Theorem 5.3 is thus proved.

The second requirement arises from the definition of $\underline{w}(k)$ and of the inequality.

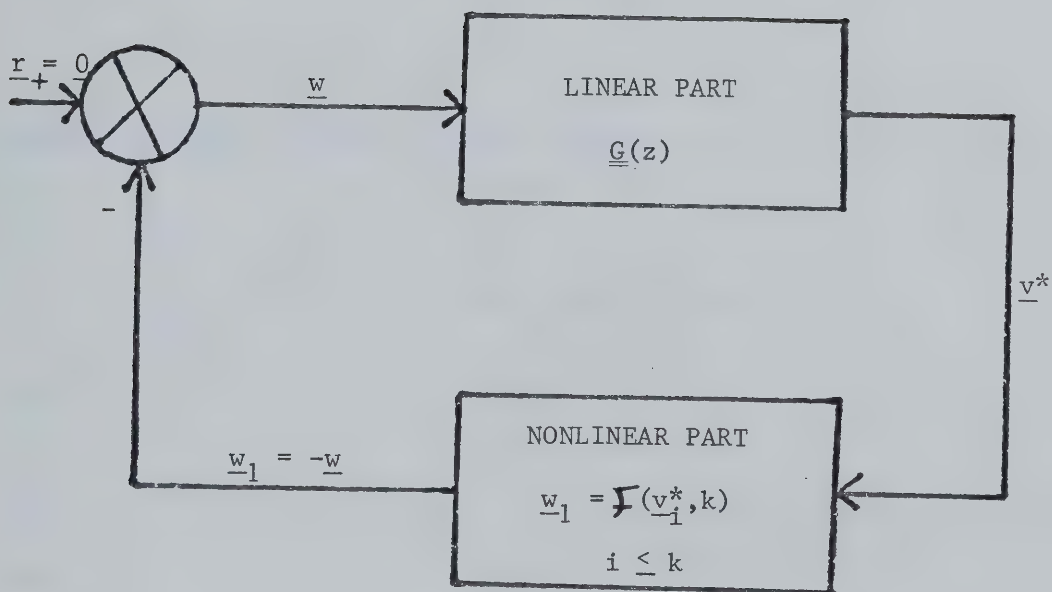


FIGURE 5.9 NONLINEAR, AUTONOMOUS FEEDBACK SYSTEM FOR IDENTIFICATION

It is now required to prove that the given adaptation laws result in the satisfaction of inequality (37).

For inequality (37) to be satisfied, it is sufficient that:

$$\sum_{k=k_o}^{k_1} v_t^*(k)\, w_{1_t}(k) \geq -\lambda_t^2 \qquad \forall k_1 \geq k_0$$

$$t = 1 \rightarrow n$$

where λ_t only depends on the initial state of the system, or:

$$\sum_{k=k_o}^{k_1} (\hat{v}_t(k) + 1/2\lambda\, w_{1_t}(k))\, w_{1_t}(k) \geq -\lambda_t^2$$

and finally:

$$\sum_{k=k_o}^{k_1} \hat{v}_t(k)\, w_{1_t}(k) \geq -\lambda_{1_t}^2 \qquad \forall k_1 \geq k_0 \ \ldots\ldots\ldots\ldots\ldots (38a)$$

and:

$$\sum_{k=k_o}^{k_1} 1/2\lambda\, w_{1_t}^2(k) \geq -\lambda_{2_t}^2 \qquad \forall k_1 \geq k_0 \ \ldots\ldots\ldots\ldots\ldots (38b)$$

$$t = 1 \rightarrow n$$

The inequality (38b) is always satisfied since λ is strictly non-negative. It remains to prove (38a) is also

always satisfied.

Considering the scalar representation of $\underline{w}_1(k)$, (38a) becomes:

$$\sum_{k=k_o}^{k_1} \sum_{t=1}^{n} \hat{v}_t(k) \quad \{ \sum_{i=1}^{h} \sum_{q=1}^{n} (a_{itq} - \hat{a}_{itq}(k)) \, \hat{y}_q(k-i) +$$

$$\sum_{j=1}^{f} \sum_{q=1}^{m} (b_{jtq} - \hat{b}_{jtq}(k)) \, u_q(k-j) \} \geq - \lambda_{1t}^2 \quad \ldots\ldots\ldots(39)$$

Sufficient conditions such that inequality (39) can be verified are:

$$\sum_{k=k_o}^{k_1} (a_{itq} - \hat{a}_{itq}(k)) \, \hat{v}_t(k) \, \hat{y}_t(k-i) \geq - \lambda_{1aitq}^2$$

$i = 1 \rightarrow h, \; t = 1 \rightarrow n, \; q = 1 \rightarrow n$

and:

$$\sum_{k=k_o}^{k_1} (b_{jtq} - \hat{b}_{jtq}(k)) \, \hat{v}_t(k) \, u_q(k-j) \geq - \lambda_{1bjtq}^2$$

$j = 1 \rightarrow f, \; t = 1 \rightarrow n, \; q = 1 \rightarrow m$

$$\forall k_1 \geq k_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(40)$$

λ_{1aitq} and λ_{1bjtq} are finite constants which are independent of time.

Chapter Five

Introducing the adaptation laws, of the theorem, the inequalities (40) become:

$$\sum_{k=k_o}^{k_1} (a_{itq} - \alpha_{itq}(k-1)\,\hat{v}_t(k)\,\hat{y}_q(k-i) - \hat{a}_{itq}(k-1))\,\hat{v}_t(k)$$

$$\hat{y}_q(k-i) \geq -\lambda^2_{1aitq} \quad \text{.......................................(41a)}$$

$$i = 1 \rightarrow h,\ t = 1 \rightarrow n,\ q = 1 \rightarrow n$$

$$\sum_{k=k_o}^{k_1} (b_{jtq} - \beta_{jtq}(k-1)\,\hat{v}_t(k)\,u_q(k-j) - \hat{b}_{jtq}(k-1))\,\hat{v}_t(k)$$

$$u_q(k-j) \geq -\lambda^2_{1bjtq}$$

$$j = 1 \rightarrow f,\ t = 1 \rightarrow n,\ q = 1 \rightarrow m$$

$$\forall k_1 \geq k_0 \quad \text{.......................................(41b)}$$

Inequalities (41) can be shown to be verified by using the positive real lemma [14]. Consider for example, inequality (41a).

Let:

$$z(k-1) = a_{itq} - \hat{a}_{itq}(k-1)$$

$$\nu = \hat{v}_t(k)$$

and:

$$\mu(k) = \alpha^*_{itq}(k - 1)\ \hat{v}_t(k)\ \hat{y}^2_q(k - i) + (a_{itq} - \hat{a}_{itq}(k - 1))$$

$$\hat{y}_q(k - i)$$

where, $\alpha_{itq}{}^*(k-1) = -\alpha_{itq}(k-1)$ ie. $\alpha_{itq}{}^*(k-1)$ is strictly positive $\forall k$.

So:

$$\mu_1(k) = 1/2\ \alpha^*_{itq}(k - 1)\ \nu(k)\ \hat{y}^2_q(k - i) + z(k - 1)$$

$$\hat{y}_q(k - i) \ldots\ldots (42a)$$

$$\mu_2(k) = 1/2\ \alpha^*_{itq}(k - 1)\ \nu(k)\ \hat{y}^2_q(k - i) \ldots\ldots (42b)$$

Equation (42b) always satisfies a relation of the form:

$$\sum_{k=k_o}^{k_1} \mu(k)\ \nu(k) \geq -\lambda_0^2 \qquad \forall k_1 \geq k_0$$

where λ_o is a finite constant, since $\alpha_{itq}{}^*$ is always positive. It remains to prove that equation (42a) also verifies a relation of this form. Consider the correspondence:

$$\underline{x}(k) \rightarrow z(k - 1),\ \underline{A}(k) \rightarrow 1,\ \underline{B}(k) \rightarrow \alpha^*_{itq}(k - 1)\ \hat{y}_q(k - i)$$

$\underline{C}(k) \to \hat{y}_q(k - i)$, $\underline{J}(k) \to 1/2\ \alpha^*_{itq}(k - 1)\ \hat{y}^2_q(k - i)$

$\underline{P}(k) \to 1/\alpha^*_{itq}(k - 1)$

where, from the relation which describes the variation of $\alpha_{itq}(k)$ with time (see Theorem 5.3), one has that:

$$\alpha_{itq}(k) = \alpha_{itq}(k - 1) + 1/\lambda[(\alpha_{itq}(k - 1)\ \hat{y}_q(k - i))^2 /$$

$$(1 - 1/\lambda\ \alpha_{itq}(k - 1)\ \hat{y}^2_q(k - i))]$$

$$1/\alpha_{itq}(k) = 1/\alpha_{itq}(k - 1) - 1/\lambda\ \hat{y}^2_q(k - i)$$

or:

$$1/\alpha^*_{itq}(k) = 1/\alpha^*_{itq}(k - 1) + 1/\lambda\ \hat{y}^2_q(k - i) \quad \text{.........(43)}$$

ie. $\underline{\underline{P}}(k)$ is always positive definite.

Now one can use the positivity lemma for time-varying discrete systems.

Consider the time-varying discrete system:

$$\underline{x}(k + 1) = \underline{A}(k)\ \underline{x}(k) + \underline{B}(k)\ \underline{u}(k) \quad \text{......................(44)}$$

$$\underline{v}(k) = \underline{C}(k)\ \underline{x}(k) + \underline{J}(k)\ \underline{u}(k) \quad \text{.........................(45)}$$

where $\underline{\underline{A}}(k)$, $\underline{\underline{B}}(k)$, $\underline{\underline{C}}(k)$ and $\underline{\underline{J}}(k)$ are matrices defined $\forall k$.

The positivity lemma [14,25] states that the system represented by equations (44) and (45) is positive (which implies that an inequality of the type (37) is satisfied) if there exists two sequences of definite positive matrices $\underline{\underline{L}}(k)$ and $\underline{\underline{W}}(k)$ and a sequence of definite positive matrices, $\underline{\underline{P}}(k)$, verifying, $\forall k \geq 0$, the system equations:

$$\underline{\underline{A}}^T(k)\ \underline{\underline{P}}(k+1)\ \underline{\underline{A}}(k) - \underline{\underline{P}}(k) = -\underline{\underline{L}}(k)\ \underline{\underline{L}}^T(k) \quad \ldots\ldots\ldots\ldots(46)$$

$$\underline{\underline{B}}^T(k)\ \underline{\underline{P}}(k+1)\ \underline{\underline{A}}(k) - \underline{\underline{C}}(k) = -\underline{\underline{W}}^T(k)\ \underline{\underline{L}}^T(k) \quad \ldots\ldots\ldots\ldots(47)$$

$$\underline{\underline{W}}^T(k)\ \underline{\underline{W}}(k) = \underline{\underline{J}}(k) + \underline{\underline{J}}^T(k) - \underline{\underline{B}}^T(k)\ \underline{\underline{P}}(k+1)\ \underline{\underline{B}}(k) \quad \ldots\ldots(48)$$

If $\underline{\underline{W}}(k)$ is restricted to a sequence of invertible matrices, then (46) - (48) are reduced to:

$$\underline{\underline{A}}^T(k)\ \underline{\underline{P}}(k+1)\ \underline{\underline{A}}(k) - \underline{\underline{P}}(k) = -(\underline{\underline{A}}^T(k)\ \underline{\underline{P}}(k+1)\ \underline{\underline{B}}(k) - \underline{\underline{C}}^T(k))$$

$$(\underline{\underline{J}}^T(k) + \underline{\underline{J}}(k) - \underline{\underline{B}}^T(k)\ \underline{\underline{P}}(k+1)\ \underline{\underline{B}}(k))^{-1}$$

$$(\underline{\underline{B}}^T(k)\ \underline{\underline{P}}(k+1)\ \underline{\underline{A}}(k) - \underline{\underline{C}}(k)) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(49)$$

Theorem 5.3 is thus proved.

Unfortunately, the algorithm again contains an inherent practical difficulty, that is, it is desired to calculate $\hat{\underline{v}}(k)$ from equation (27). $\hat{\underline{\varepsilon}}(k)$ is not available, however, since it is a function of the, as yet, unknown $\hat{\underline{\underline{A}}}_i(k)$ and $\hat{\underline{\underline{B}}}_j(k)$'s. An outline of an algorithm to calculate $\hat{\underline{v}}(k)$ must, therefore, be included.

From equations (25) - (27), one obtains:

$$\hat{\underline{v}}(k) = \hat{\underline{y}}(k) - \underline{y}(k) + \sum_{i=1}^{p_1} \underline{\underline{D}}_i \, \hat{\underline{\varepsilon}}(k-i) \quad \ldots\ldots\ldots\ldots\ldots\ldots (50)$$

and:

$$\hat{\underline{v}}(k) = -\underline{y}(k) + \sum_{i=1}^{h} \hat{\underline{\underline{A}}}_i(k)\, \hat{\underline{y}}(k-i) + \sum_{j=1}^{f} \hat{\underline{\underline{B}}}_j(k)\, \underline{u}(k-j) + \sum_{i=1}^{p_1} \underline{\underline{D}}_i \, \hat{\underline{\varepsilon}}(k-i) \quad \ldots\ldots\ldots\ldots\ldots\ldots (51)$$

or in scalar form:

$$\hat{v}_t(k) = -y_t(k) + \sum_{i=1}^{h}\sum_{q=1}^{n} \hat{a}_{itq}(k)\, \hat{y}_q(k-i) + \sum_{j=1}^{f}\sum_{q=1}^{m} \hat{b}_{jtq}(k)\, u_q(k-j) + \sum_{i=1}^{p_1}\sum_{q=1}^{n} d_{itq}\, \hat{\varepsilon}_q(k-i) \quad .(52)$$

$t = 1 \rightarrow n$

Introducing the parameter adaptation laws, of

Theorem 5.3, in equation (52), one has:

$$\hat{v}_t(k) = -y_t(k) + \sum_{i=1}^{h}\sum_{q=1}^{n} (\alpha_{itq}(k-1)\,\hat{v}_t(k)\,\hat{y}_q(k-i) + \hat{a}_{itq}(k-1))\,\hat{y}_q(k-i) + \sum_{j=1}^{f}\sum_{q=1}^{m} (\beta_{jtq}(k-1)\,\hat{v}_t(k)\,u_q(k-j) + \hat{b}_{jtq}(k-1))\,u_q(k-j) + \sum_{i=1}^{p_1}\sum_{q=1}^{n} d_{itq}\,\hat{\varepsilon}_q(k-i) \quad \text{......(53)}$$

$t = 1 \rightarrow n$

So that with some algebraic manipulation:

$$\hat{v}_t(k) = X/Y \quad \text{......(54)}$$

where:

$$X = -y_t(k) + \sum_{i=1}^{h}\sum_{q=1}^{n} \hat{a}_{itq}(k-1)\,\hat{y}_q(k-i) + \sum_{j=1}^{f}\sum_{q=1}^{m} \hat{b}_{jtq}(k-1)\,u_q(k-j) + \sum_{i=1}^{p_1}\sum_{q=1}^{n} d_{itq}\,\hat{\varepsilon}_q(k-i) \quad \text{......(55)}$$

and:

$$Y = 1 - (\sum_{i=1}^{h}\sum_{q=1}^{n} \alpha_{itq}(k-1)\,\hat{y}_q^2(k-i) +$$

$$\sum_{j=1}^{f} \sum_{q=1}^{m} \beta_{jtq}(k-1)\, u_q^2(k-j)) \ldots\ldots\ldots\ldots\ldots\ldots\ldots(56)$$

5.4.2 On a SISO Closed-Loop Identification Scheme

Finally, Ljung [26] has shown that Landau's scheme obtains parameter estimates convergent to the true ones with probability one, for SISO systems, provided that:

1. the input sequence $\{u(k)\}$ and the measurement noise sequence $\{\eta(k)\}$ are stationary stochastic processes with rational spectral densities and such that all moments exist;
2. $u(t)$ is independent of $\eta(s)$, $s>t$;
3. the open-loop plant must be asymptotically stable (as well as the closed-loop system if feedback is present)
4. $D(z) = 1$.

If condition 4 is not met, it can still be shown that the output convergence is unaffected in the presence of noise obscured measurements [14].

5.5 Extensions

The theory included in the previous sections has considered an example of a practical control system, based

on prior hyperstable identification. There are certain extensions which need to be outlined for this scheme to be fully acceptable in the light of actual operational conditions. These can be categorized in the following way:

1.

The stability of the control plus identification scheme, in the presence of disturbances (measurable or not) needs to be shown.

2.

Since the hyperstability theory is based on nonlinear feedback stability work, it should be possible to include certain classes of plant nonlinearity in the formulation.

3.

The calculation of the control block parameters can become quite horrendous for large systems, in that, in the general sense the method requires the inversion of large matrix polynomials. Whilst this is technically feasible, via Leverrier's algorithm [29 - 30], the task, even for a digital computer, is not a trivial one and in an on-line calculation mode may even become critical. Some simplifications must therefore be considered and, finally,

on prior hyperstable identification. There are certain extensions which need to be outlined for this scheme to be fully acceptable in the light of actual operational conditions. These can be categorized in the following way:

1.

The stability of the control plus identification scheme, in the presence of disturbances (measurable or not) needs to be shown.

2.

Since the hyperstability theory is based on nonlinear feedback stability work, it should be possible to include certain classes of plant nonlinearity in the formulation.

3.

The calculation of the control block parameters can become quite horrendous for large systems, in that, in the general sense the method requires the inversion of large matrix polynomials. Whilst this is technically feasible, via Leverrier's algorithm [29 - 30], the task, even for a digital computer, is not a trivial one and in an on-line calculation mode may even become critical. Some simplifications must therefore be considered and, finally,

4.

The problems of right-half plane zeroes in the identification model and right-half plane poles in the plant description must be looked at from a practical point of view.

Since these points are not critical to the development, which this chapter has attempted to present, all, except the last of these questions are discussed in Appendices.

The last remark relates to the manner in which the control input is calculated. At present only one method exists for this computation, ie. the adaptive plant inverse technique. Since this method makes use of the inverse of the identification model, right half plane model zeroes will result in an unstable control input specification. This can be disasterous in practical situations although this is not a theoretical restriction.

5.6 The Apriori Design

Having noted the theoretical development as a continuing entity, with respect to new output feedback control techniques, the apriori design for the example problem can be seen to reduce to:

(i) Design of the compensators, $\underline{\underline{D}}_1(z)$ and $\underline{\underline{D}}_2(z)$;

(ii) Choice of the adaptive gains for the identification and control schemes;

(iii) Initialization of the adaptive algorithms and finally,

(iv) Specification of the reference model.

5.6.1 Compensator Designs

For the general scheme, depicted as Figure 5.8, two compensators or linear filters are required. The design of each is slightly different so that it is expedient to consider them as two separate problems.

There are essentially three possible design methods for the filters:

(i) algebraic positivity conditions for continuous time functions obtained via the bilinear transformation, $z=1+w/1-w$, may be considered.[1]

---

[1]

Karmarkar [31 - 32] has presented such a compensator design algorithm which ensures that the operation of the discrete model reference adaptive system is insensitive to initial parameter estimates, within a maximized hypercube in the model parameter space.

(ii) state representation of the equivalent linear block, of Figure 5.4, and use of the positivity lemma [25], plus the solution of a Liapunov equation. This approach has been used in most of the practical work reported by Bethoux, Courtiol and Landau [15 - 16] and,

(iii) Adhoc relationships which guarantee that $\underline{\underline{G}}(z)$ will be known positive real, $\forall k$.

5.6.1(a) The Primary Control System

The two methods that will be considered for the design of the linear compensator, $\underline{\underline{D}}_1(z)$, are the approach taken by Karmarkar, and possible adhoc solutions.

$\underline{\underline{D}}_1(z)$ is designed in order that:

$$\underline{\underline{D}}_1(z)\,(\underline{\underline{I}} - \sum_{i=1}^{h_1} \underline{\underline{A}}_{im}\, z^{-i})^{-1}$$

be strictly positive real; where $\underline{\underline{A}}_{im}$ are diagonal, known reference model matrices.

From the definition of $\underline{v}(k)$ (Equation (7)), it is possible to state that $\underline{\underline{D}}_1(z)$ is diagonal.

a) Systematic Design

From the nature of the problem, it is obvious that the design reduces to n SISO procedures, since for

$\underline{\underline{G}}(z)$ (diagonal) to be strictly positive real, it is sufficient that $g_{ii}(z)$, $(i=1 \rightarrow n)$ be strictly positive real.

Consider a typical element:

$$g(z) = \sum_{i=o}^{p_1} \alpha_i \ z^{-i} / (1 - \sum_{j=1}^{h_1} a_j \ z^{-j}) \quad \text{.................(57)}$$

If the bilinear transformation:

$$z = 1 + w/1 - w \quad \text{.....................................(58)}$$

is utilized, then equation (57) becomes:

$$g(w) = \sum_{i=o}^{h_1} e_i \ w^i / \sum_{j=o}^{h_1} b_j \ w^j \quad \text{..........................(59)}$$

Theorem 5.4 may then be stated as:

<u>Theorem 5.4</u> [31 - 33]

For g(w) to be strictly positive real, it is necessary and sufficient that:

(i) g(w) be non-degenerate;

(ii) the polynomial $\sum_{j=o}^{h_1} b_j w^j$ be strictly Hurwitz and,

(iii) Re $g(jx)>0$, $\forall x \geq 0$.

Condition (ii) implies that the determinants Δ_{h_1}, Δ_{h_1-1}, ..., Δ_1 must all be greater than zero, where:

$$\Delta_{h_1} = \begin{bmatrix} b_{h_1-1} & b_{h_1} & 0 & \cdots & & & & 0 \\ b_{h_1-3} & b_{h_1-2} & b_{h_1-1} & b_{h_1} & 0 & \cdots & & 0 \\ \cdot & & & & & & & \cdot \\ \cdot & & & & & & & \cdot \\ \cdot & & & & & & & \cdot \\ \cdot & & & & & & & \cdot \\ \cdot & & & & & & & \cdot \\ b_1 & 0 & \cdots & & 0 & b_0 & b_1 & b_2 \\ 0 & \cdots & & & & & 0 & b_0 \end{bmatrix}$$

and Δ_{h_1-1} may be obtained from Δ_{h_1} by removing the last row and the last column of Δ_{h_1}. The convention is also adopted, here, that all elements with negative subscripts are zero. This approach presupposes that b_{h_1} is at least positive [34].

In the practical approach, this procedure need only be done once, and merely signifies that the reference model is stable.

The third condition results in the inequality:

$$\P(x^2) = \sum_{i=o}^{m} c_i \, x^{2i} > 0 \qquad \forall x \geq 0$$

$$m = 2h_1 \quad \text{.......................................(60)}$$

Chapter Five

Sufficient, though perhaps restrictive, conditions such that (60) is satisfied are:

$$c_k = \sum_{j=o}^{N} (-1)^j \, b_{N-j} \, e_j > 0 \qquad k \text{ even}$$

and:

$$c_k = \sum_{j=o}^{N} (-1)^j \, b_{N-j} \, e_j < 0 \qquad k \text{ odd}$$

$$N = 2k, \; k = 0, 1, \ldots, h_1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(61)$$

It is obvious, from the relation between (58) and (59), that:

$$t_j = f(a_i) \qquad i = 0 \rightarrow h_1, \; j = 0 \rightarrow h_1$$

and:

$$e_i = f(d_j) \qquad i = 0 \rightarrow h_1, \; j = 0 \rightarrow p_1$$

thus:

$$c_k = f_k(a_i, d_j) \qquad k = 0 \rightarrow h_1, \; i = 0 \rightarrow h_1, \; j = 0 \rightarrow p_1$$

where $f(a_i)$, $f(d_j)$ and $f_k(a_i, d_j)$ are all linear functions, since the a_i's are known.

Karmarkar, at this point, formulates the design problem as a mathematical programming exercise:

Minimize $(-r)$ subject to:

$$f_k(a_i - lr, d_j - mr) > 0 \qquad i = 0 \rightarrow h_1,\ j = 0 \rightarrow p_1$$

and

k even

$$f_k(a_i - lr, d_j - mr) < 0 \qquad k \text{ odd}$$

$$k = 0 \rightarrow h_1$$

where l and m take on all possible combinations among (-1,0,1). Thus, each function is evaluated at all the vertices of the hypercube.

The resultant solution vector is:

$$(r^*, d_j^*) \qquad j = 0 \rightarrow p_1$$

d_j^* represents the desired compensator parameters and $2r^*$ is the maximized hypercube edge in the model parameter space.

It will be noticed, that strictly any $\underline{\underline{D}}_1(z)$ which satisfies the inequalities (61) would suffice for the control system (provided that the reference model is stable) and, thus, the mathematical programming procedure is, strictly, unnecessary. However, the approach also indicates

over what range of model-space the compensator is valid and thus may be of help if the model parameters were to be changed.

b) Adhoc Design

Since the reference model parameter matrices can be assumed to be known, $\forall k$, it is obvious that $\underline{\underline{D}}_1(z)$ could be chosen such that:

$$\underline{\underline{X}}(z) = \underline{\underline{D}}_1(z)\left(\underline{\underline{I}} - \sum_{i=1}^{h_1} \underline{\underline{A}}_{im} z^{-i}\right)^{-1}$$

is strictly positive real, where $\underline{\underline{X}}(z)$ is known. In particular, if $\underline{\underline{D}}_1 = (\underline{\underline{I}} - \sum_{i=1}^{h_1} \underline{\underline{A}}_{im} z^{-i})$ then, $\underline{\underline{X}}(z) = \underline{\underline{I}}$.

5.6.1(b) Identification Scheme

a) Systematic Design

This time, it is required to design a compensator, $\underline{\underline{D}}_2(z)$, such that [26]:

$$\underline{\underline{G}}(z) = \underline{\underline{D}}_2(z)\left(\underline{\underline{I}} - \sum_{i=1}^{h} \underline{\underline{A}}_i z^{-i}\right)^{-1} - 1/2\lambda\, \underline{\underline{I}}$$

$$(\lambda > 0.5)$$

is strictly positive real.

Once again the $\underline{\underline{A}}_i$'s and $\underline{\underline{D}}_2(z)$ are diagonal, so that it is possible to consider n individual SISO problems. On this

(ii) $\Delta_h, \Delta_{h-1}, \ldots\ldots, \Delta_1$ are all greater than zero, where:

$$\Delta_h = \begin{bmatrix} b_{h-1} & b_h & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ b_{h-3} & b_{h-2} & b_{h-1} & b_h & 0 & \cdots & \cdots & 0 \\ \cdot & & & & & & & \cdot \\ \cdot & & & & & & & \cdot \\ \cdot & & & & & & & \cdot \\ \cdot & & & & & & & \cdot \\ \cdot & & & & & & & \cdot \\ b_1 & 0 & \cdots & \cdots & 0 & b_0 & b_1 & b_2 \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & b_0 \end{bmatrix}$$

and Δ_{h-1} is obtained from Δ_h by removing the last row and column of Δ_h. Also $b_h>0$ and, finally,

(iii) conditions (61) hold.

The inequalities defined by conditions (ii) and (iii) may be written as:

$$s_l = f(a_i) \qquad l = 1 \rightarrow h,\ i = 0 \rightarrow h$$

and:

$$c_k = f_k(a_i, d_j) > 0 \qquad k \text{ even}$$

$$c_k = f_k(a_i, d_j) < 0 \qquad k \text{ odd}$$

$$k = 0 \rightarrow h, \quad i = 0 \rightarrow h, \quad j = 0 \rightarrow p_1$$

where, in general, $f_l(a_i)$ are linear functions and $f_k(a_i, d_j)$ are nonlinear functions, since the a_i's and d_j's are unknown.

There is a simplification, however, in the procedure to be considered in this chapter. Firstly, for the identification scheme $\underline{\underline{D}}_2(z)$ is assumed known and:

$$\underline{\underline{D}}_2(z) = \underline{\underline{I}}$$

ie. for each diagonal element, of the compensator $d_j=1$ and $d_j=0$, $(j=1 \rightarrow p_1)$.

Thus, the problem can be cast as a mathematical programming exercise:

Minimize $(-r)$ subject to:

$$f_l(a_i - lr) > 0 \qquad l = 1 \rightarrow h, \; i = 0 \rightarrow h$$

and:

$$f_k(a_i - pr, d_j - mr) > 0 \qquad k \text{ even}$$

$$f_k(a_i - pr, d_j - mr) < 0 \qquad k \text{ odd}$$

$$k = 0 \rightarrow h, \; i = 0 \rightarrow h, \; j = 0 \rightarrow p_1$$

where l, p, and m take on all possible combinations among (-1,0,1).

The solution vector is (r^*, a_j^*), $(j=1 \rightarrow h)$ where a_j^* represent the "best" starting estimates for the identification model parameters.

b) Time-Varying Filter

By choosing $\underline{\underline{D}}_2(z)$ as a sufficiently good apriori estimate of $(\underline{\underline{I}} - \sum_{i=1}^{h} \underline{\underline{A}}_i z^{-i})$, the condition that:

$$\underline{\underline{G}}(z) = \underline{\underline{D}}_2(z)\,(\underline{\underline{I}} - \sum_{i=1}^{h} \underline{\underline{A}}_i\, z^{-i})^{-1} - 1/2\,\lambda\,\underline{\underline{I}}$$

be strictly positive real discrete, can always be satisfied.

A rather natural way of using this idea is to let $\underline{\underline{D}}_2(z)$ be time-varying and let it equal the current estimate of $(\underline{\underline{I}} - \sum_{i=1}^{h} \underline{\underline{A}}_i z^{-i})$ at each sampling instant. This approach has, indeed been shown to have a beneficial effect on the convergence properties of the algorithm [26].

5.6.2 <u>Reference Model Design and the Specificiation of the Adaptive Gains With Consideration of the Adaptive Model Initialization Problem.</u>

The specifications of these three parameter groups are,

theoretically, quite arbitrary. In practice, however, there will be certain choices of the parameters which will give an optimum performance, in some sense. It is presumed that a performance index has been chosen to regulate this design somewhat, in light of the accepted practice for individual applications.

Some general points which are process-independent can be stated as desired objectives.

1. The reference model would normally be chosen to represent a rational and realizable dynamical representaion of the plant. It would not, for instance, be sensible to specify a reference model which has a response many times faster than the open-loop plant. Further, in view of the control configuration envisaged, it would be expected that the steady-state response of the reference model would correspond to the setpoint input vector. This implies of course, that the dynamic response of the model is stable although, theoretically, this need not be so. $\underline{\underline{D}}_1(z)$ may still be designed in such a way that the linear part of the equivalent nonlinear feedback system is pcsitive real even though the reference model is unstable.

2. As a general rule, the identification scheme is required to converge faster than the outer primary control adaptive loop. This requirement would need to be considered when specifying the adaptive loop gains, although some freedom is allowed, in that, the gains of the outer loop may be changed at discrete intervals on-line.

3. Finally, as much information about the plant as is known apriori should be included in the system description. It is not recommended that the adaptive loops be initialized in a random manner, for even though the overall system is stable convergence times can be theoretically, infinite.

There is one more practical problem that remains to be discussed before the relationship between the input-output formulations of Chapter Four and the general scheme of this chapter may be investigated.

5.6.3 Unstable, Non-Minimum Phase Systems

The design presented above, is strictly applicable (as are all designs of this nature) to stable plants. There are, however, some adhoc methods of utilizing these techniques when this condition is violated. This is usually at the cost of more apriori information.

Firstly, if the plant is unstable, but an approximate knowledge of the poles is available (as a result of some open-loop identification runs, say), then it is possible to analytically "shift" the open-loop plant into a stable region via the so-called "pole-shifting technique" [34,35].

For SISO systems, this procedure may be briefly described as consisting of the transformation:

$$u^*(t) = u(t) - ay(t)$$

where y is the output of the plant and a is a constant negative feedback gain. The linear open-loop transfer function, G(z), is then transformed to:

$$G_a(z) = G(z)/(1 + aG(z)) \quad \text{.............................(64)}$$

The design may then proceed as before. This technique has also received a good deal of attention, in the literature, for multivariable systems [36 - 37].

Unfortunately it is not possible to shift right half plane zeroes, analytically, using the adaptive configuration as shown in Figure 5.8 and usually recourse is taken to pole-zero cancellation methods [20, 34, 38]. For the present adaptive control scheme, however, there are some

adhoc considerations which may be taken into account. Firstly, if it may be assumed that accurate parameter identification is not a prime goal then the identification model may be constrained to a minimum-phase region. This would allow the output convergence of the identification scheme without the risk of an unstable control input being generated. Secondly, the gains of the outer adaptive loop may be adjusted to achieve the same ends. Both inherently, govern how fast the system approaches the desired trajectory.

5.7 The Relationship Between the General Augmented Output Control Technique and the Method of Martin-Sanchez

The similarities between the general augmented output control technique and the method of Martin-Sanchez can best be outlined with the use of a block diagram. Several simplifications must first, however, be included in the mathematical development of the general scheme:

1. the adaptive gains for the identification block must be considered, constant, strictly negative quantities;
2. the reference model must admit the same dynamical form as the driver block and,

3. the control action should be obtained via the equation:

$$\underline{u}(k) = \hat{\underline{\underline{B}}}_1^{-1}(k)[\underline{y}'(k+1) - \sum_{i=1}^{h} \hat{\underline{\underline{A}}}_i\, \underline{y}(k-i+1) - \sum_{j=2}^{f} \hat{\underline{\underline{B}}}_j(k)\, \underline{u}(k-j+1)] \quad \ldots\ldots(65)$$

where $\underline{y}'(k+1)$ is given by:

$$\underline{y}'(k+1) = \sum_{i=1}^{h'} \underline{\underline{A}}_i'\, \underline{y}'(k-i+1) + \sum_{j=1}^{f'} \underline{\underline{B}}_j'\, \underline{r}(k-j+1) + \quad \ldots\ldots(66)$$

$\underline{y}'(k-i+1)$ are the n dimensional driver block output vectors,

$\underline{r}(k-j+1)$ represent the n dimensional setpoint vectors.

$\underline{\underline{A}}_i'$, $\underline{\underline{B}}_j'$ and $\underline{\underline{D}}_1'$ are parameter matrices of appropriate order.

Given these simplifications, the general scheme can be depicted as in Figure 5.10.

The approach can then be shown to reduce to Martin-Sanchez's scheme for:

(i) $\underline{\underline{D}}_1(z)=0$

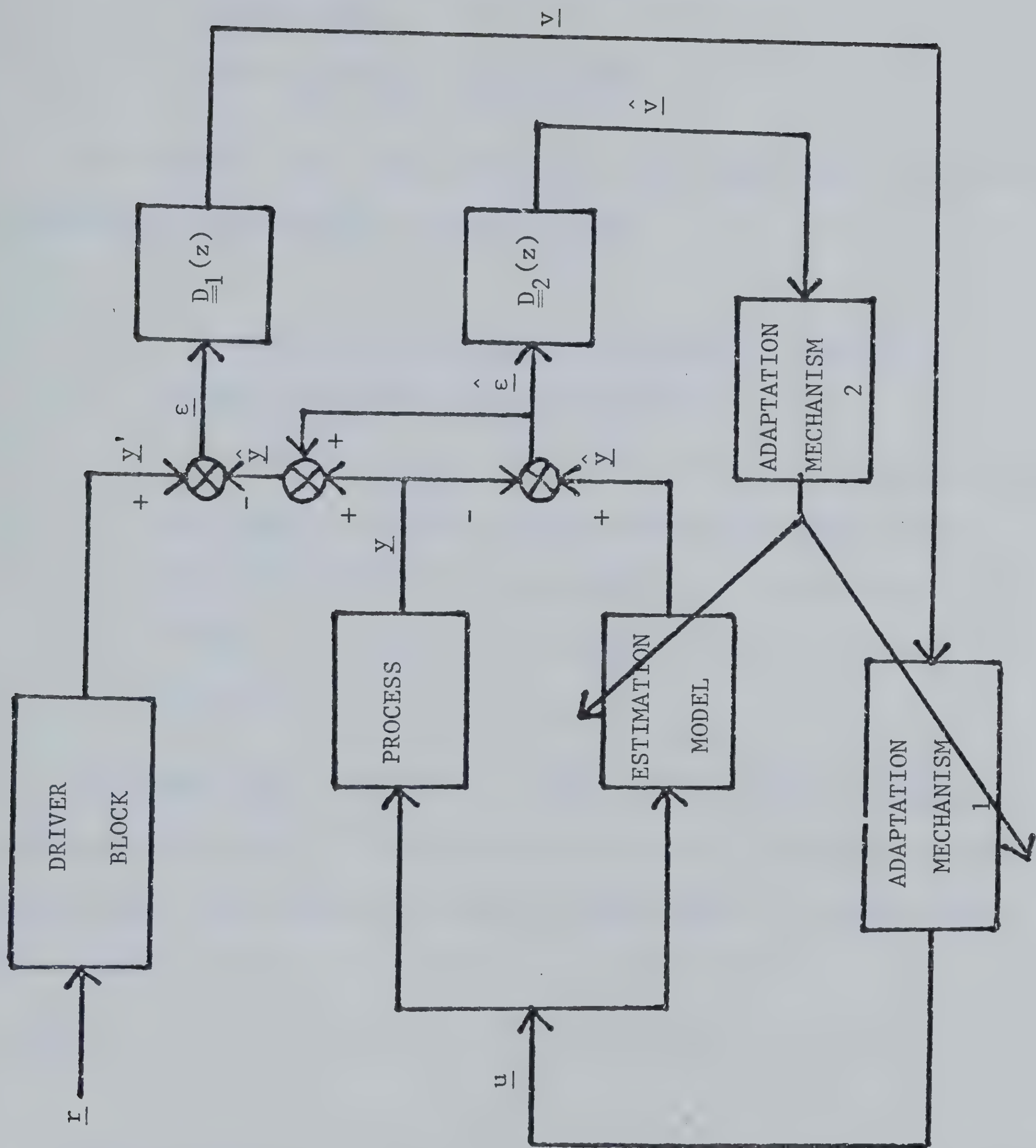


FIGURE 5.10 SIMPLIFIED AUGMENTED OUTPUT MODEL REFERENCE ADAPTIVE CONTROL SYSTEM

(ii) $\underline{y}'(k-i+1)$ and $\hat{\underline{y}}(k-i)$ considered to be plant outputs and,

(iii) $\underline{\underline{D}}_2(z)=(\underline{\underline{I}} - \sum_{i=1}^{h} \hat{\underline{\underline{A}}}_i(k)z^{-i})$.

As before, the conclusion is that this system will be asymptotic hyperstable provided that:

1) the open-loop plant contains no right-half plane poles (ie. it is stable);

2) the identification model has order greater than or equal to that of the open-loop plant, and

3) the identification model does not enter a region in which it contains right half plane zeroes.

Although only the case of control parameter adaptation has been detailed, it is possible to carry out the control task using the signal synthesis technique of Chapter Four, such that:

$$\underline{u}(k) = \hat{\underline{B}}_1^{-1}(k)[\ \underline{y}_d(k+1) - \sum_{i=1}^{h} \hat{\underline{A}}_i(k)\ \hat{\underline{y}}(k-i+1) - \sum_{j=2}^{f} \hat{\underline{B}}_j(k)\ \underline{u}(k-j+1) - \sum_{l=1}^{g} \hat{\underline{D}}_l(k)\ \underline{\xi}(k-l+1)\ ] \quad \ldots\ldots(67)$$

where $\underline{y}_d(k+1)$ is the desired output at the next sampling instant. The latter may be obtained from the model

reference output or via the primary adaptation loop parameters. In equation (67), the influence of measurable disturbances has been added for generality.

It may be noted that these two techniques are identical for the case in which the adaptation of all variables is carried out at each sampling instant. The former method, however, has the advantage of allowing for differing sample times between the adaptation loops and the primary control loop. This has important ramifications for industrial implementation although no analytical proof of stability, in between adaptation times, is available at present.

5.8 Conclusions

A very general adaptive control configuration has been described, in this chapter. This technique was considered as a subset of the overall conceptual approach which may be described as apriori identification + control. The scheme is as general as possible, and several practical aspects have been discussed.

The proof of hyperstability, for the general scheme, is a powerful result implying as it does, asymptotic stability in the large [22].

A list of practical advantages of the proposed method

could include:

1. The entire control plus identification scheme has been designed with guaranteed hyperstability.

2. In the absence of unmeasurable disturbances, an explicit identification of the open-loop plant can be obtained (although no guarantee of parameter convergence for MIMO systems is available).

3. Little apriori information is required.

4. The flexibility of the transfer function approach is maintained through use of the input-output formulation. This negates, effectively, any problems of state inaccessibility, etc.

5. The method is able to handle square or non-square plants.

6. At the cost of a little more information, unstable plants may be included.

7. $\underline{\underline{K}}_1$ allows the possibility of dynamic decoupling of

the closed-loop plant (see Appendix 5.2 -- equation (23)).

8. Disturbances do not affect stability, although unmeasurable disturbances will upset the identification problem.

9. The controls to the plant can be constrained [55].

10. Known information about the plant may be incorporated since the adaptation loops can be initialized using this data and the gains adjusted accordingly.

11. Some nonlinear behaviour of the plant can be allowed for and, finally,

12. The provision has been allowed for different sampling intervals for the primary control loop and the adaptive loops.

CHAPTER SIX

Simulation of a Hyperstable Adaptive Control Scheme

6.1 Introduction

The salient features of the design of hyperstable systems are difficult to pinpoint using a purely mathematical presentation and hence, recourse has been taken, here, to simulation studies.

In this chapter, there will be considered five systems -- two single-input single-output (SISO) and three multi-input multi-output (MIMO) configurations.

Because of the complicated nature in which even simple MIMO systems interact, the major effort has been directed towards the SISO simulation study; the MIMO examples being included for the sake of completeness. This does not restrict the presentation in any way, since the effects of the design parameters should be more apparent with this approach.

6.2 Organization of the Chapter

This chapter is best read throughout, rather than piecemeal, since interpretation of the simulation results depends, to a certain extent, on an understanding of the implications of each of the parameter sets, investigated.

As was mentioned in the introduction, the major thrust has been directed towards an investigation of the effects of the design parameters on the performance of the adaptive scheme. As such, the SISO examples are considered first, utilizing three modes of explanation.

Firstly, in each case, the major parameter effect to be considered will be mentioned as a subtitle. Secondly, reference will be drawn to two or more particular sets of figures which show the effect in question. Lastly, a textual interpretation of the diagrams will be presented with brief references to design implications.

The MIMO examples appear towards the end of the chapter. Three systems have been referred to:

(i) A second order system[1] [1] with open-loop transfer matrix:

---

[1]

Note this formulation is equivalent to a state-space representation.

$$\underline{G}_1(s) = \begin{bmatrix} 2/(s+1) & 1/(s+1) \\ 1/(s+1) & 1/(s+1) \end{bmatrix}$$

and poles at (-1,-1);

(ii) A fourth order system with open-loop transfer matrix:

$$\underline{G}_2(s) = \begin{bmatrix} 2/(s+1) & 1/(s+1) \\ 1/(s+1) & 1/(s+10)^2 \end{bmatrix}$$

and poles at (-1,-1,-10,-10) and zeroes being $-9 \pm \sqrt{7}j$, respectively, and lastly,

(iii) A sixth order system with open-loop transfer matrix:

$$\underline{\underline{G}}_3(s) = \begin{bmatrix} 1/(s+1)(5s+1) & 1/(0.1s+1) \\ 1/(0.1s+1) & 1/(5s+1)(0.2s+1) \end{bmatrix}$$

and poles at (-1,-0.2,-0.2,-5,-10,-10) and zeroes at (0.,-0.5,-0.9,-5,0).

Before proceeding, however, it would be practicable to review the adaptive system configuration and the computer algorithm design.

6.3 Hyperstable Adaptive System Configuration

The simulations have been carried out using a configuration such as that depicted in Figure 6.1

In all cases, $\underline{\underline{D}}_1(z)$ and $\underline{\underline{D}}_2(z)$ are equal to the identity matrix, $\underline{\underline{I}}$, and the control action has been calculated using an adaptive inverse technique. The majority of the

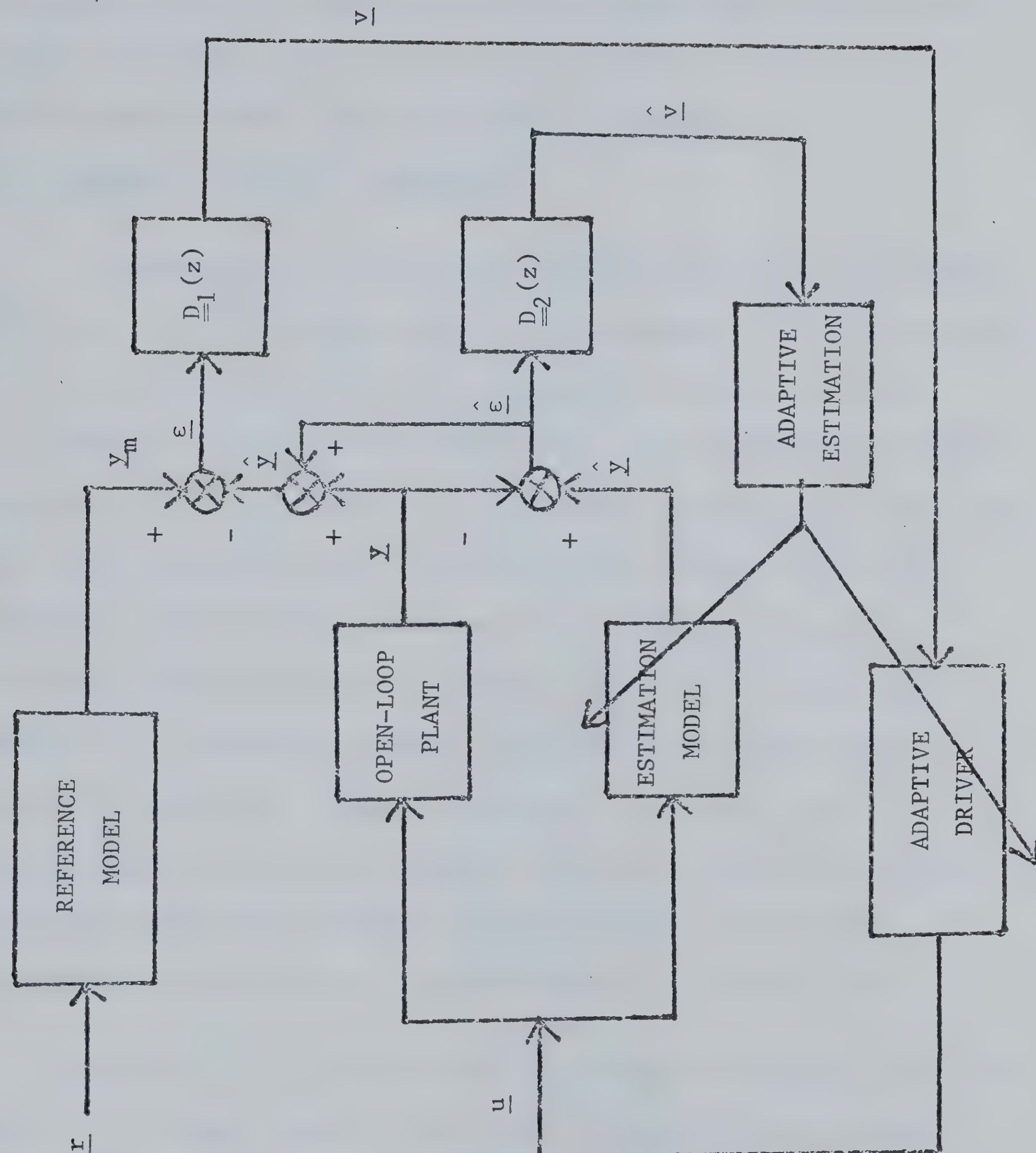


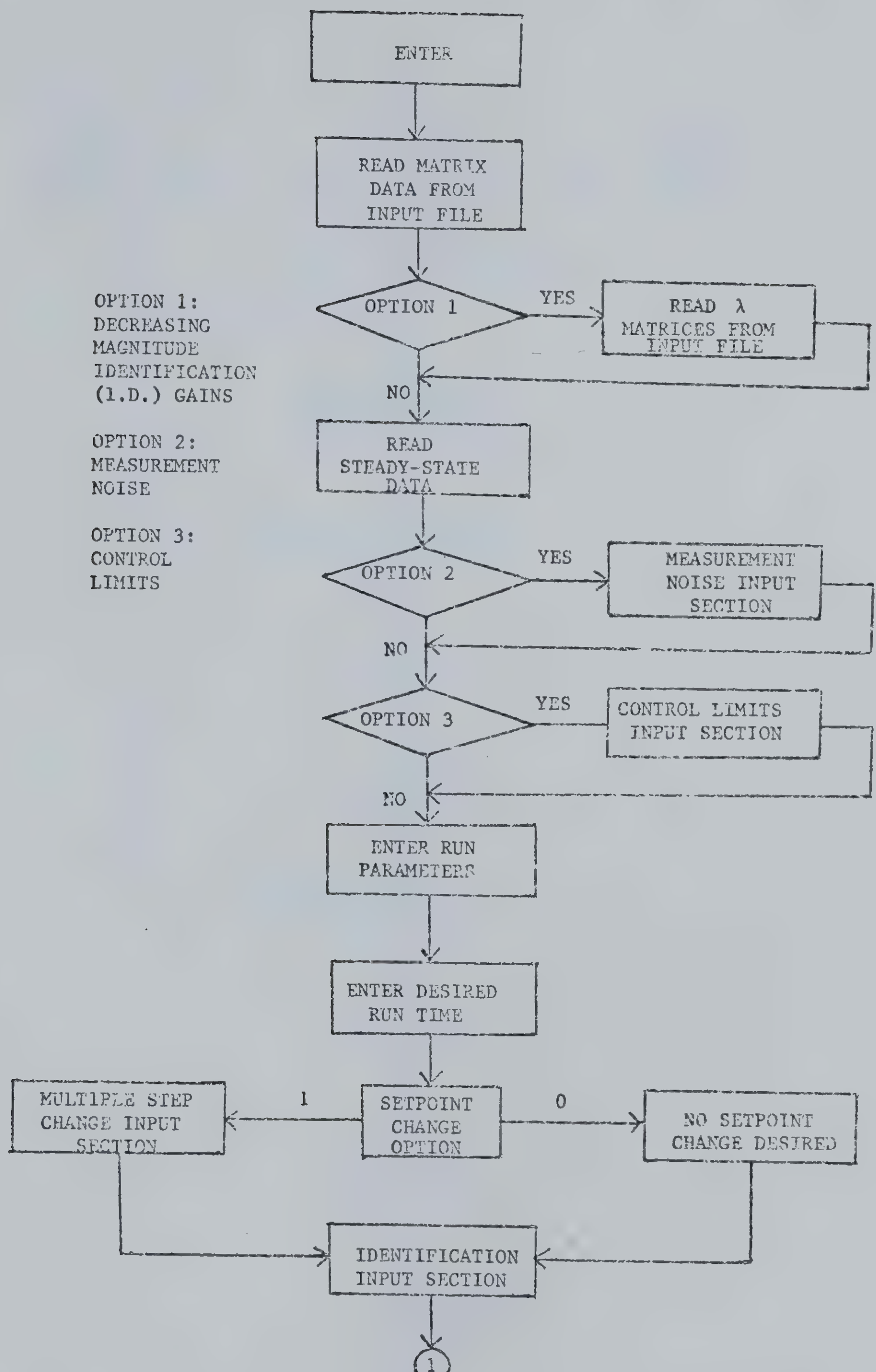
FIGURE 6.1 ADAPTIVE SYSTEM CONFIGURATION USED FOR SIMULATION STUDIES

simulations have also included a measurement noise input which is simply added to the actual plant output before being used by the identification mechanism.

6.4 Adaptive Control Programs

A series of user-oriented control programs have been written for use on the University's Amdahl 470 V/6 computer. In particular, two sets of programs -- IDENT1 -- and -- LANDAU -- were used to implement the simulation studies reported in this chapter. The former program was used only for SISO examples, as it was designed solely for this purpose. The latter, however, is a general purpose multivariable simulation program which allows the user a number of important options, including multiple step setpoint changes, control constraint options, and an interface secticn to a general plotting program -- PLOT. An illustrative flow diagram demonstrating the options presently available is included here as Figure 6.2.

The user is prompted by the program tc enter relevant data at various points and error diagnostics have been included to flag certain illegal actions. The data file may then be editted and the program restarted. At present -- PLOT -- must be run as a separate module from -- LANDAU -- as it has been designed for general plotting tasks not


ENTER
READ MATRIX DATA FROM INPUT FILE
OPTION 1: DECREASING MAGNITUDE IDENTIFICATION (I.D.) GAINS
OPTION 2: MEASUREMENT NOISE
OPTION 3: CONTROL LIMITS
OPTION 1
YES
READ λ MATRICES FROM INPUT FILE
NO
READ STEADY-STATE DATA
OPTION 2
YES
MEASUREMENT NOISE INPUT SECTION
NO
OPTION 3
YES
CONTROL LIMITS INPUT SECTION
NO
ENTER RUN PARAMETERS
ENTER DESIRED RUN TIME
MULTIPLE STEP CHANGE INPUT SECTION
1
SETPOINT CHANGE OPTION
0
NO SETPOINT CHANGE DESIRED
IDENTIFICATION INPUT SECTION
1

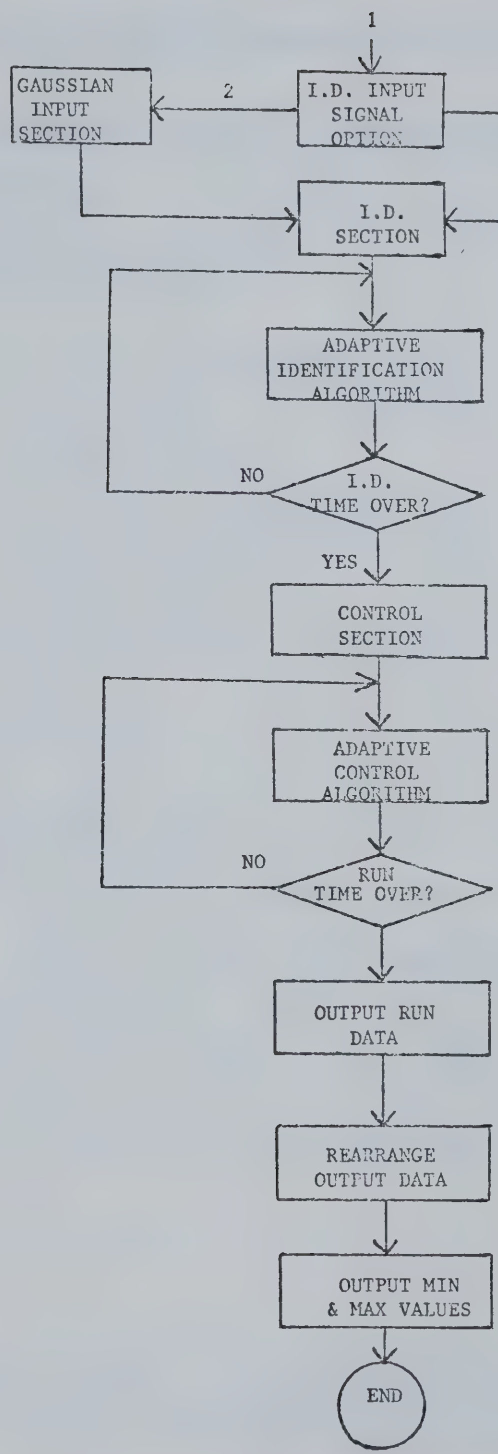


FIGURE 6.2 FLOWSHEET OF THE GENERAL MULTIVARIABLE ADAPTIVE CONTROL/IDENTIFICATION PROGRAM
-- LANDAU --

specific to this application.[1]

6.4.1 Adaptive Identification and Control Algorithm

The algorithm used by the adaptive control programs can best be described as a number of sequential steps executed at each sampling instant:

1. The plant output, $\underline{y}(k)$, is measured (calculated).
2. If the measurement noise flag is set, the program calculates a measurement noise component and adds this to the plant output obtained in step 1.
3. An estimate of the identification model output, $\underline{y}^o(k)$, is calculated from past values of the noise corrupted plant outputs, $\underline{y}$, the plant inputs, $\underline{u}$, and the identification model parameters, $\hat{\underline{\underline{A}}}_i(k-1)$ and $\hat{\underline{\underline{B}}}_j(k-1)$, $(i=1 \rightarrow h,\ j=1 \rightarrow f)$ from the previous instant.
4. The identification error estimate,

1

Further details of programs and options that are available for use in adaptive simulation studies appears in a separate program documentation manual [2].

$\underline{\varepsilon}^o(k)$, can then be calculated as the difference between $\underline{y}^o(k)$ and the present value of the plant output (plus measurement noise).

5. The program is now able to calculate a value of $\hat{\underline{v}}(k)$[1], the equivalent identification system state.

6. Using the adaptation laws of Theorem 5.3, the values of $\hat{\underline{\underline{A}}}_i(k)$ and $\hat{\underline{\underline{B}}}_j(k)$ $(i=1 \rightarrow h,\ j=1 \rightarrow f)$ can be calculated.

7. The reference model output, $\underline{y}_m(k)$, is calculated from known past values of $\underline{y}_m$ and setpoint values, $\underline{r}$.

8. $\hat{\underline{y}}(k)$, the identification model output, can be determined from the updated parameters calculated in step 6.

9. A control error, $\underline{\varepsilon}(k)$, is then determined as the difference between the reference model output and the identification model output at step 8.

10. The equivalent control system state at the $(k+1)^{th}$ instant, $\underline{v}(k+1)$, is then calculated by

---

1 This is accomplished by using the relation:

$$\hat{\underline{v}}(k)=\hat{\underline{\varepsilon}}(k) + \sum_{i=1}^{P1} \underline{\underline{D}}_i\hat{\underline{\varepsilon}}(k-i)$$

using equations (18)-(20) of Chapter Five.

11. The adaptive model closed-loop parameter matrices $\underline{\underline{A}}_{ip}(k+1)$ and $\underline{\underline{B}}_{jp}(k+1)$, $(i=1 \rightarrow h,\ j=1 \rightarrow f)$ can be calculated by using the adaptation rules of Theorem 5.2.

12. The desired output at the $(k+1)^{th}$ instant, $\underline{y}_d(k+1)$, is calculated using the parameter matrices determined in step 11. and present and past values of $\hat{\underline{y}}$ and $\underline{u}$.

13. The control input, $\underline{u}(k)$, is determined using the adaptive inverse approach described in Chapter Five (equation (67)).

14. Step 1. is then iterated upon at the next sampling instant.

6.5 Simulation Results

This section will discuss the results obtained using the two SISO systems described above. The explanation has been divided into two subsections because of the inherent differences in the examples chosen.

The major effects studied included:

(i) measurement noise;

(ii) variation in the (constant) outer loop gains;

(iii) changing the rate of decrease of the

magnitude of the identification loop gains;

(iv) effects of choosing different initial identification loop gains and,

(v) reference model choice.

A summary of all the SISO runs, presented, appears in Tables 6.1 and 6.2.

6.5.1 System 1 -- 1.75/14s+1 [8]

a) Measurement Noise

A direct indication of the effect of measurement noise on the system, can be obtained by examining Figures 6.4 and 6.5. In particular, Figures 6.4(a) and 6.5(a) demonstrate the filtering effect that is apparent when the magnitude of the identification loop gains are decreased. In both cases, the adaptive identification loop is only able to utilize the noise corrupted measurements shown in Figures 6.4(b) and 6.5(b). Both these runs were carried out using a very large noise input; the maximum variation being approximately 0.6. This represents a noise level of 60% of the final desired output.

Whilst the response is clearly dependent on a number of factors, including λ, the overall conclusion is that decreasing magnitude gains must be used in a noisy

SYSTEM: 1.75/(14s + 1)
RUN TIME = 500 SAMPLING INSTANTS

RUN NO.	REFERENCE MODEL	MEASUREMENT NOISE (S.D.,MEAN)	DECREASING MAGNITUDE I.D. LOOP GAINS (λ)	INITIAL I.D. LOOP PARAMETERS (INITIAL GAINS)	INITIAL OUTER LOOP PARAMETERS (GAINS)	EXCITATION MODE
1	$\frac{1}{20s + 1}$	YES (0.2,0.0)	YES (λ = 10)	-10 $\begin{bmatrix}0\\0.1\\0\end{bmatrix}$	10 $\begin{bmatrix}0\\0\end{bmatrix}$	SETPOINT CHANGE 1 @ 0
2	$\frac{1}{20s + 1}$	YES (0.2,0.0)	NO	-10 $\begin{bmatrix}0\\0.1\\0\end{bmatrix}$	100 $\begin{bmatrix}0\\0\end{bmatrix}$	"
3	$\frac{1}{20s + 1}$	YES (0.2,0.0)	YES (λ = 1)	-10 $\begin{bmatrix}0\\0.1\\0\end{bmatrix}$	100 $\begin{bmatrix}0\\0\end{bmatrix}$	"
3a	$\frac{1}{20s + 1}$	YES (0.2,0.0)	YES (λ = 10)	-10 $\begin{bmatrix}0\\0.1\\0\end{bmatrix}$	100 $\begin{bmatrix}0\\0\end{bmatrix}$	"
3b	$\frac{1}{20s + 1}$	YES (0.2,0.0)	YES (λ = 1)	-100 $\begin{bmatrix}0\\0.1\\0\end{bmatrix}$	100 $\begin{bmatrix}0\\0\end{bmatrix}$	"
4	$\frac{1}{100s + 1}$	YES (0.2,0.0)	YES (λ = 1)	-10 $\begin{bmatrix}0.9\\0.1\\0\end{bmatrix}$	100 $\begin{bmatrix}0.9\\0.01\end{bmatrix}$	SETPOINT CHANGE 1 @ 50
4a	$\frac{1}{100s + 1}$	YES (0.2,0.0)	YES (λ = 1)	-1 $\begin{bmatrix}0.9\\0.1\\0\end{bmatrix}$	1 $\begin{bmatrix}0.9\\0.01\end{bmatrix}$	"

TABLE 6.1 SIMULATION RUNS FOR SISO SYSTEM 1.75/(14s + 1)

SYSTEM: 1.75/(14w + 1)
RUN TIME = 500 SAMPLING INSTANTS
SETPOINT CHANGE 1.0 @ 20

RUN NO.	REFERENCE MODEL	MEASUREMENT NOISE (S.D.,MEAN)	DECREASING MAGNITUDE I.D. LOOP GAINS (λ)	INITIAL I.D. LOOP PARAMETERS (INITIAL GAINS)	INITIAL OUTER LOOP PARAMETERS (GAINS)
1	$\frac{1}{20s + 1}$	NO	YES (λ = 10)	$-10\ \begin{bmatrix}0\\0.1\\0\end{bmatrix}$	$10\ \begin{bmatrix}0.9\\0.1\end{bmatrix}$
2	$\frac{1}{20s + 1}$	YES (0.2,0.0)	YES (λ = 10)	$-10\ \begin{bmatrix}0\\0.1\\0\end{bmatrix}$	$10\ \begin{bmatrix}0.9\\0.1\end{bmatrix}$
3	$\frac{1}{20s + 1}$	YES (0.2,0.0)	NO	$-10\ \begin{bmatrix}0\\0.1\\0\end{bmatrix}$	$10\ \begin{bmatrix}0.9\\0.1\end{bmatrix}$
3a	$\frac{1}{20s + 1}$	YES (0.2,0.0)	YES (λ = 10)	$-10\ \begin{bmatrix}0\\0.1\\0\end{bmatrix}$	$1\ \begin{bmatrix}0.9\\0.1\end{bmatrix}$
4	$\frac{1}{20s + 1}$	YES (0.2,0.0)	YES (λ = 10)	$-10\ \begin{bmatrix}0\\0.1\\0\end{bmatrix}$	$0.1\ \begin{bmatrix}0.9\\0.1\end{bmatrix}$
4a	$\frac{1}{20s + 1}$	YES (0.2,0.0)	YES (λ = 10)	$-10\ \begin{bmatrix}0\\0.1\\0\end{bmatrix}$	$0.01\ \begin{bmatrix}0.9\\0.1\end{bmatrix}$
4b	$\frac{1}{20s + 1}$	YES (0.2,0.0)	YES (λ = 10)	$-10\ \begin{bmatrix}0\\0.1\\0\end{bmatrix}$	$100\ \begin{bmatrix}0.9\\0.1\end{bmatrix}$
4c	$\frac{1}{20s + 1}$	YES (0.2,0.0)	NO	$-10\ \begin{bmatrix}0\\0.1\\0\end{bmatrix}$	$100\ \begin{bmatrix}0.9\\0.1\end{bmatrix}$
5	$\frac{1}{20s + 1}$	YES (0.2,0.0)	YES (λ = 1)	$-10\ \begin{bmatrix}0\\0.1\\0\end{bmatrix}$	$100\ \begin{bmatrix}0.9\\0.1\end{bmatrix}$

TABLE 6.2 SIMULATION RUNS FOR SISO SYSTEM 1.75/(14w + 1)

environment; if not, an oscillatory response about the desired output will result.

b) Outer Loop Gains

The outer loop of Figure 6.1 has been included to allow some insurance against excessive control action being generated as a result of an incorrect driving model specification. This loop also guarantees a stable approach to the desired output transient. The results depicted in Figures 6.3 and 6.6 have been employed to investigate the use of the outer loop gains as a means of controlling the rate of approach to the desired output. In both runs, a moderate amount of noise filtering was employed (λ =10).

From the responses shown in Figure 6.3(a) and Figure 6.6(a), it can be concluded that the outer loop gains do not significantly affect the adaptive trajectory.

c) Effect of Different Reference Models

The results of Figures 6.5 and 6.8 indicate the effect of changing the manner in which the outer adaptive loop is excited. It was thought that a very slow reference model transient would allow the identification system more time to converge and at the same time reduce the effect of large control action. However, from the simulation results

obtained, there does not seem to be any advantage in specifying a reference model with a very large time constant relative to the plant, although this may depend, to some extent, on the amount of measurement noise present.

d) Variation of the Rate of Decrease of the Magnitude of the Identification Loop Gains

Figures 6.5 and 6.6 show the effects of a change in λ from 1 to 10. This action is to allow less filtering of the noise corrupted plant output measurements for the second simulation run. This step has the expected result, in that the actual plant output of Figure 6.6(a) exhibits much more oscillation about the desired output level than does the response of Figure 6.8(a).

e) Choice of Initial Identification Loop Gains

Theoretically, the initial identification loop gains may be as large as desired, although it would be expected that, because of the initially faster adaptation of the identification loop parameters, the actual plant behaviour would exhibit a larger deviation from the desired transient at the start of the run. Figures 6.5(a) and 6.7(a) demonstrate this effect. It is also noted that the convergence to the desired trajectory is faster when larger initial gains are used. The choice, therefore, of these

parameters will depend on how much initial oscillation can be borne.

Finally, Figures 6.9 demonstrate the response obtained with a large amount of filtering, small adaptation gains in both loops, and a reference model with a large time constant. The actual plant output characteristic, Figure 6.9(a), demonstrates an oscillatory behaviour, although the initial response does not mirror the extreme deviations of Figure 6.8(a). The converse is true with respect to the latter stages of the trajectory where the convergence exhibited by the system of run 4 is almost complete.

6.5.2 System 2 -- 1.75/14w+1

This system injects numerator dynamics into the adaptive configuration. The example, therefore, cannot be viewed as a simple state-space formulation since, by definition, it incorporates incomplete state information.

a) Measurement Noise

As with the previous example, the effect of measurement noise on this system can be quite dramatic, particularly since previous control inputs are inherently present in the system description. Figures 6.11 and 6.12, and Figures 6.16

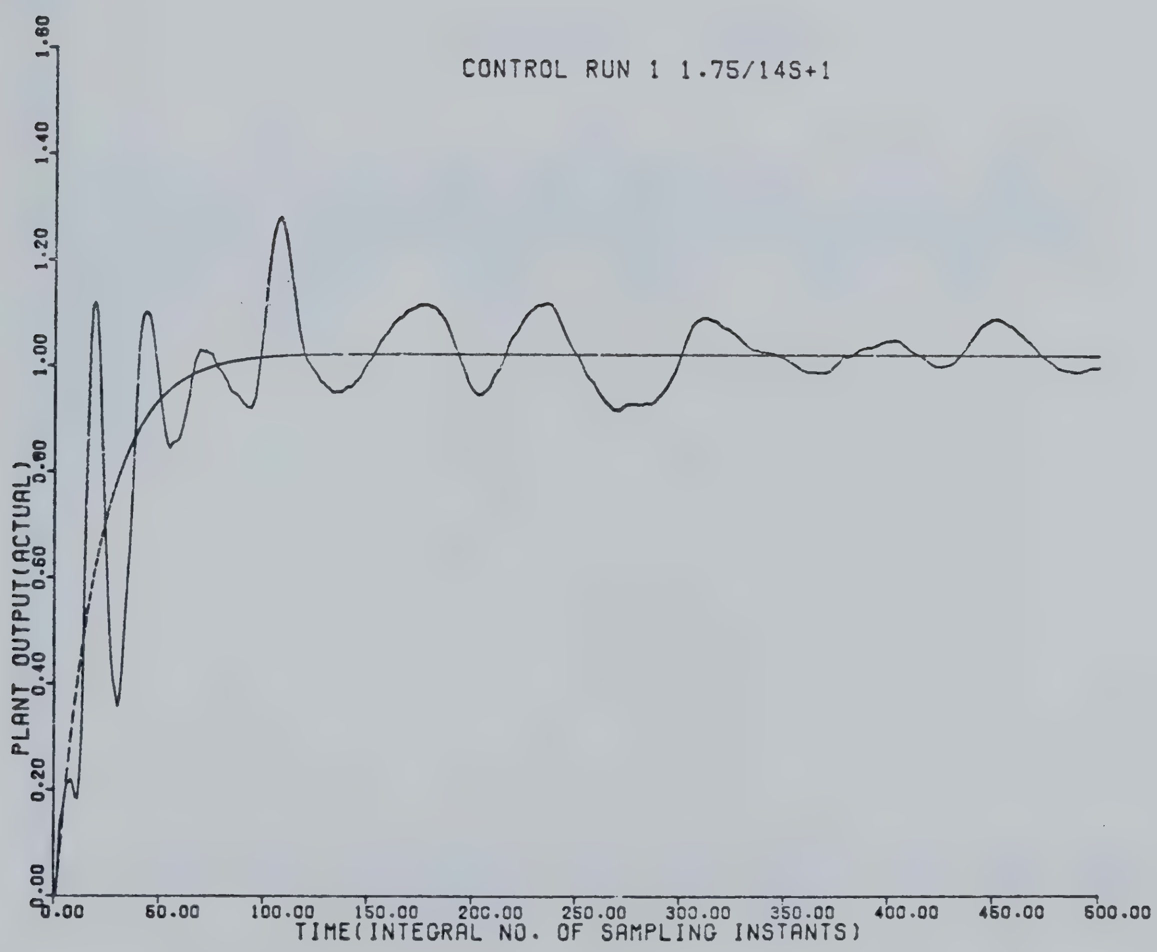


FIGURE 6.3(a): SISO SYSTEM 1 1.75/(14s + 1)
PLANT OUTPUT (ACTUAL) VS TIME

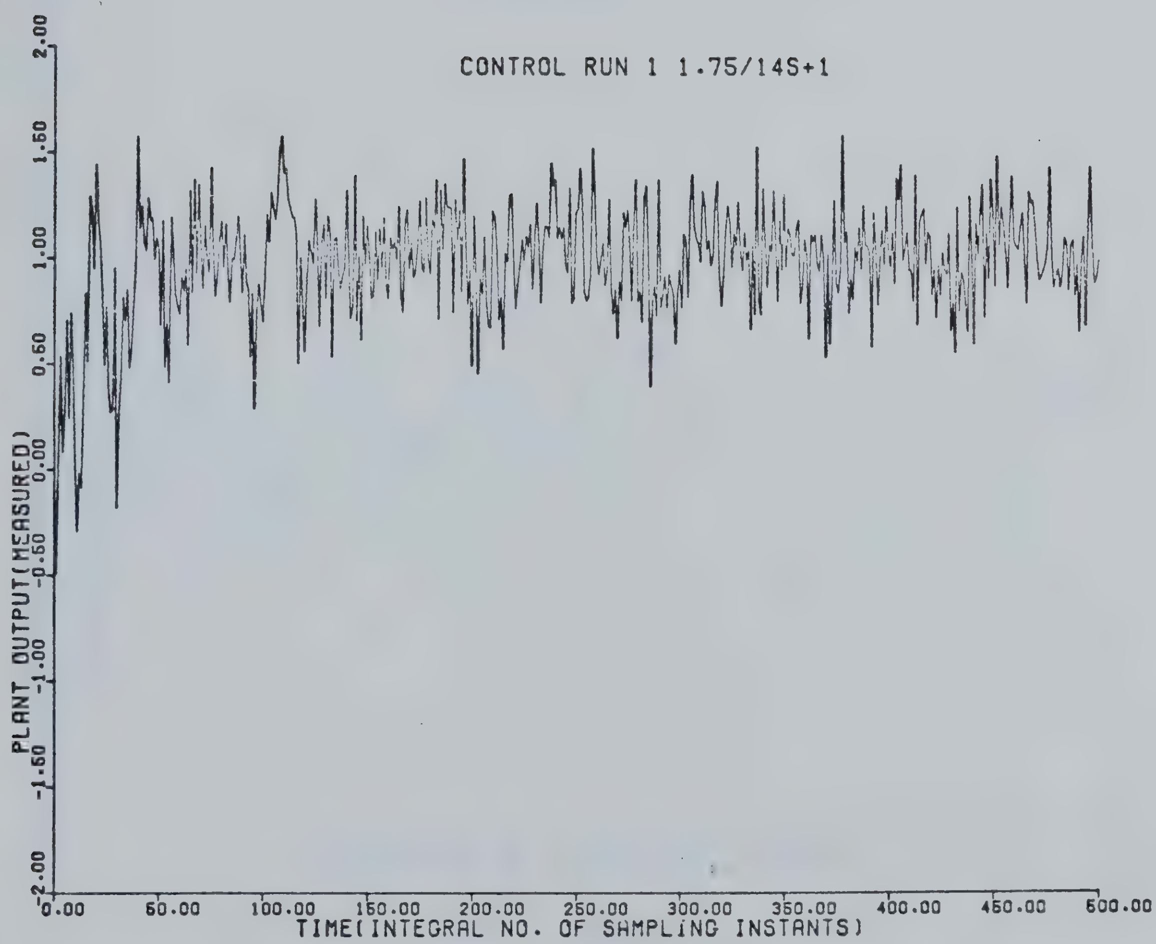


FIGURE 6.3(b): SISO SYSTEM 1 1.75/(14s + 1)
PLANT OUTPUT (MEASURED) VS TIME

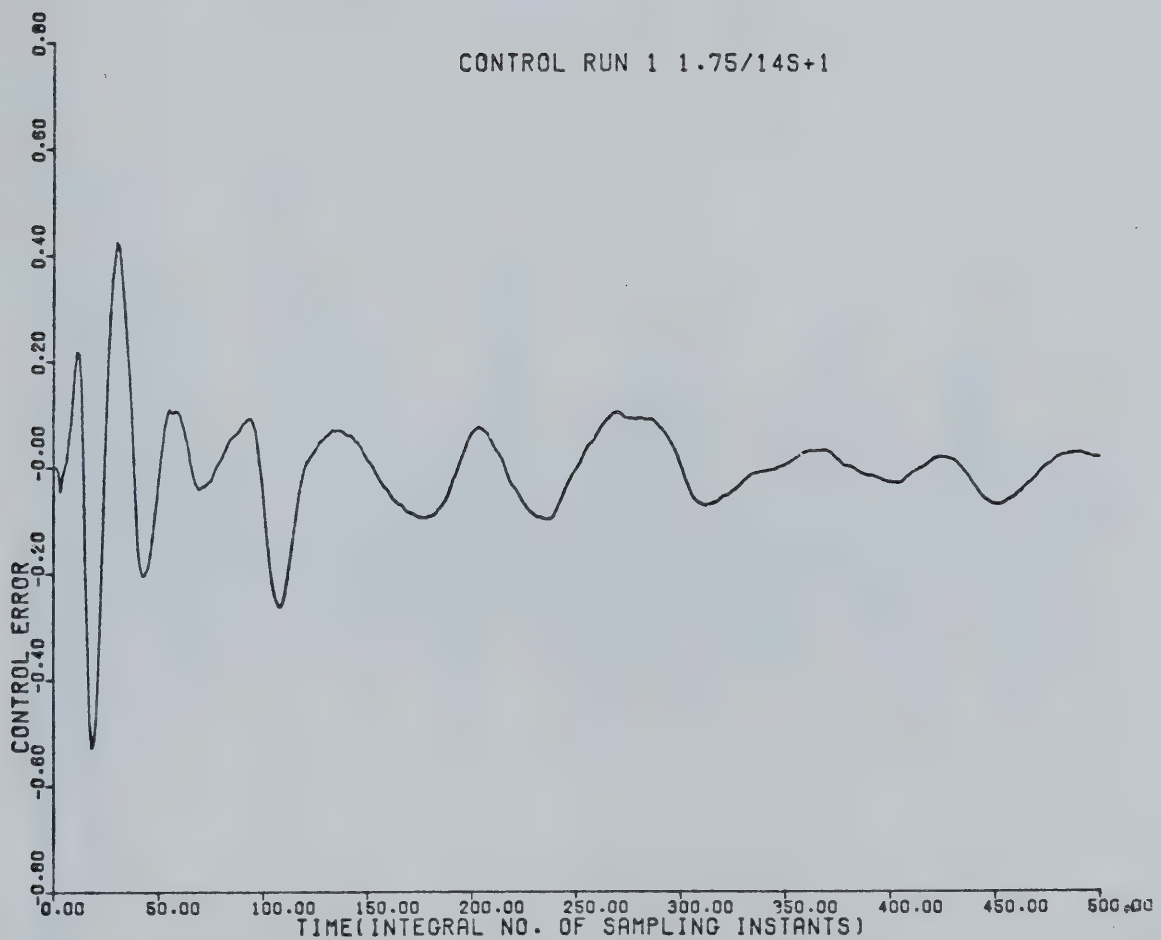


FIGURE 6.3(c): SISO SYSTEM 1 1.75/(14s + 1)
CONTROL ERROR VS TIME

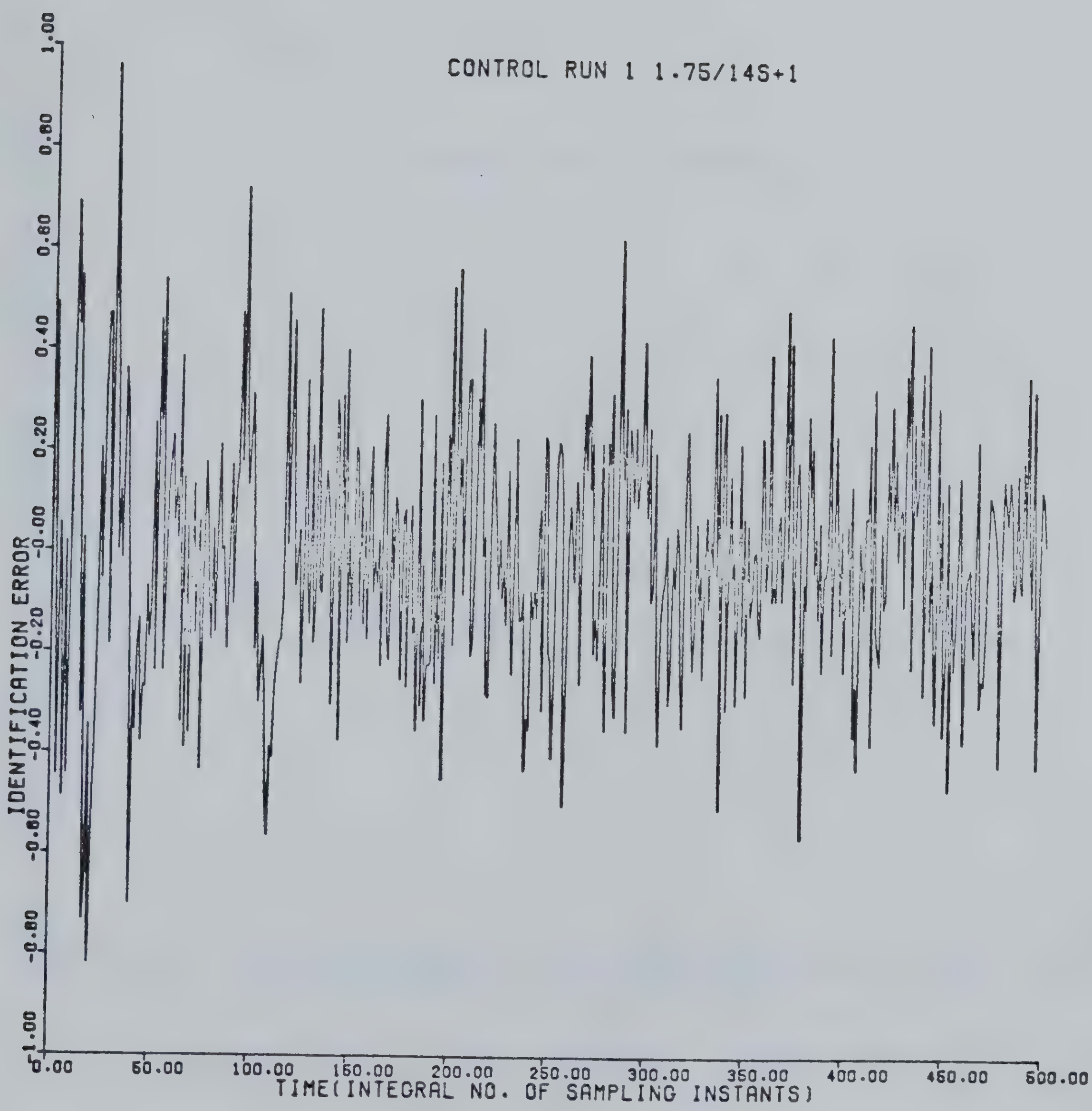


FIGURE 6.3(d): SISO SYSTEM 1 1.75/(14s + 1)
IDENTIFICATION ERROR VS TIME

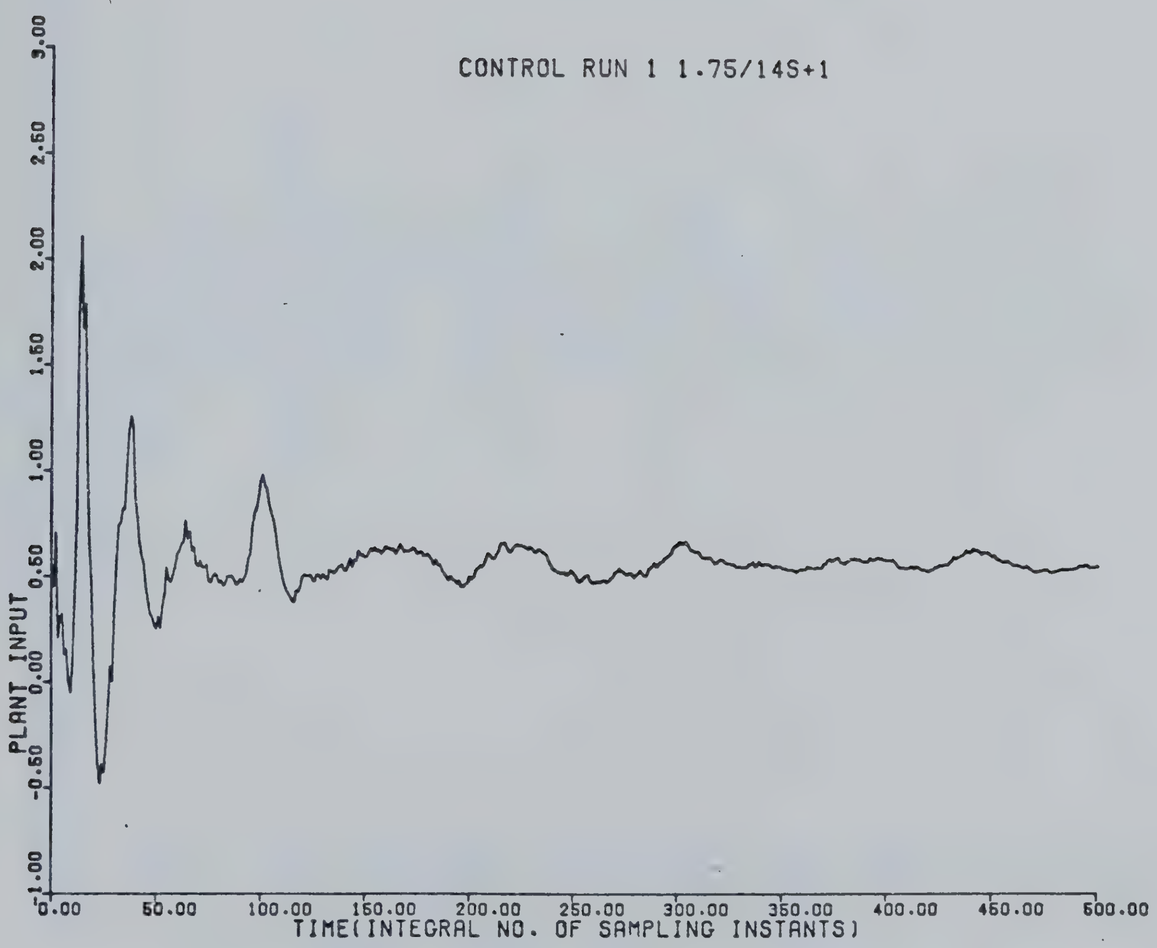


FIGURE 6.3(e): SISO SYSTEM 1 1.75/(14s + 1)
PLANT INPUT VS TIME

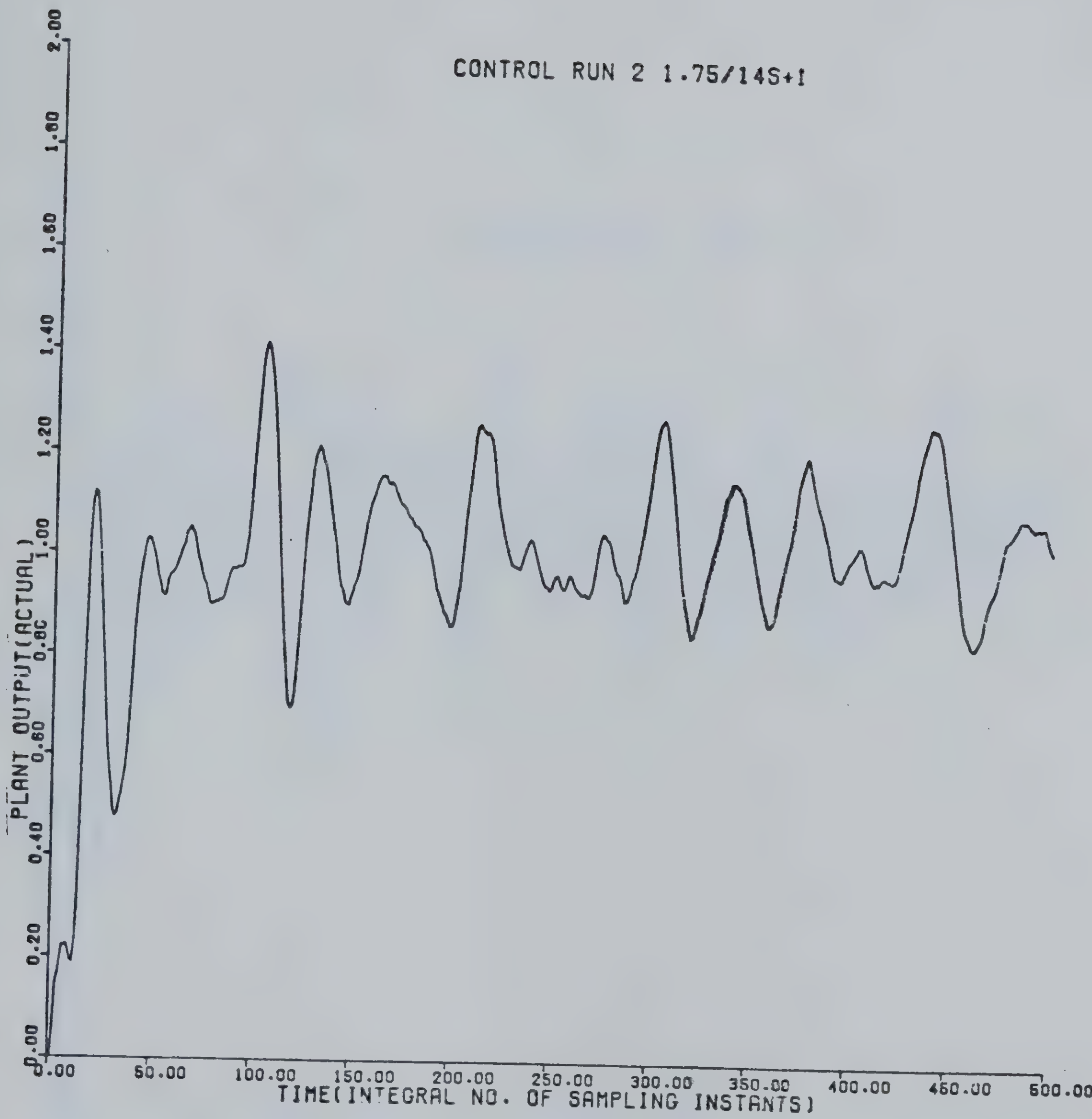


FIGURE 6.4(a): SISO SYSTEM 1 1.75/(14s + 1)
PLANT OUTPUT (ACTUAL) VS TIME

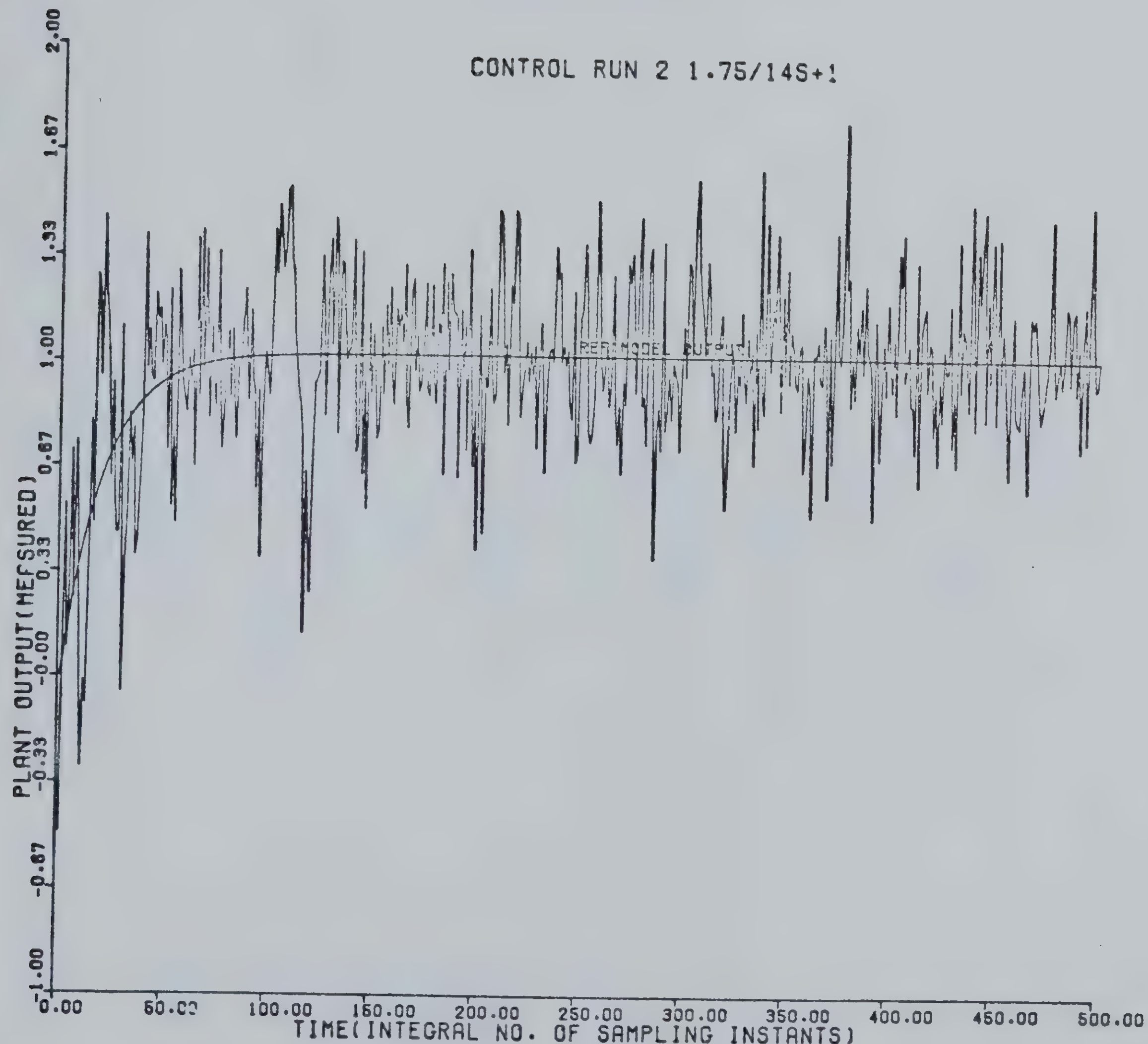


FIGURE 6.4(b): SISO SYSTEM 1 1.75/(14s + 1)
PLANT OUTPUT (MEASURED) VS TIME

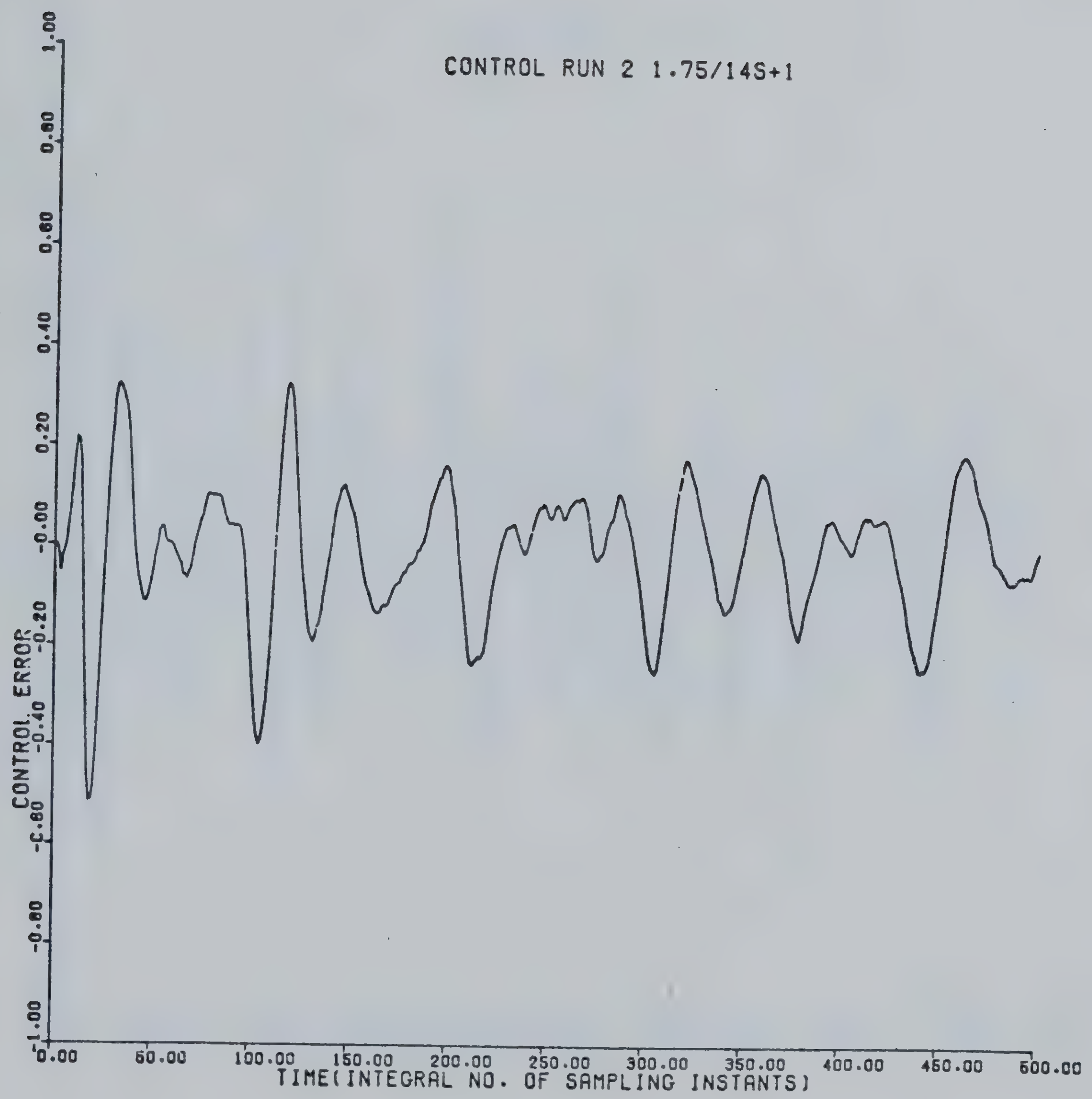


FIGURE 6.4(c): SISO SYSTEM 1 1.75/(14s + 1)
CONTROL ERROR VS TIME

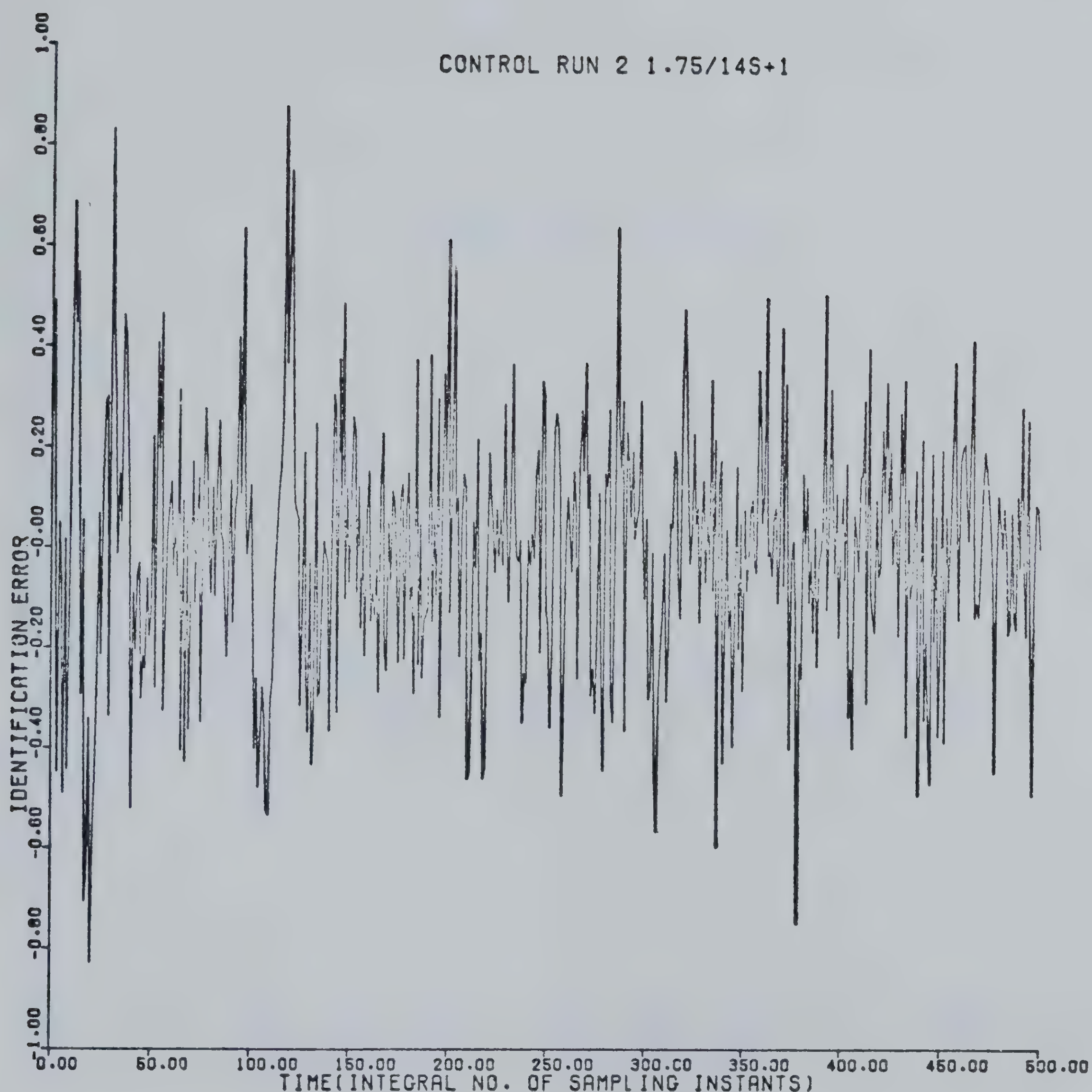


FIGURE 6.4(d): SISO SYSTEM 1 1.75/(14s + 1)
IDENTIFICATION ERROR VS TIME

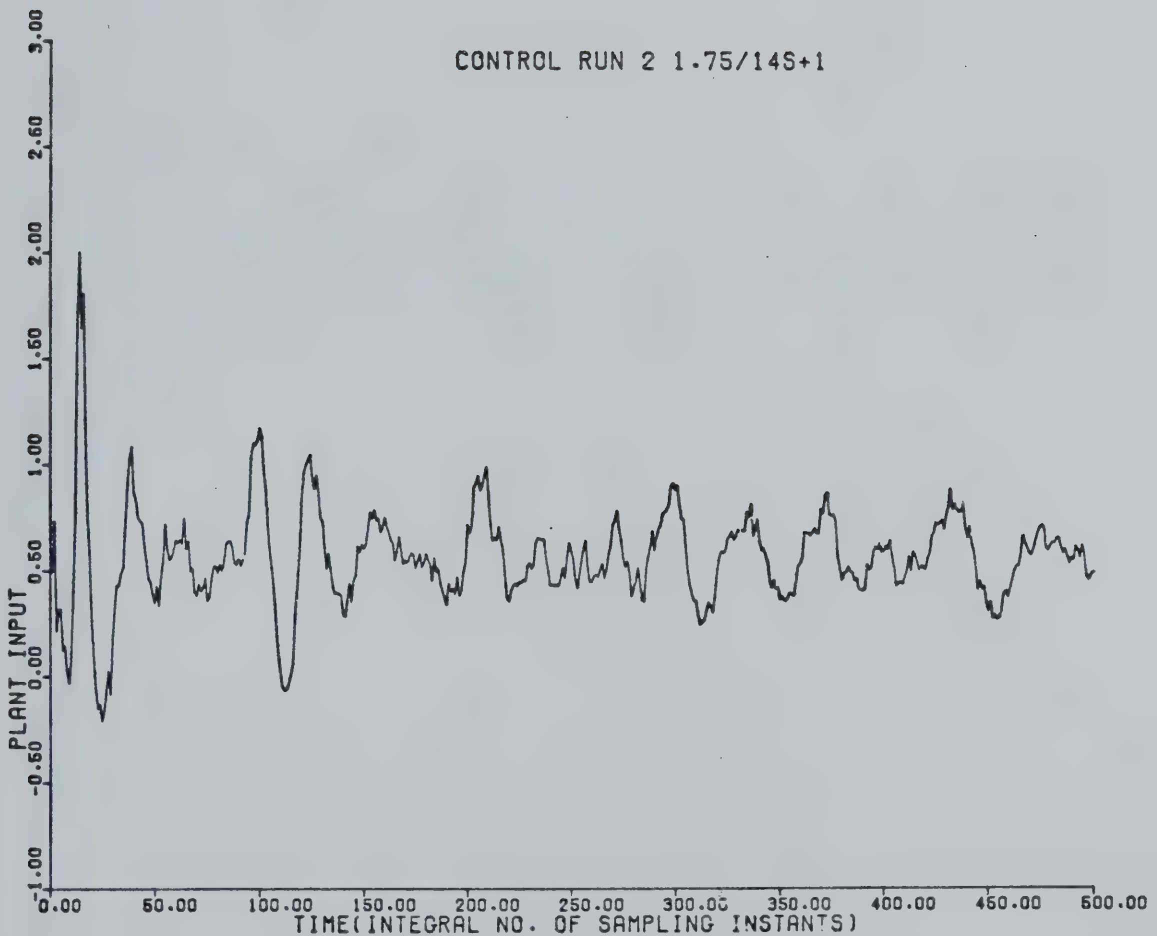


FIGURE 6.4(e): SISO SYSTEM 1 1.75/(14s + 1)
PLANT INPUT VS TIME

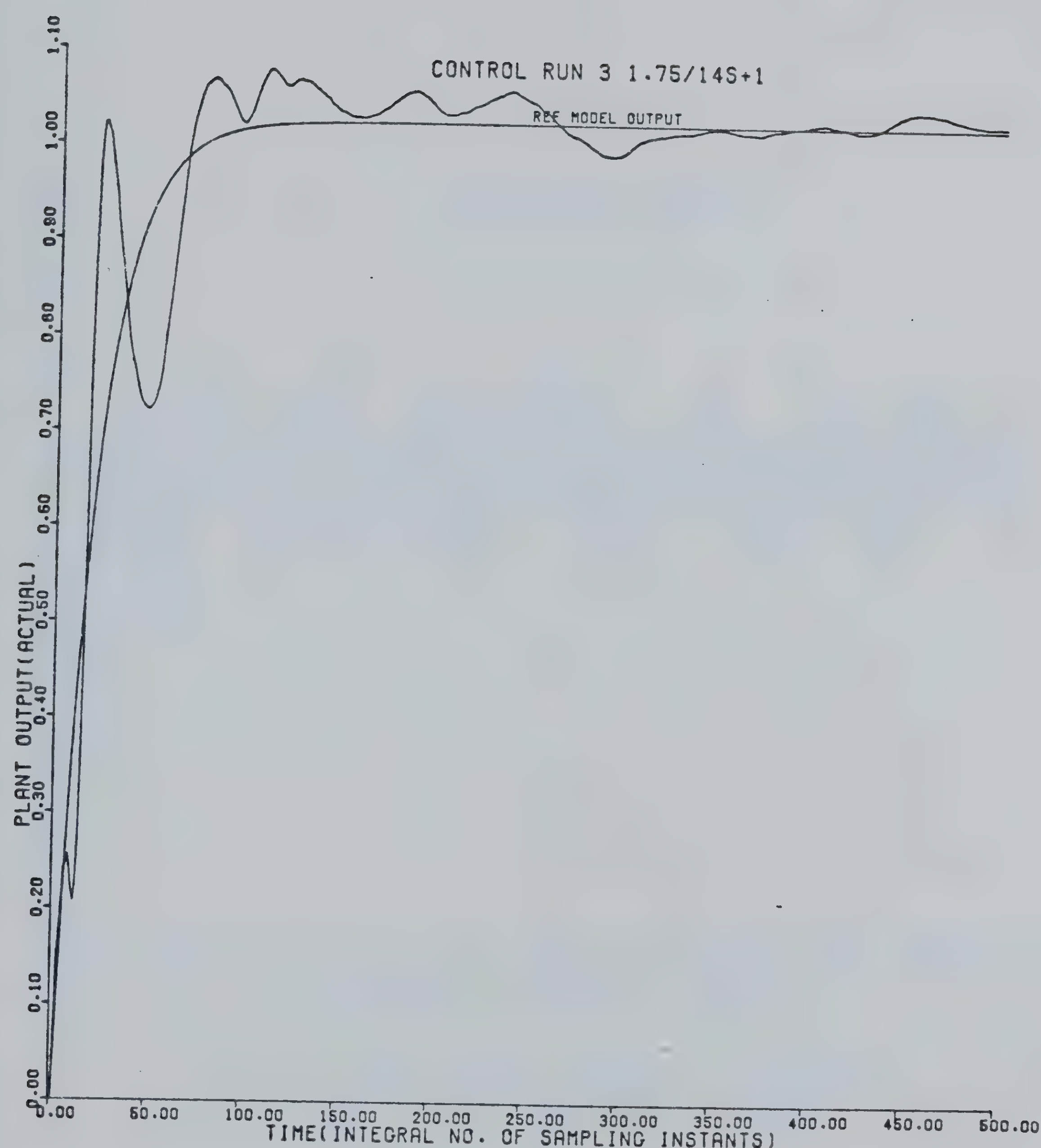


FIGURE 6.5(a): SISO SYSTEM 1 1.75/(14s + 1)
PLANT OUTPUT (ACTUAL) VS TIME

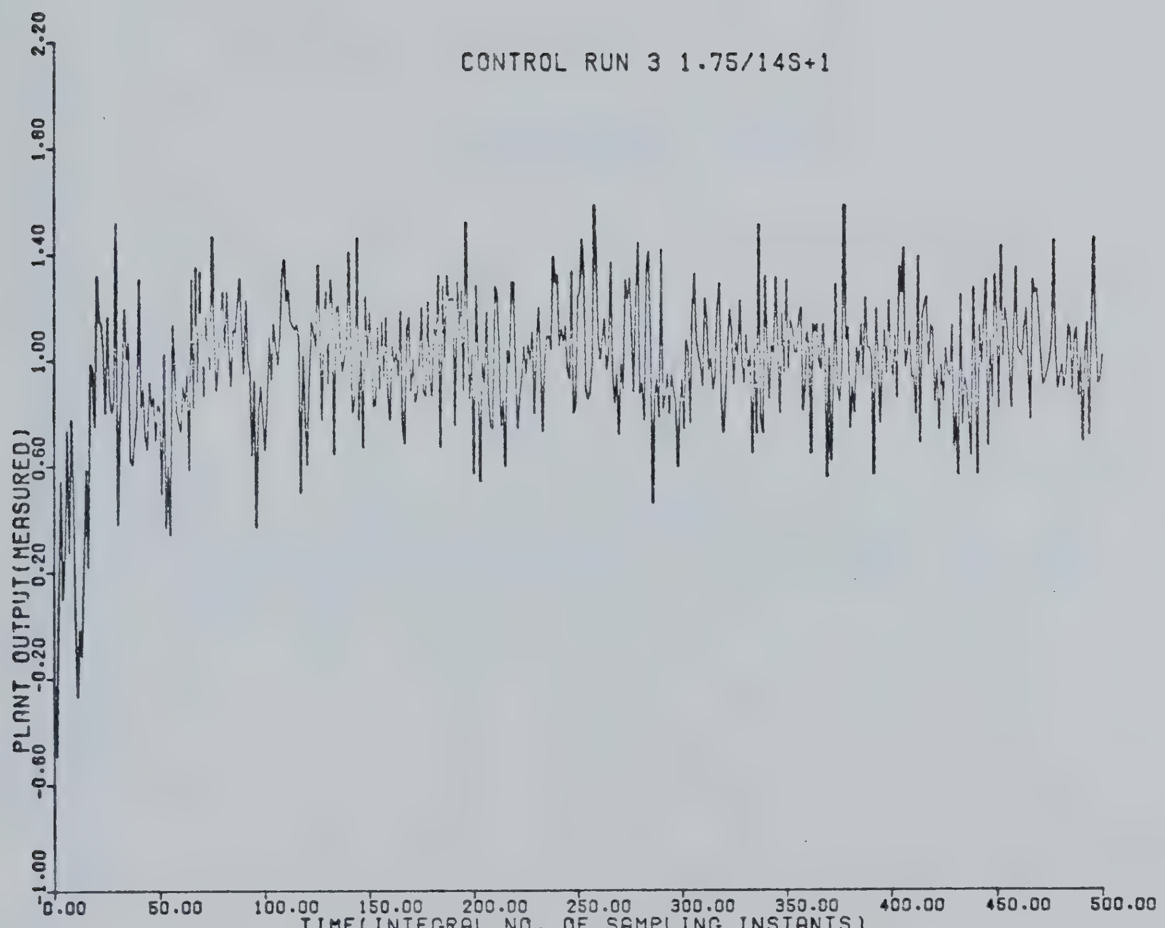


FIGURE 6.5(b): SISO SYSTEM 1 1.75/(14s + 1)
PLANT OUTPUT (MEASURED) VS TIME

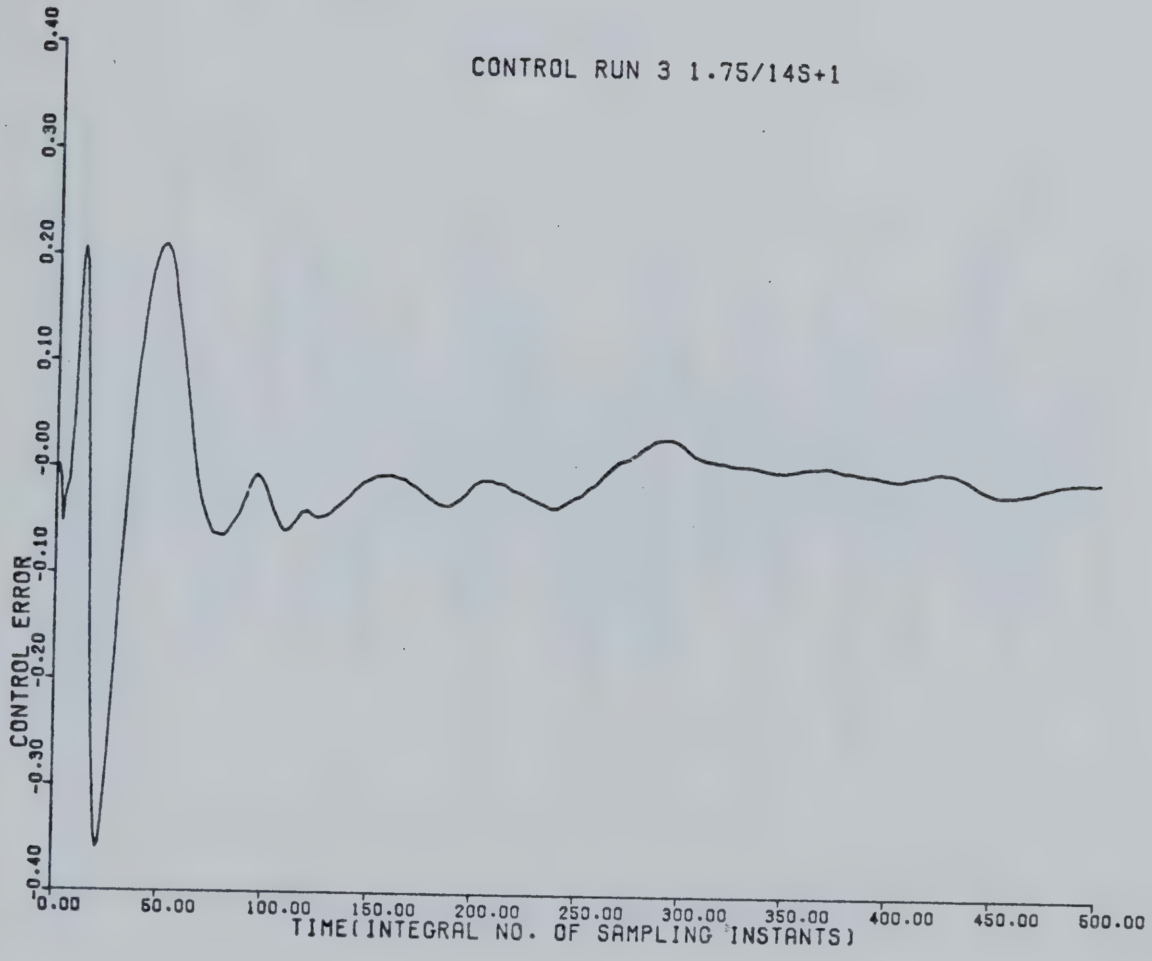


FIGURE 6.5(c): SISO SYSTEM 1 1.75/(14s + 1)
CONTROL ERROR VS TIME

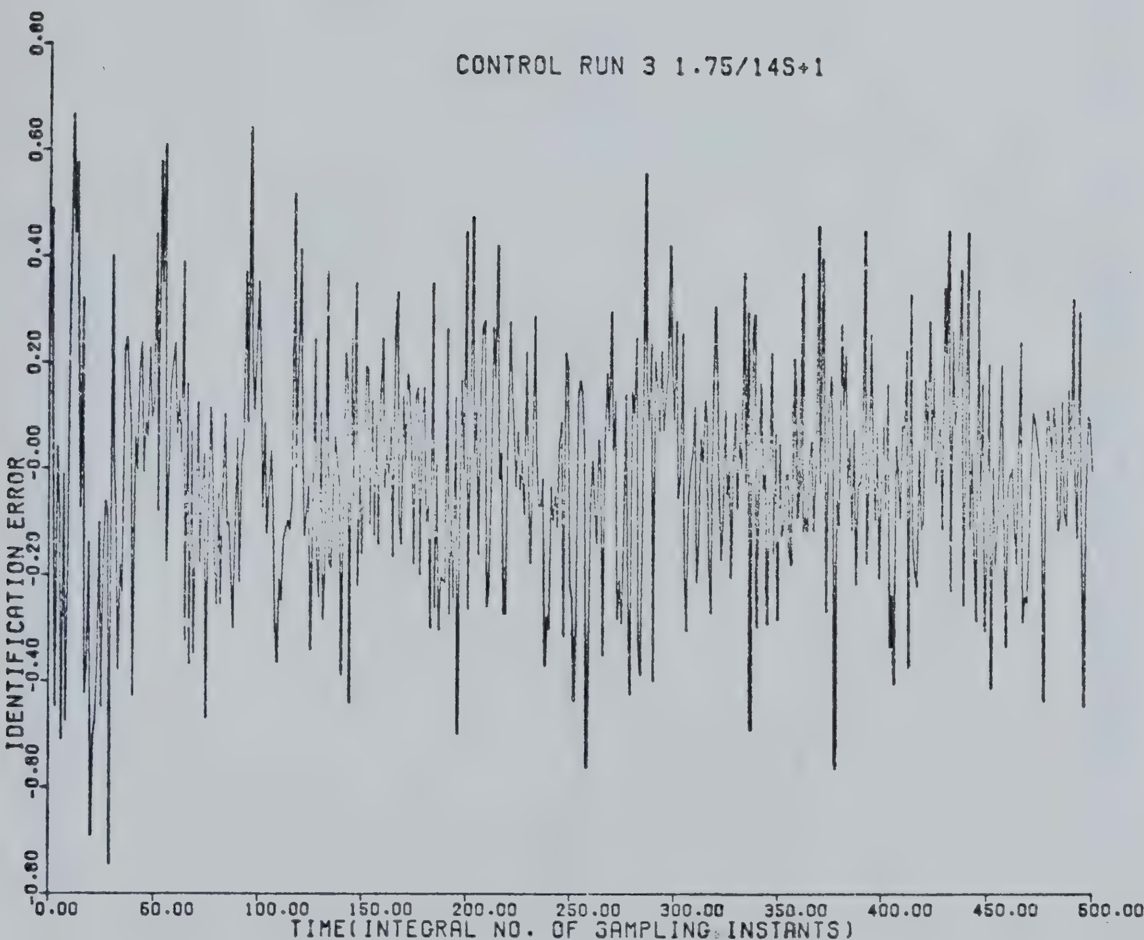


FIGURE 6.5(d): SISO SYSTEM 1 1.75/(14s + 1)
IDENTIFICATION ERROR VS TIME

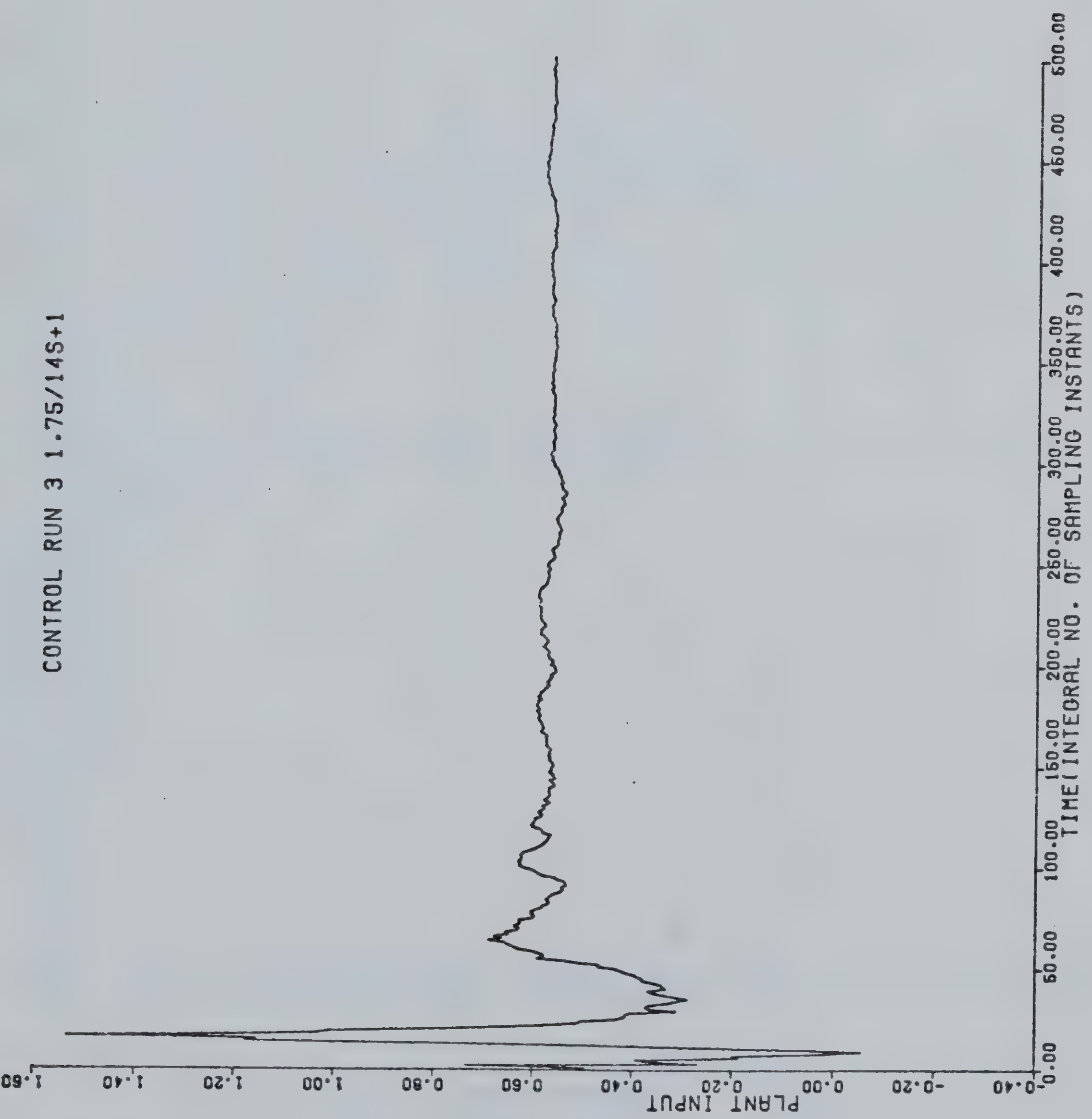


FIGURE 6.5(e): SISO SYSTEM 1 1.75/(14s + 1)
PLANT INPUT VS TIME

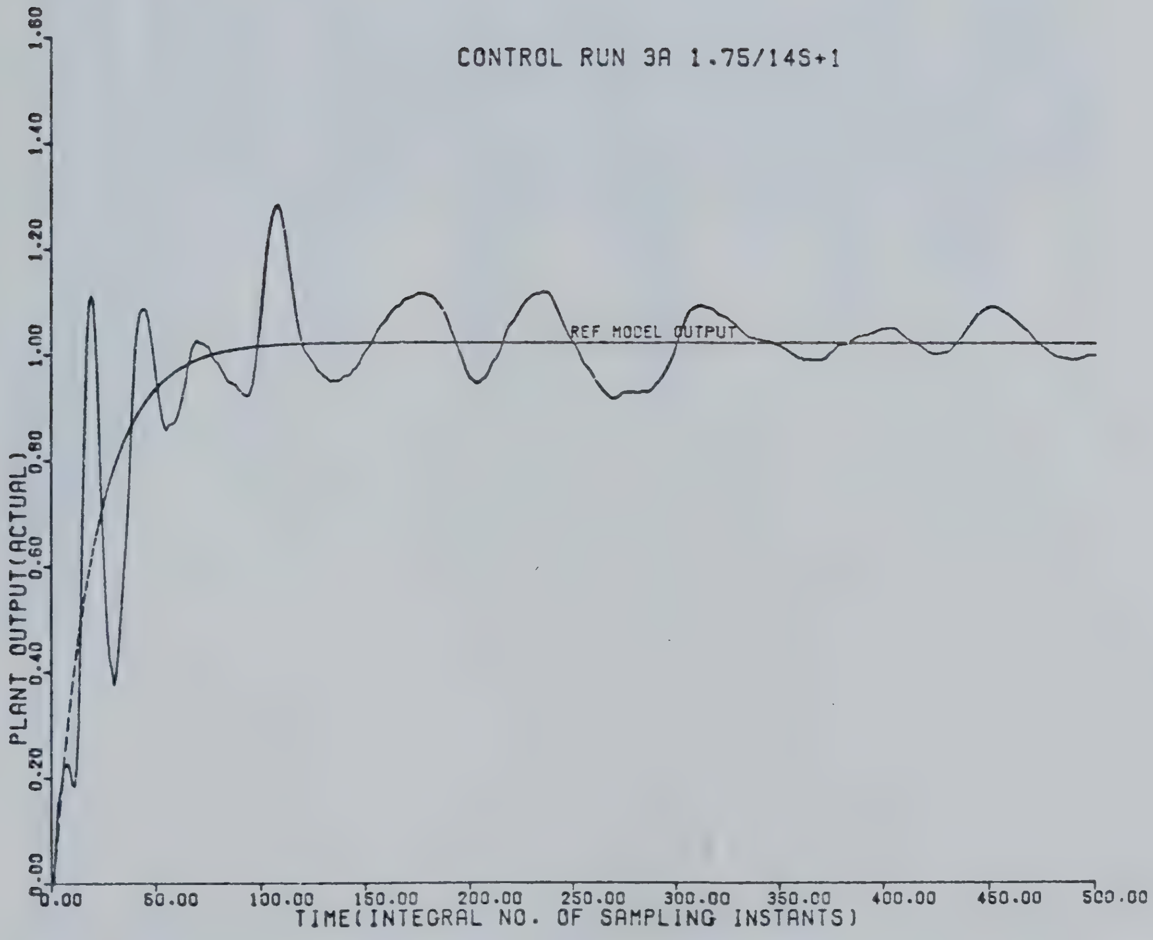


FIGURE 6.6(a): SISO SYSTEM 1 1.75/(14s + 1)
PLANT OUTPUT (ACTUAL) VS TIME

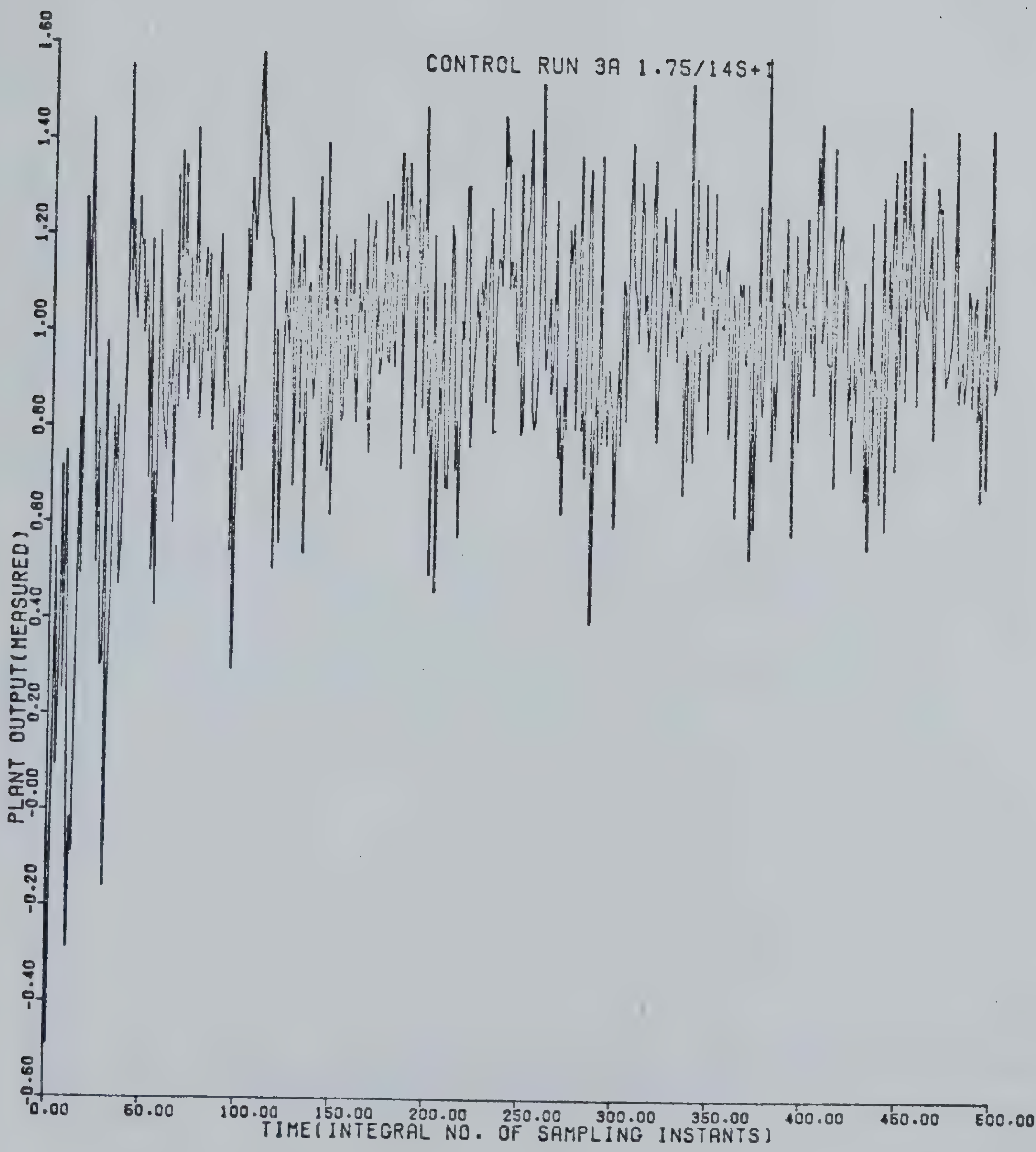


FIGURE 6.6(b): SISO SYSTEM 1 1.75/(14s + 1)
PLANT OUTPUT (MEASURED) VS TIME

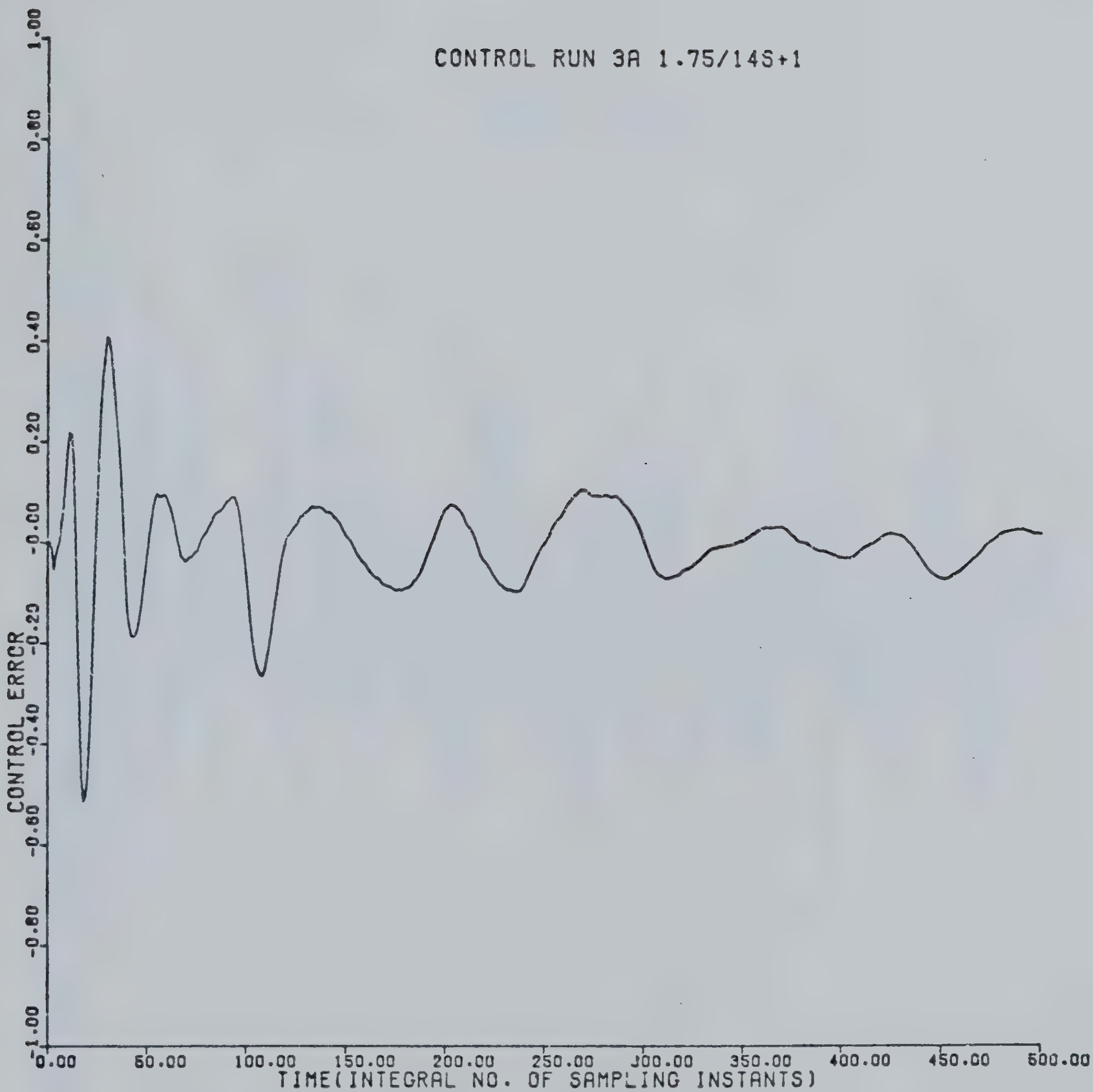


FIGURE 6.6(c): SISO SYSTEM 1 1.75/(14s + 1)
CONTROL ERROR VS TIME

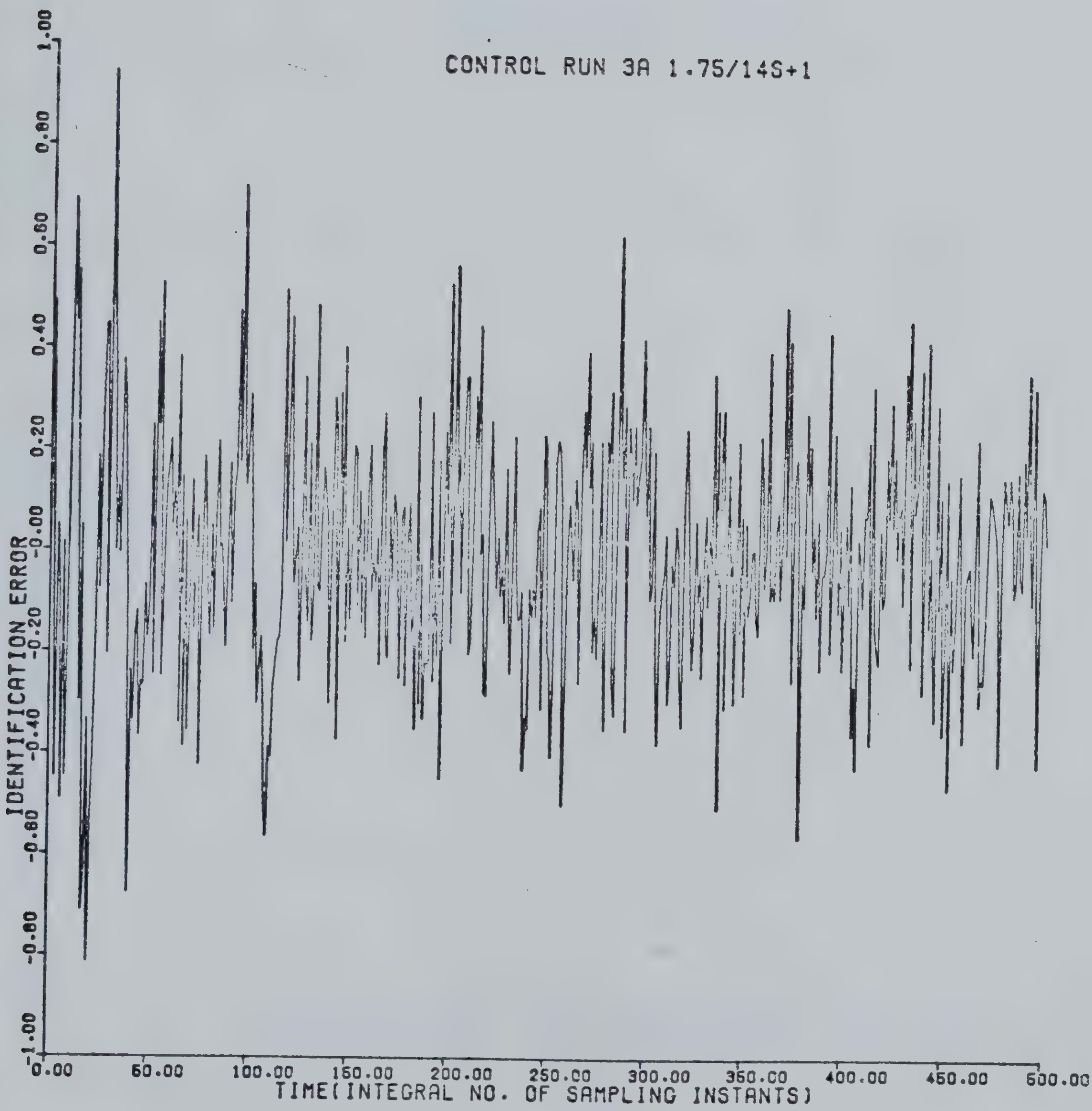


FIGURE 6.6(d): SISO SYSTEM 1 1.75/(14s + 1)
IDENTIFICATION ERROR VS TIME

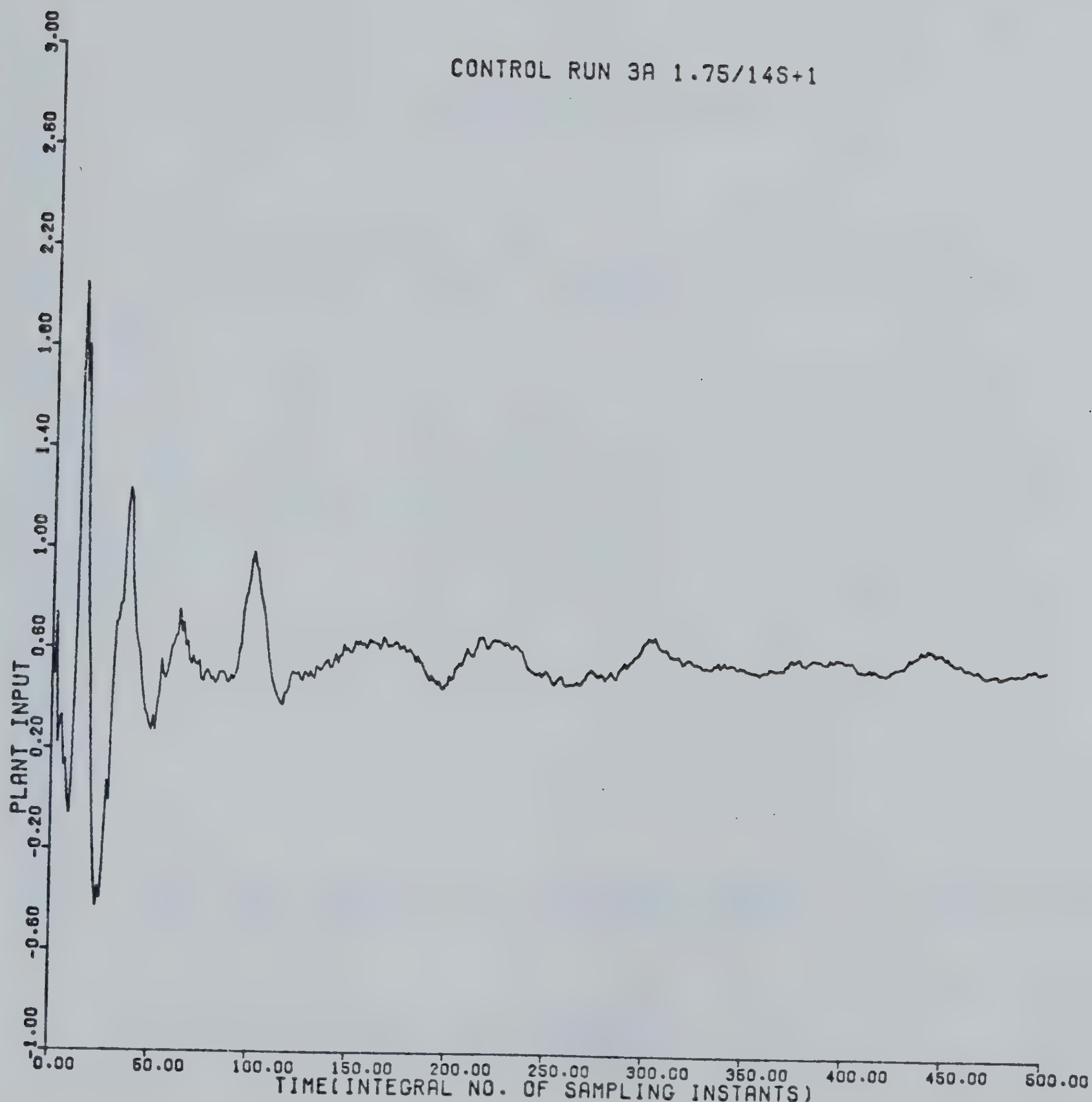


FIGURE 6.6(e): SISO SYSTEM 1 1.75/(14s + 1)
PLANT INPUT VS TIME

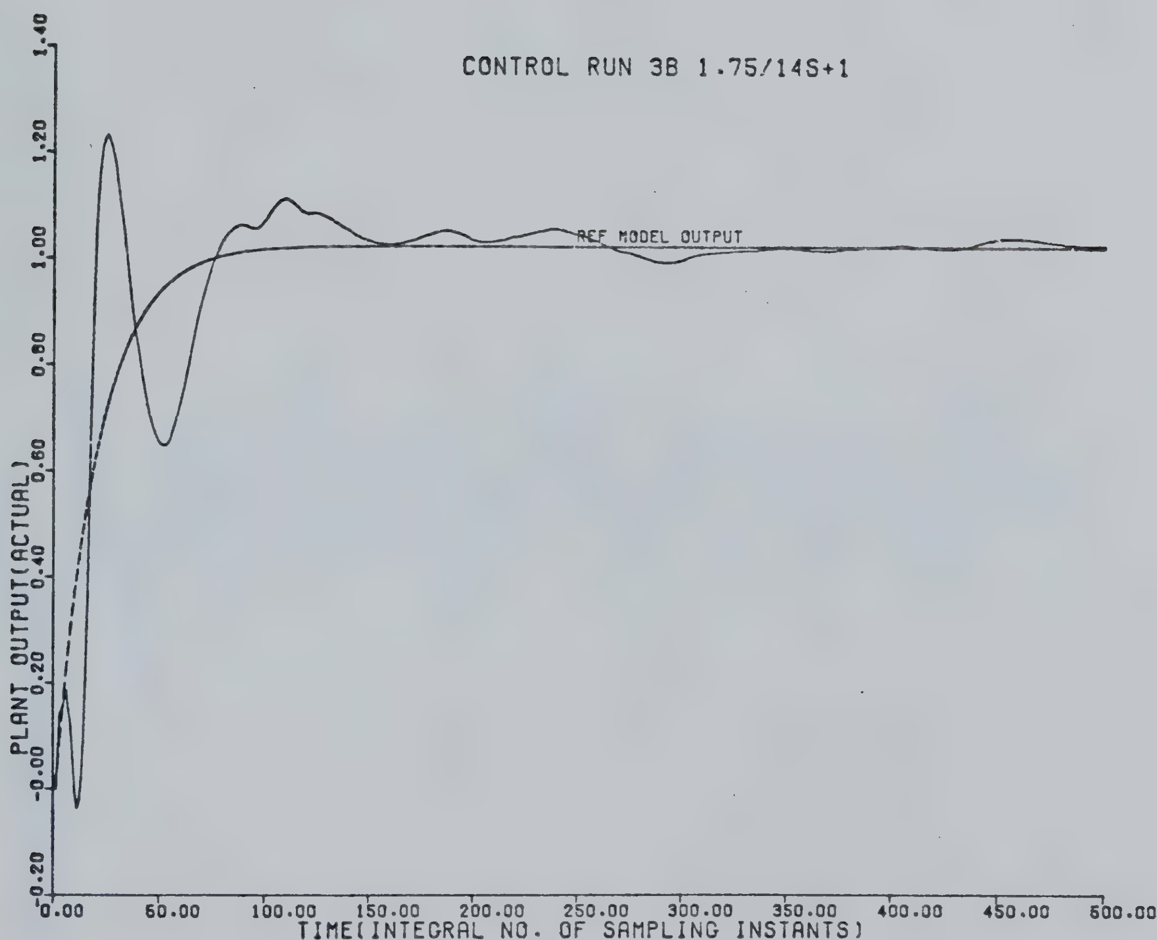


FIGURE 6.7(a): SISO SYSTEM 1 1.75/(14s + 1)
PLANT OUTPUT (ACTUAL) VS TIME

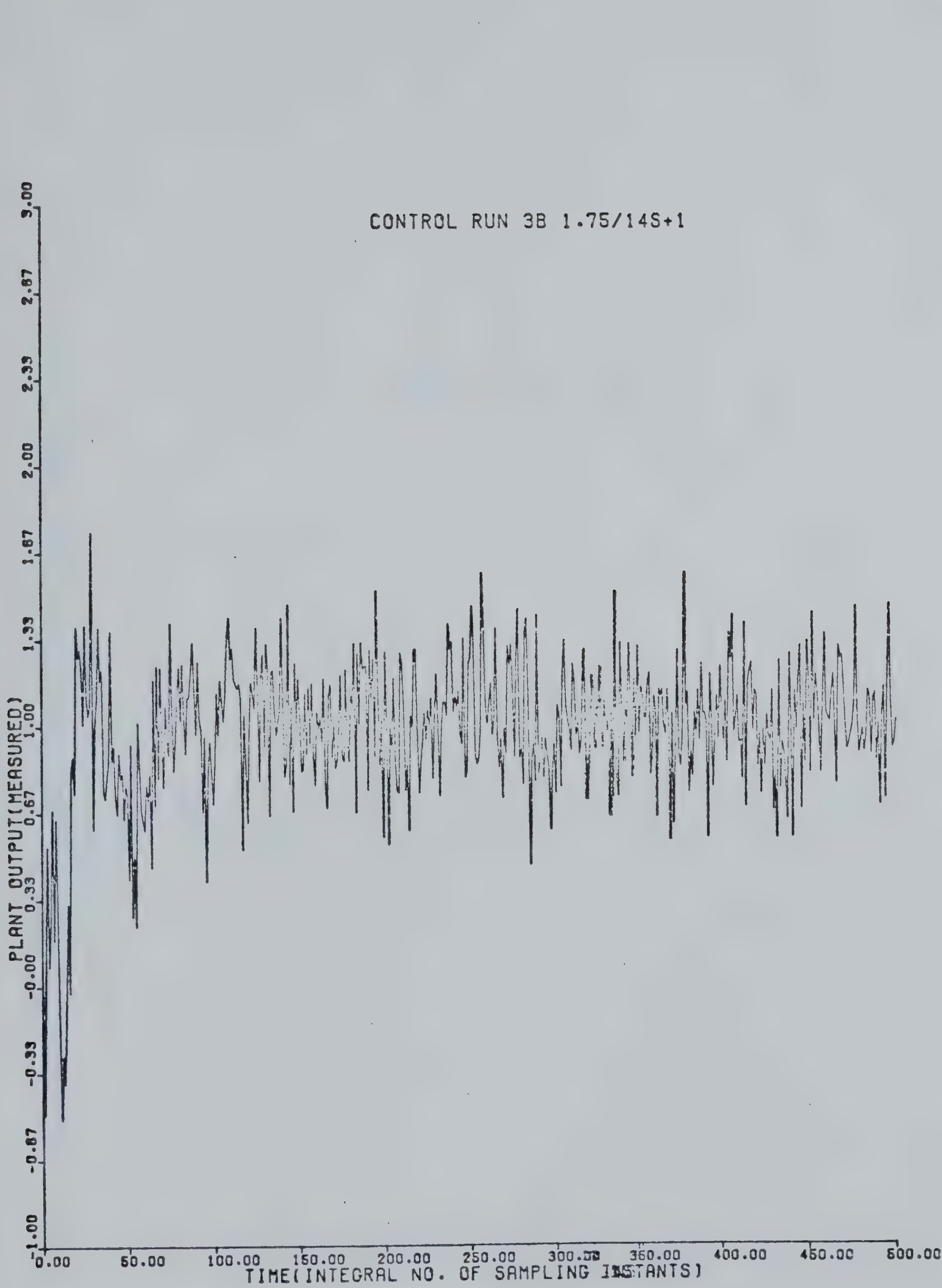


FIGURE 6.7(b): SISO SYSTEM 1 1.75/(14s + 1)
PLANT OUTPUT (MEASURED) VS TIME

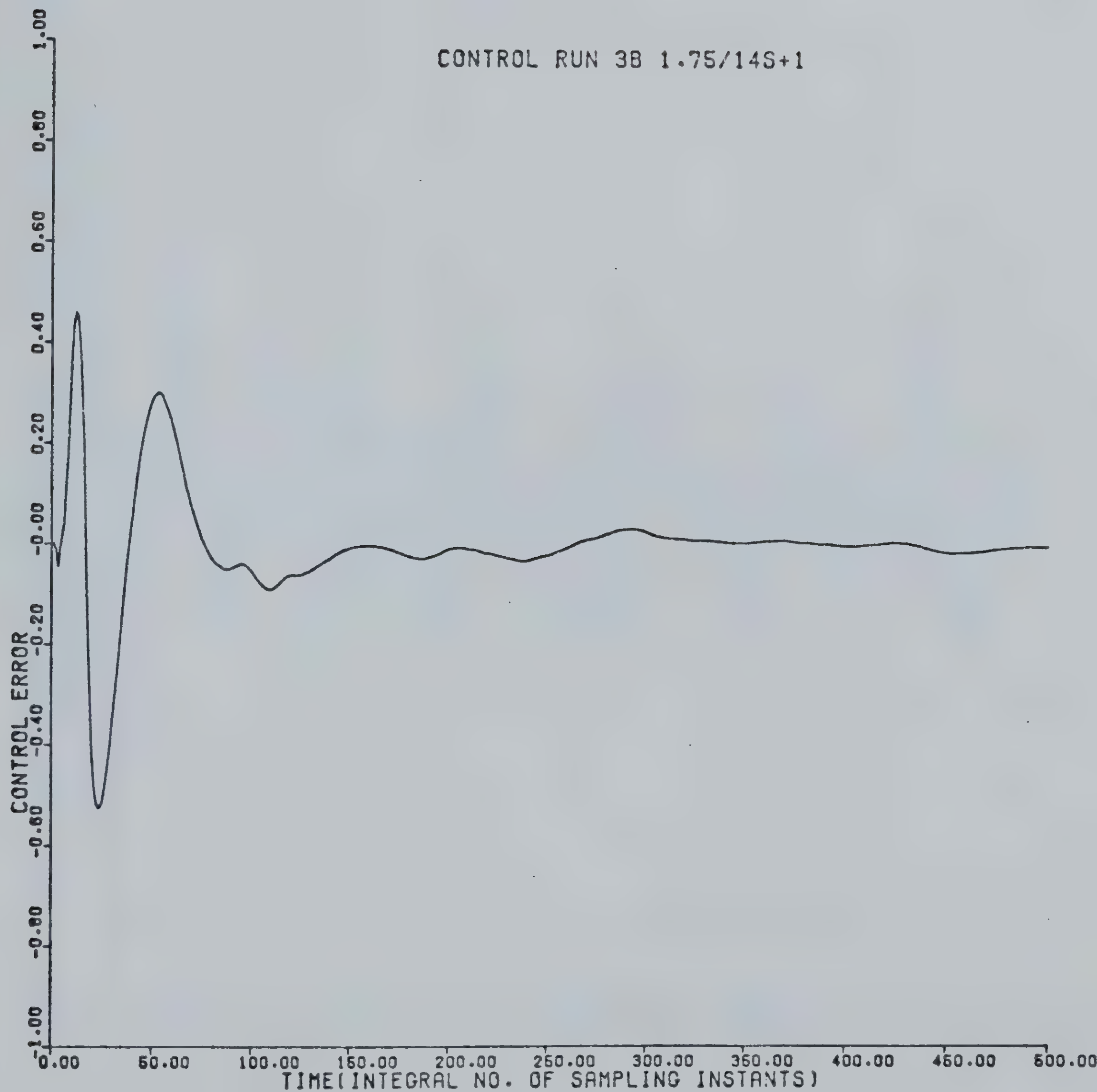


FIGURE 6.7(c): SISO SYSTEM 1 1.75/(14s + 1)
CONTROL ERROR VS TIME

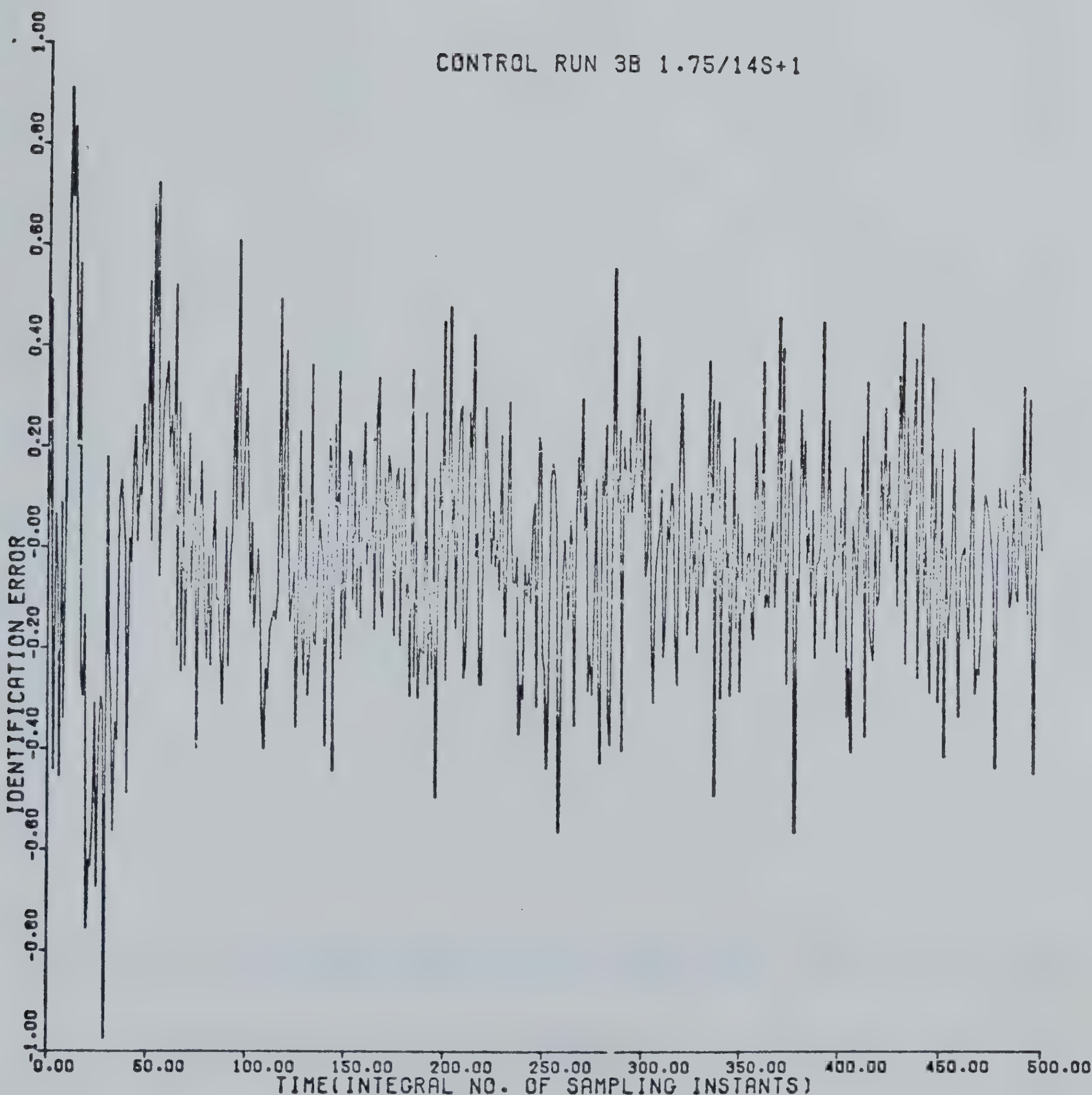


FIGURE 6.7(d): SISO SYSTEM 1 1.75/(14s + 1)
IDENTIFICATION ERROR VS TIME

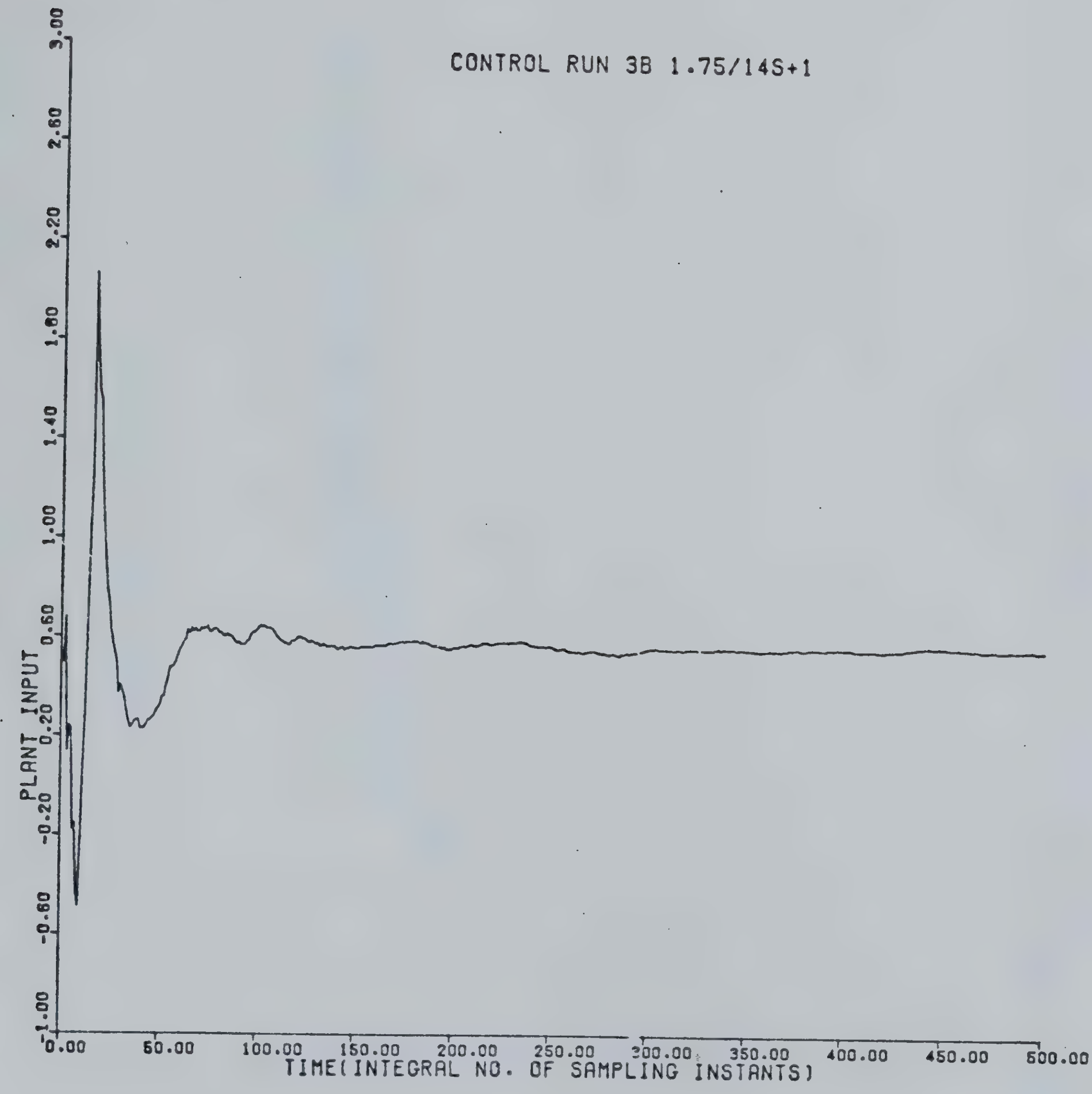


FIGURE 6.7(e): SISO SYSTEM 1 1.75/(14s + 1)
PLANT INPUT VS TIME

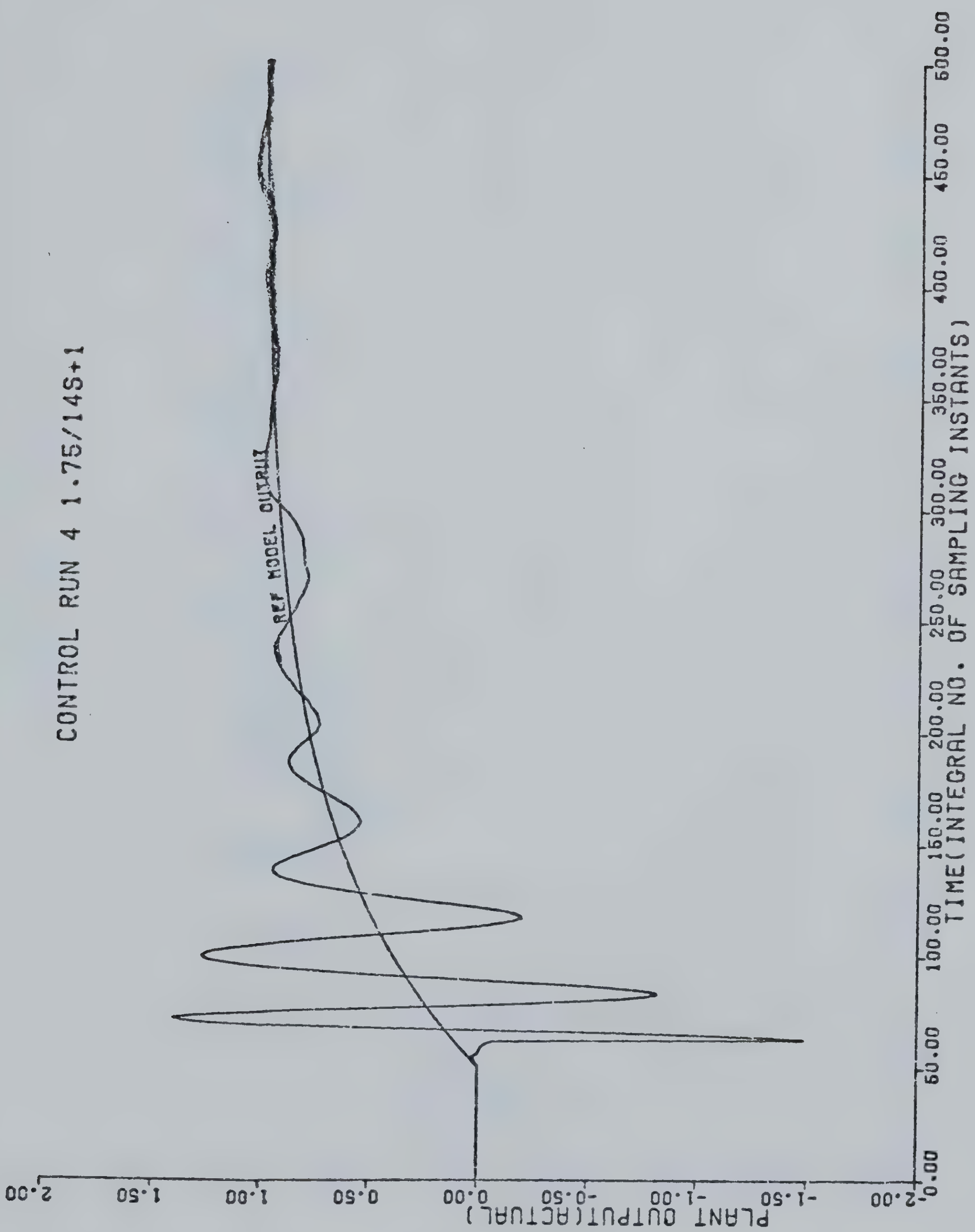


FIGURE 6.8(a): SISO SYSTEM 1 1.75/(14s + 1)
PLANT OUTPUT (ACTUAL) VS TIME

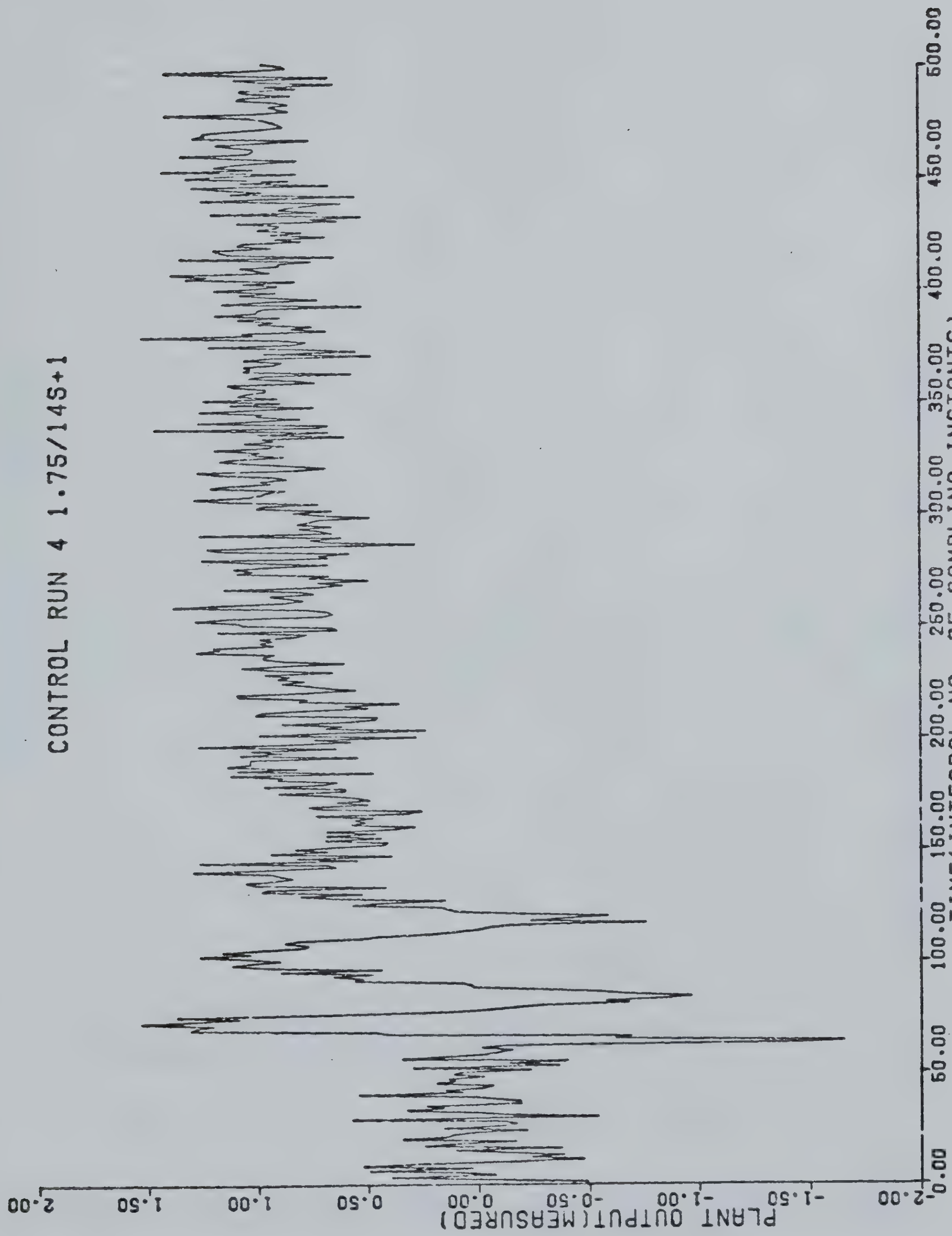


FIGURE 6.8(b): SISO SYSTEM 1 1.75/(14s + 1)
PLANT OUTPUT (MEASURED) VS TIME

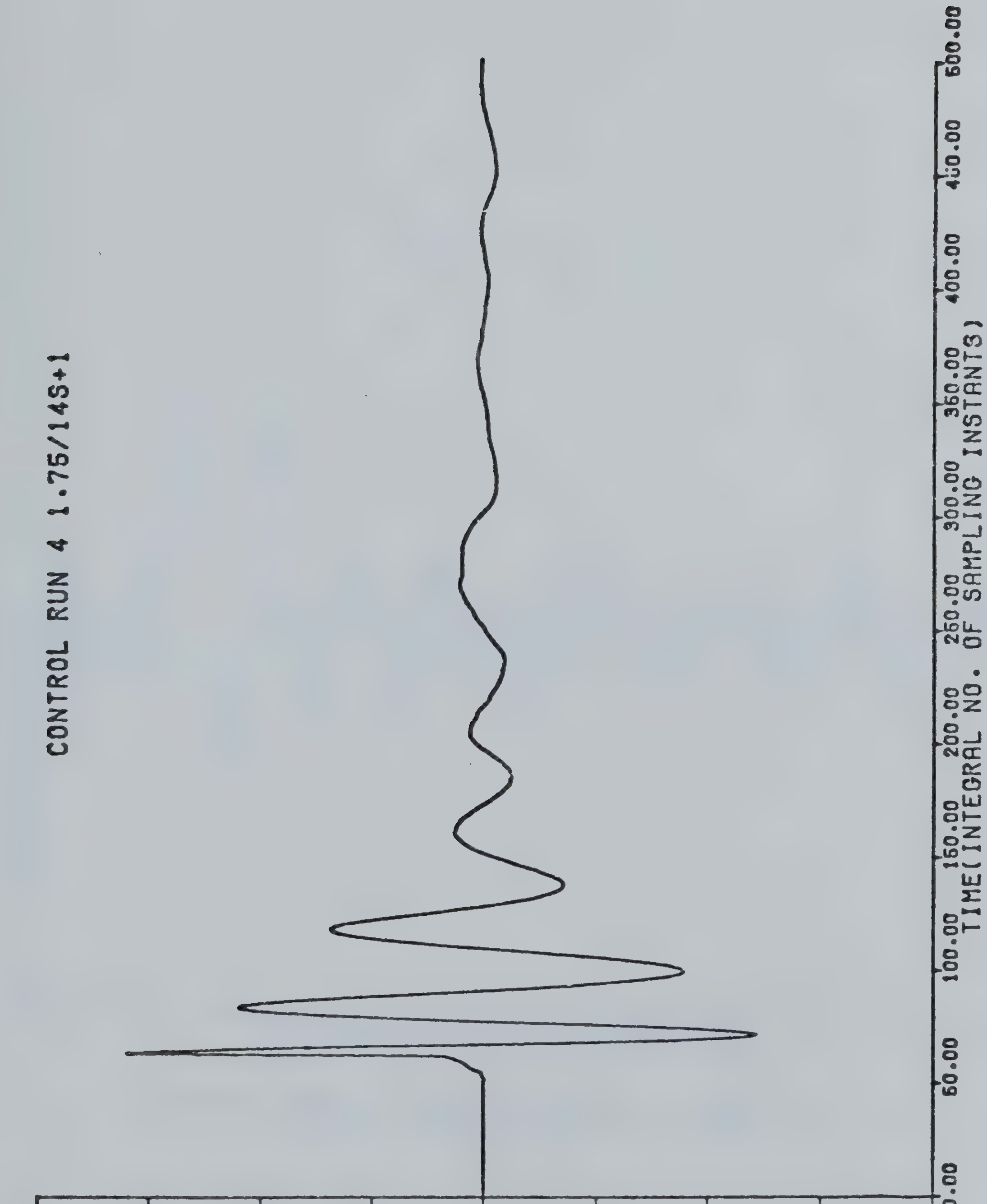


FIGURE 6.8(c): SISO SYSTEM 1 1.75/(14s + 1)
CONTROL ERROR VS TIME

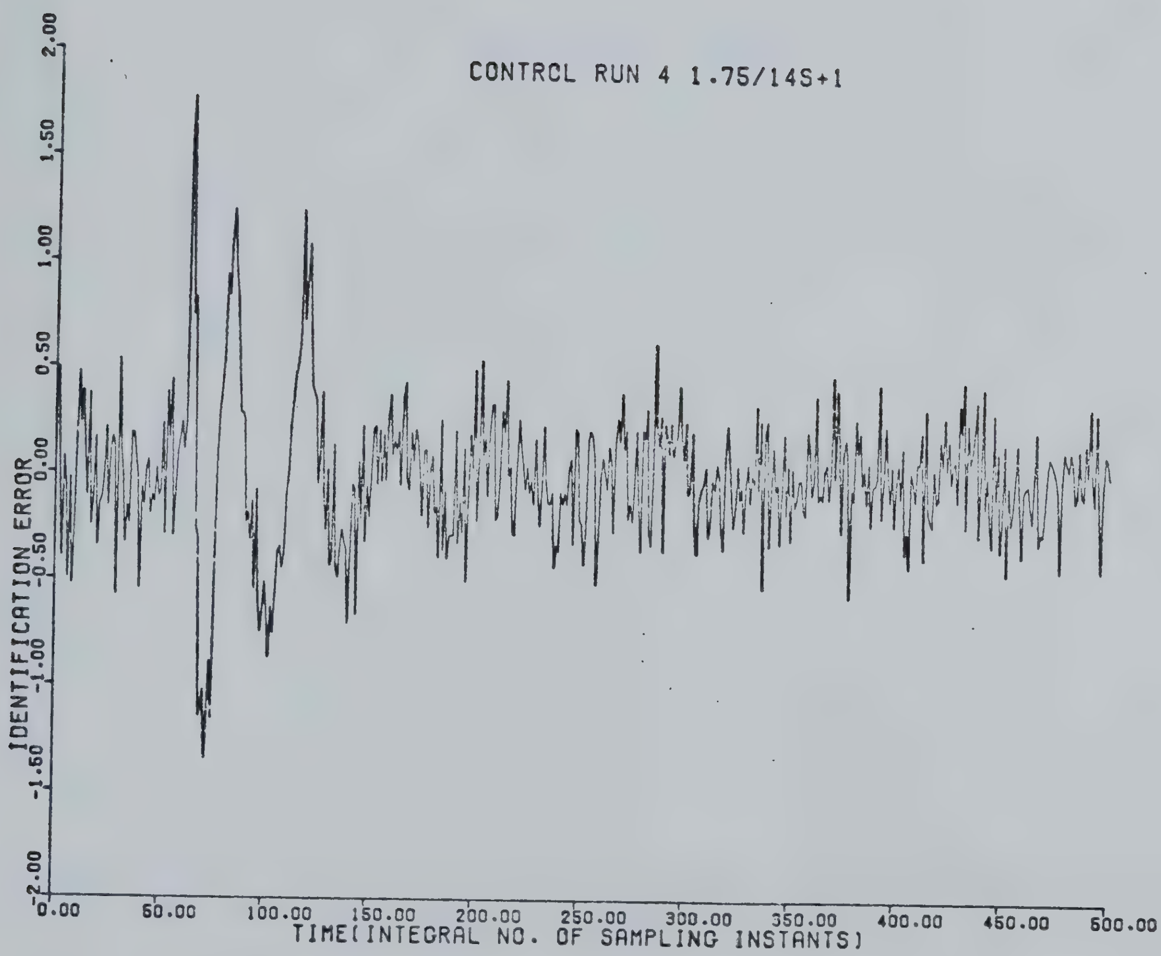


FIGURE 6.8(d): SISO SYSTEM 1 1.75/(14s + 1)
IDENTIFICATION ERROR VS TIME

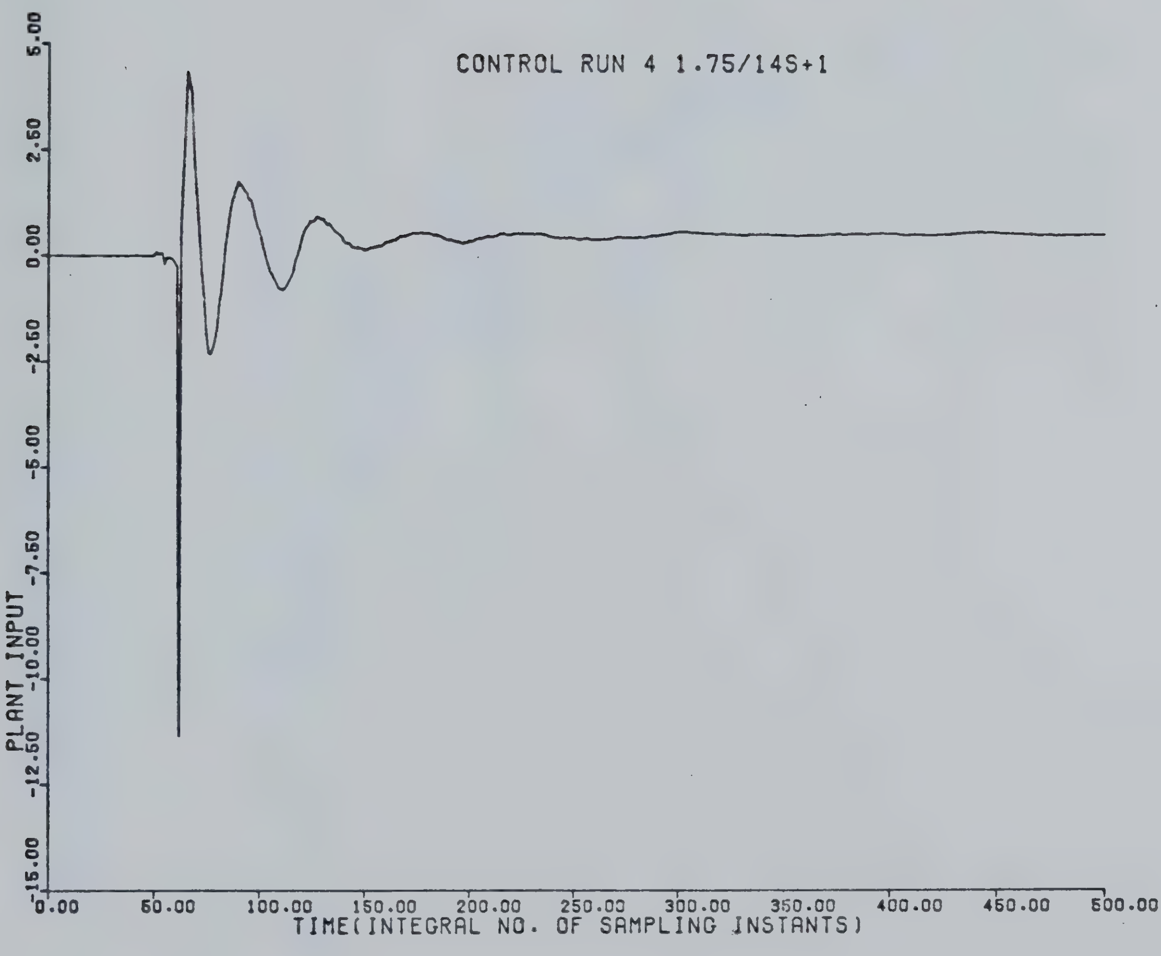


FIGURE 6.8(e): SISO SYSTEM 1 1.75/(14s + 1)
PLANT INPUT VS TIME

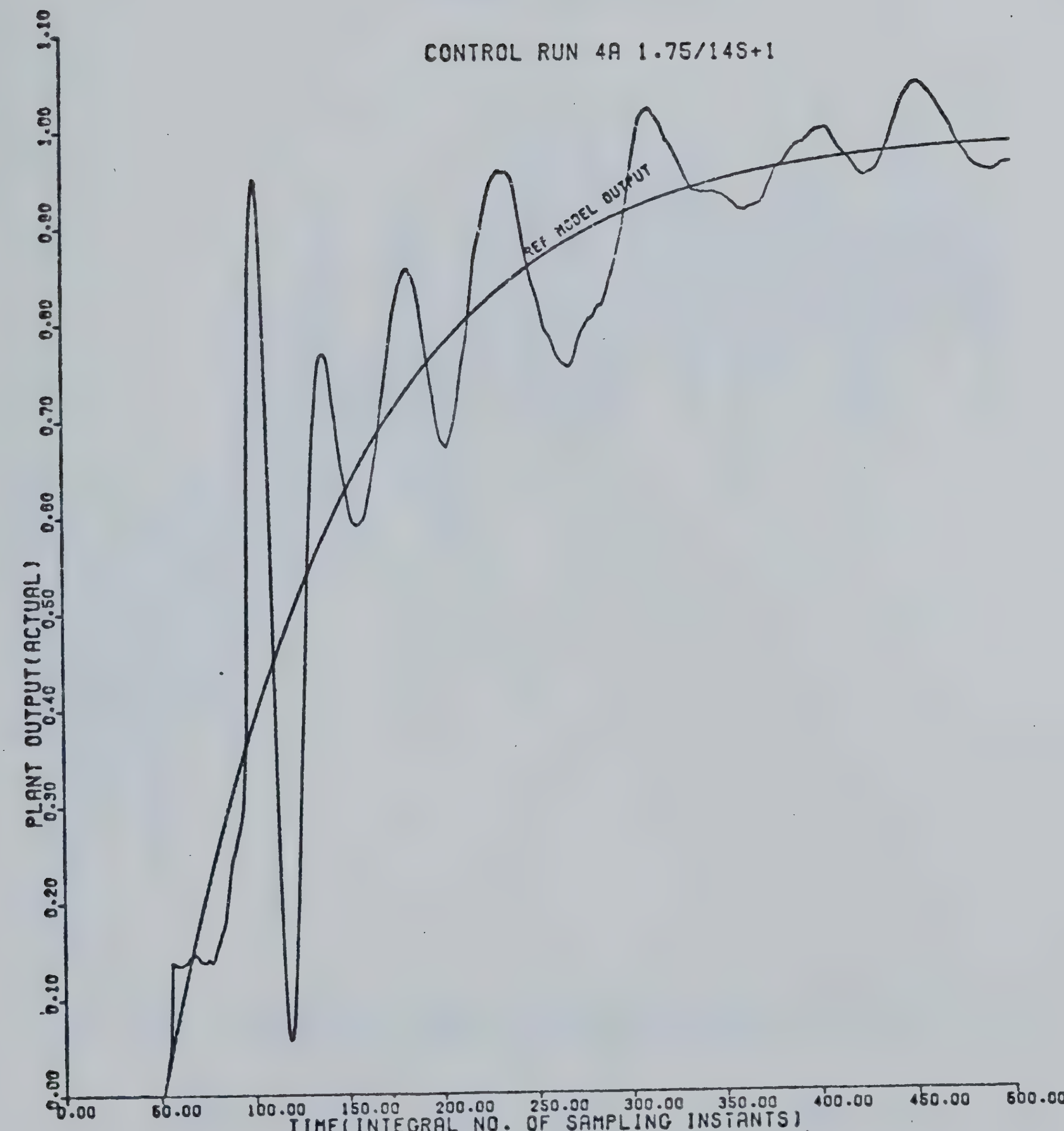


FIGURE 6.9(a): SISO SYSTEM 1 1.75/(14s + 1)
PLANT OUTPUT (ACTUAL) VS TIME

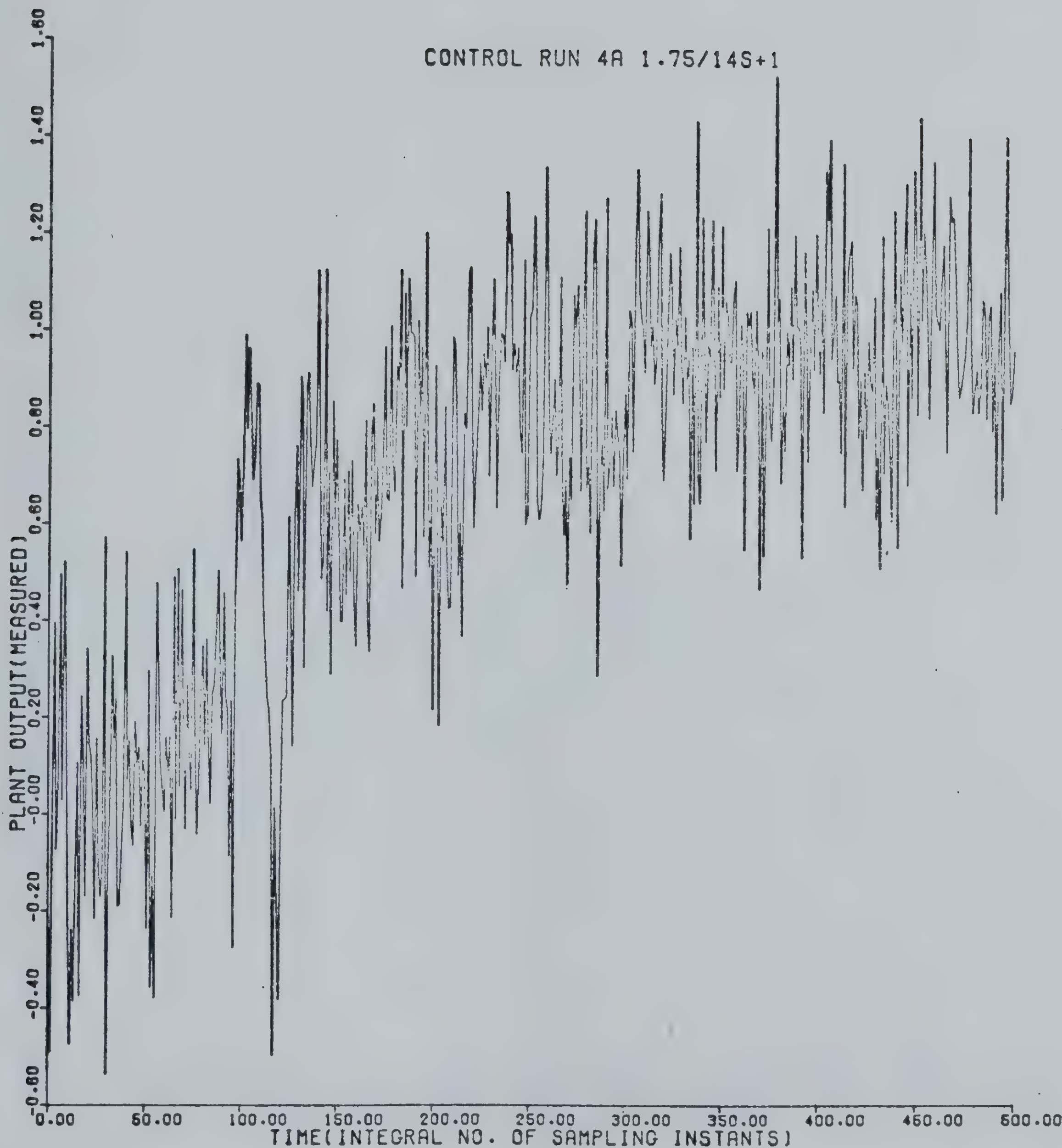


FIGURE 6.9(b): SISO SYSTEM 1 1.75/(14s + 1)
PLANT OUTPUT (MEASURED) VS TIME

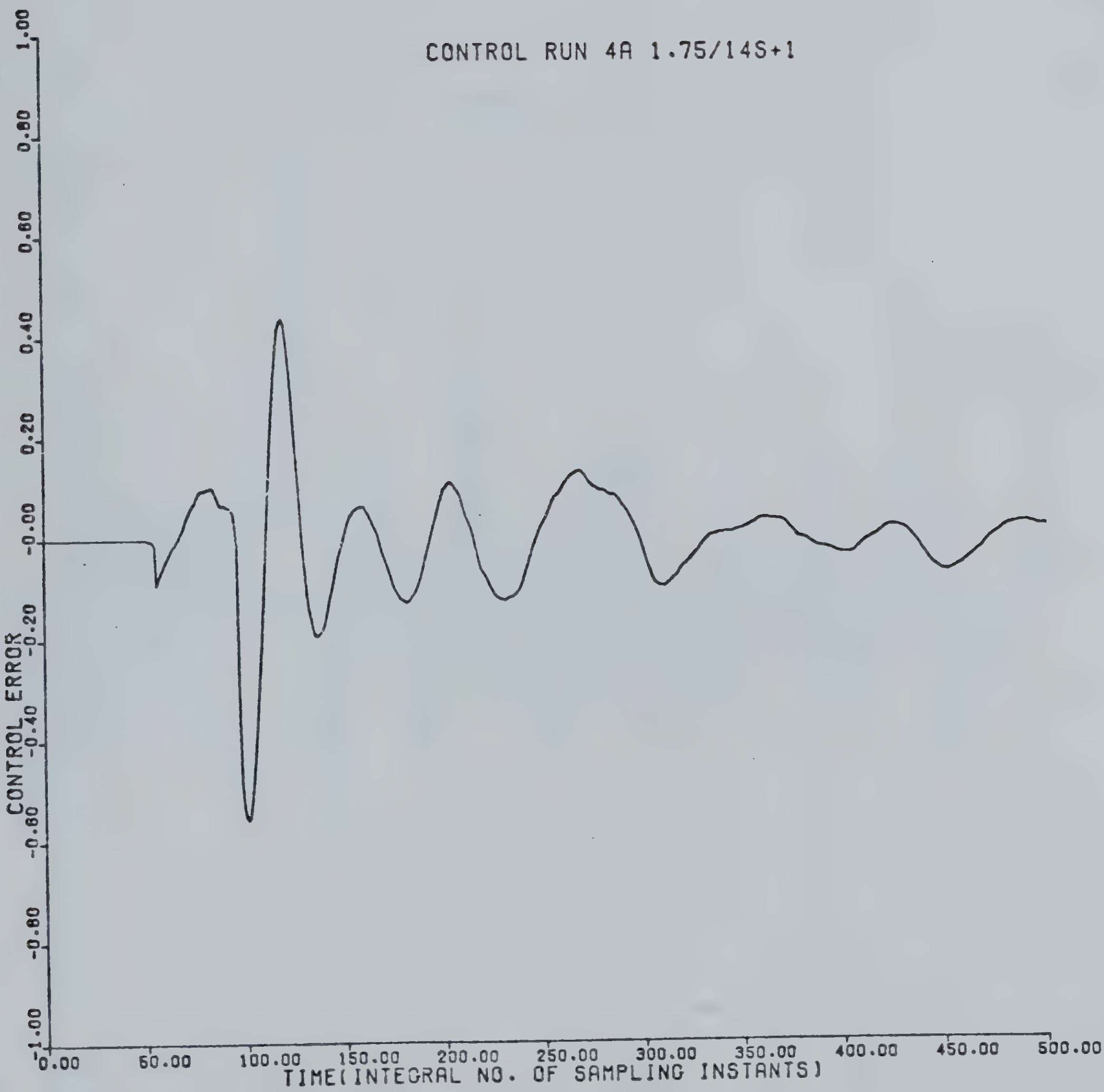


FIGURE 6.9(c): SISO SYSTEM 1 1.75/(14s + 1)
CONTROL ERROR VS TIME

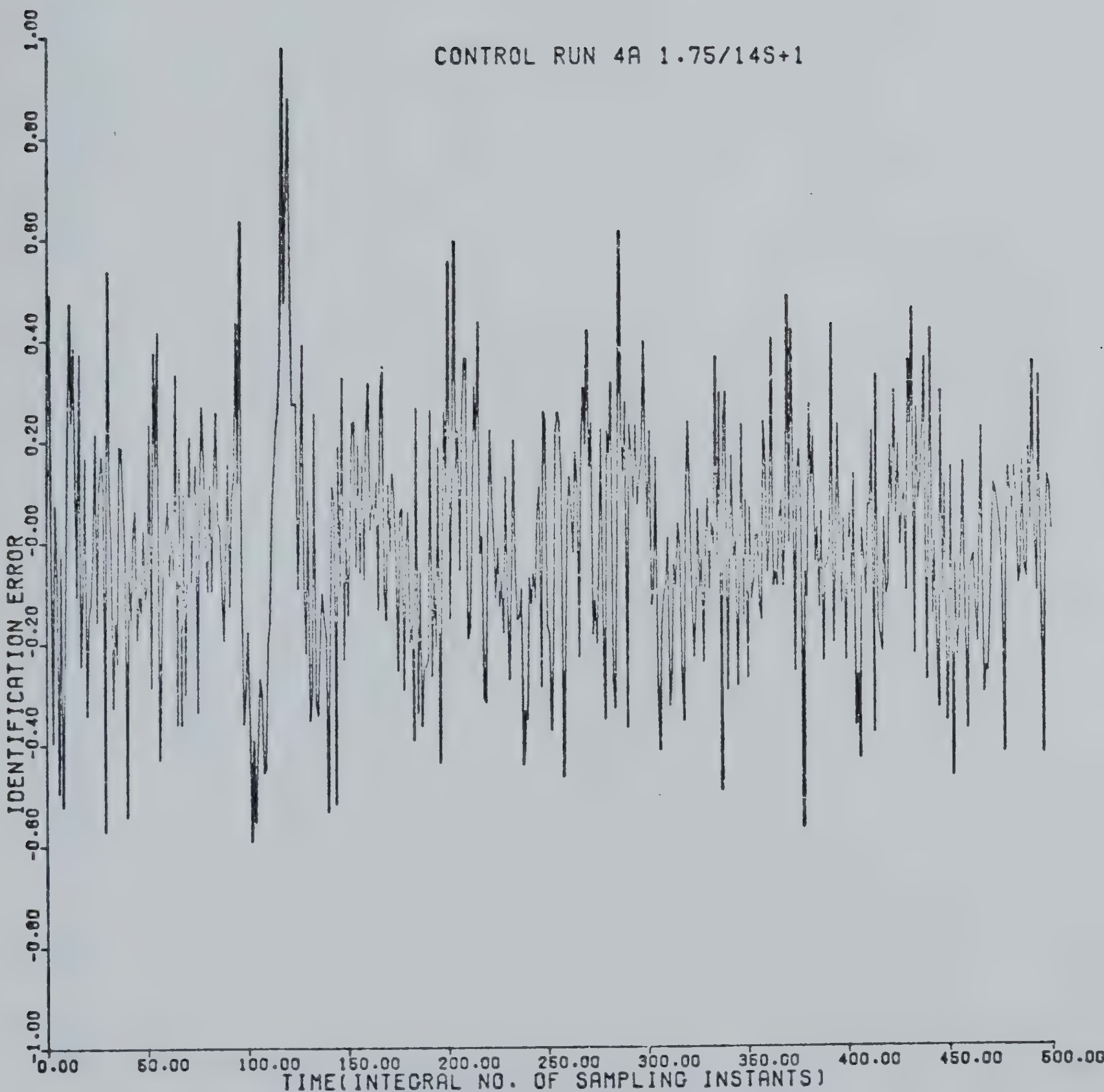


FIGURE 6.9(d): SISO SYSTEM 1 1.75/(14s + 1)
IDENTIFICATION ERROR VS TIME

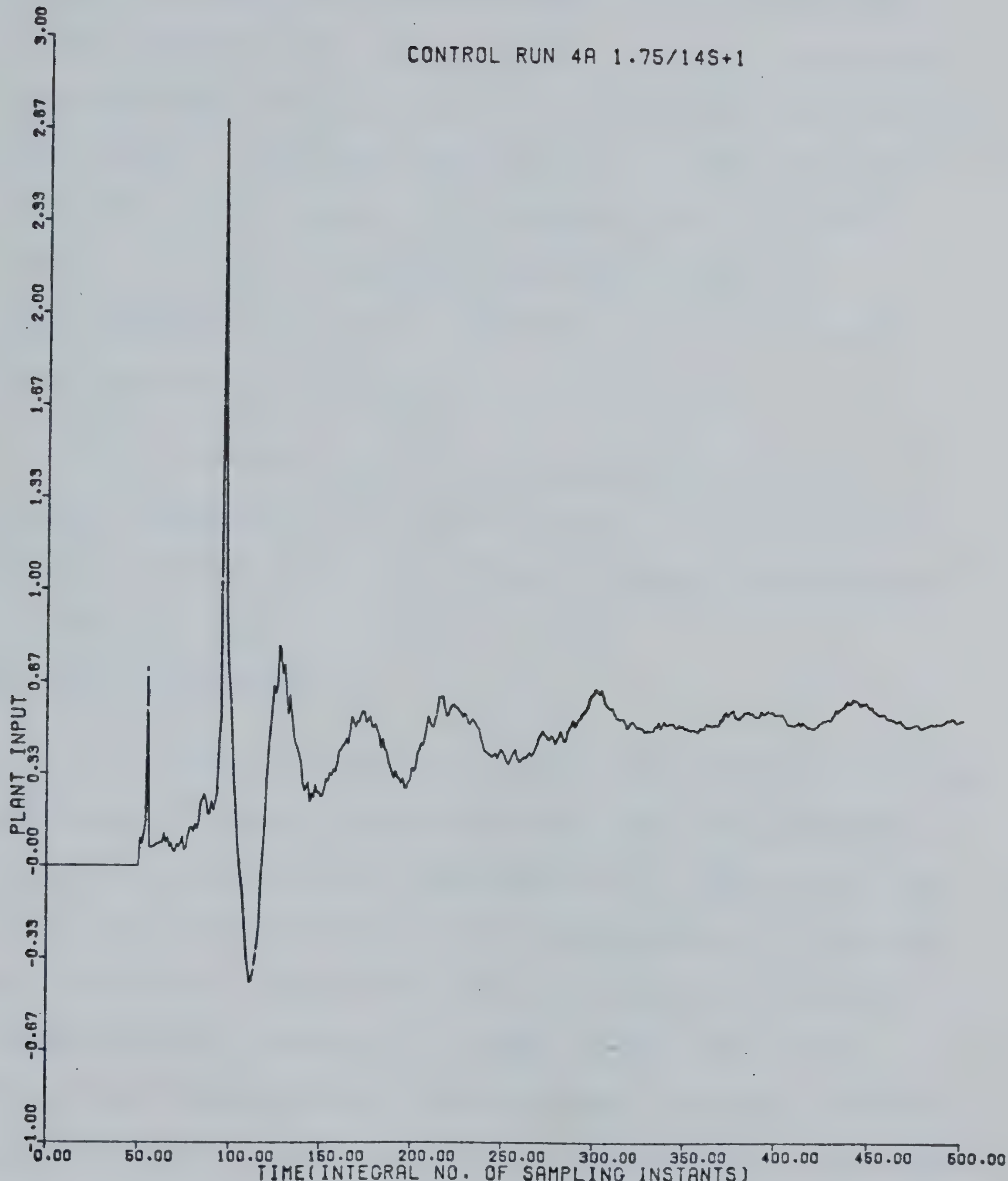


FIGURE 6.9(e): SISO SYSTEM 1 1.75/(14s + 1)
PLANT INPUT VS TIME

and 6.17 may be compared, respectively. In the first response of each set, the trajectory of the actual plant output approaches the desired value. However, the second responses show unacceptable oscillation abcut the reference model output. Although these results are obtained using very large noise inputs, the recommendation is clearly towards the use of decreasing magnitude adaptive identification gains in the presence of noise corrupted measurements.

Run 1 (Figures 6.10) show the responses obtained without measurement noise being present. The actual plant output depicted in Figure 6.10(a) is rapidly convergent to the desired output.

b) Outer Loop Gains

It is dubious whether the outer loop gains are a viable design parameter for this system, at all. The responses shown in the two sets of runs depicted by Figures 6.11 and 6.14, and Figures 6.15 and 6.16, respectively, do not show any significant improvement over one another. It can be argued that the response of Figure 6.16(a) shows some improvement over that of Figure 6.15(a); however, the outer loop gains vary by three orders of magnitude for these runs and the effect must be considered, at best, insensitive.

c) Variation of the Rate of Decrease of the Magnitude of the Identificaticn Loop Gains

As with the previous design parameter choice, this decision does not seem to be a critical one for this system. Indeed, the results shown in Figures 6.16 and 6.18 show little difference. In spite of this, the general inclination would be to specify a large filtering action for large measurement noise inputs.

d) Choice of Initial Identificaticn Loop Gains

Figures 6.13 and 6.15 show the response characteristics when one changes the initial identification loop gains tenfold. In general terms, the actual plant output characteristic shown in Figure 6.13(a), shows less overshoot than does Figure 6.15(a), especially in the initial stages of the run. The latter part of the response, however, indicates a closer convergence to the desired trajectory for Figure 6.15(a). An explanation for this effect can be advanced if it is assumed that the adaptive gains behave as do normal conventional controller gains, ie. the larger the initial gains are, the more oscillatory will be the initial output. At the same time, because of the larger identification gains, the tracking of the plant output will occur more quickly and hence, the final convergence will be

closer.

6.6 Summary of the Single-Input Single-Output Results

The simulation results discussed in the above section have demonstrated several effects of the design parameters of the hyperstable adaptive control system of Figure 6.1.

For the two particular examples discussed, several conclusions can be drawn:

(i) Constant identification loop gains cannot cope with noise corrupted measurements and decreasing magnitude gains should be used wherever unfiltered measurement noise is a problem.

(ii) The overall system appears to be quite insensitive to the magnitude of the outer adaptation loop gains and these are a relatively unimportant design specification.

(iii) The amount of filtering of noise corrupted measurements (as determined by the values of the parameters, λ) is important in certain cases. Generally, it would be recommended that the amount of filtering action used, should reflect the magnitude of noise present in the plant output, and,

(iv) The larger the initial identification loop gains, the more oscillatory will be the initial stages of the plant response. Commeasurate with this effect, is that higher

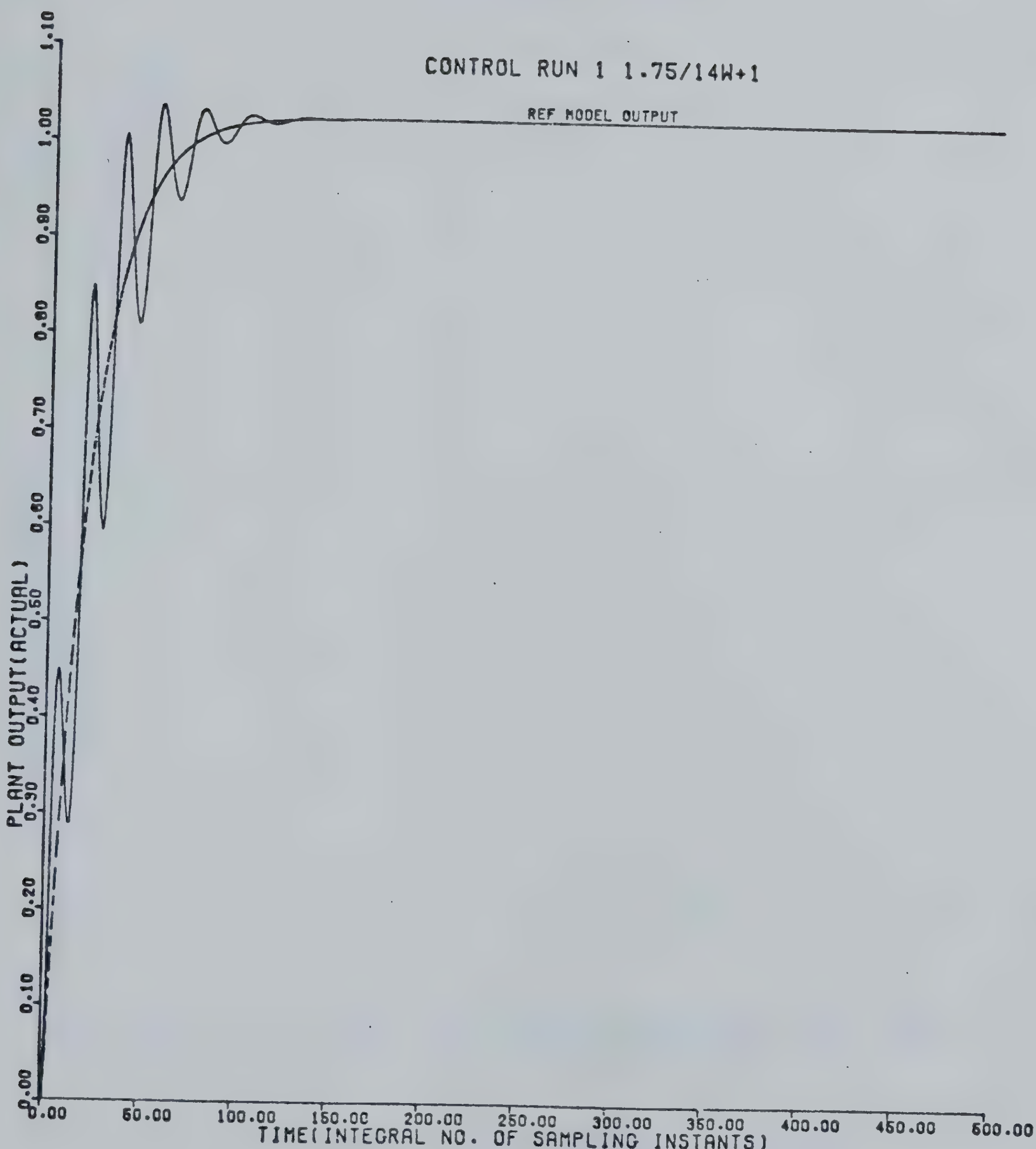


FIGURE 6.10(a): SISO SYSTEM 2 1.75/(14w + 1)
PLANT OUTPUT (ACTUAL) VS TIME

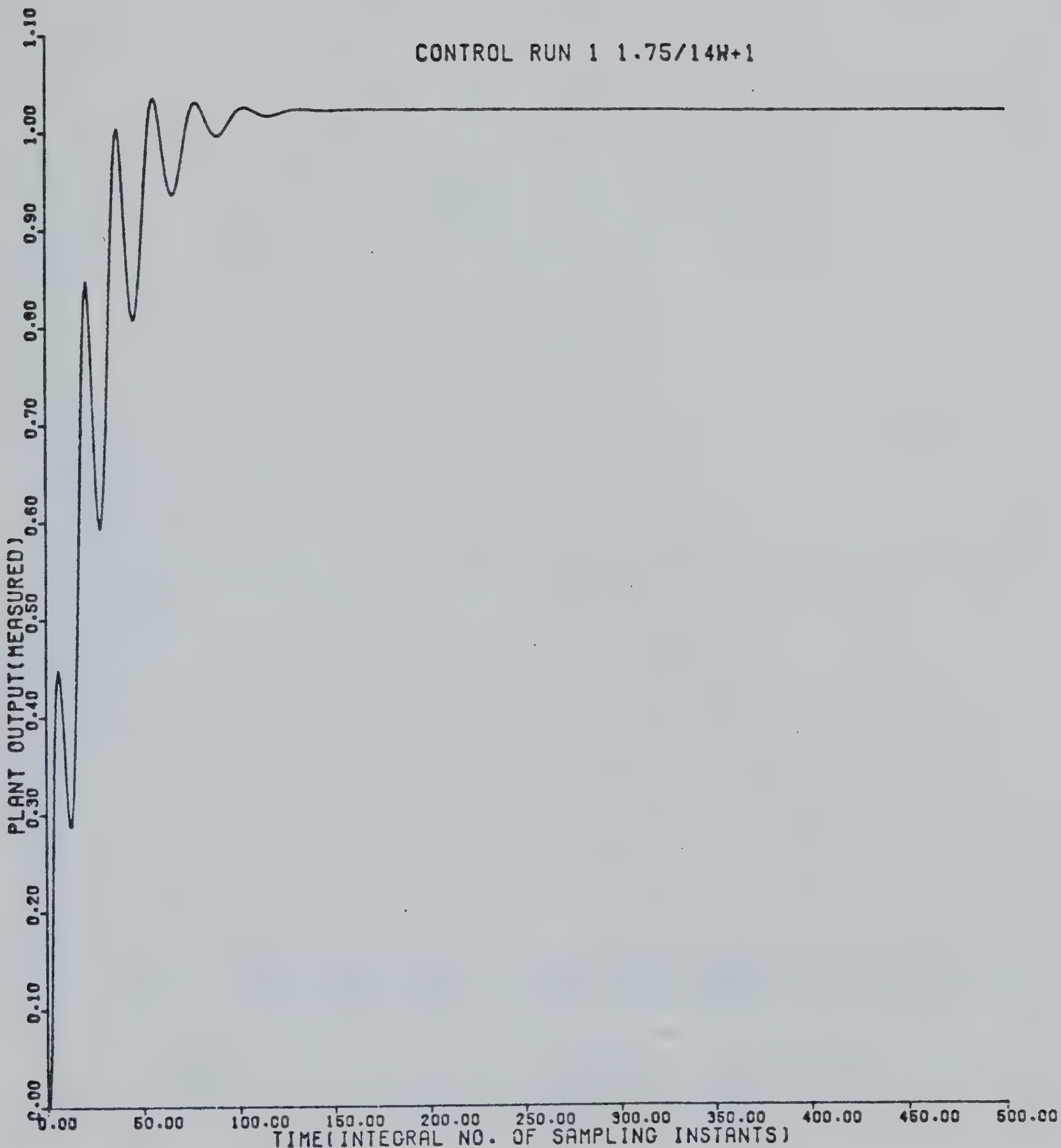


FIGURE 6.10(b): SISO SYSTEM 2 1.75/(14w + 1)
PLANT OUTPUT (MEASURED) VS TIME

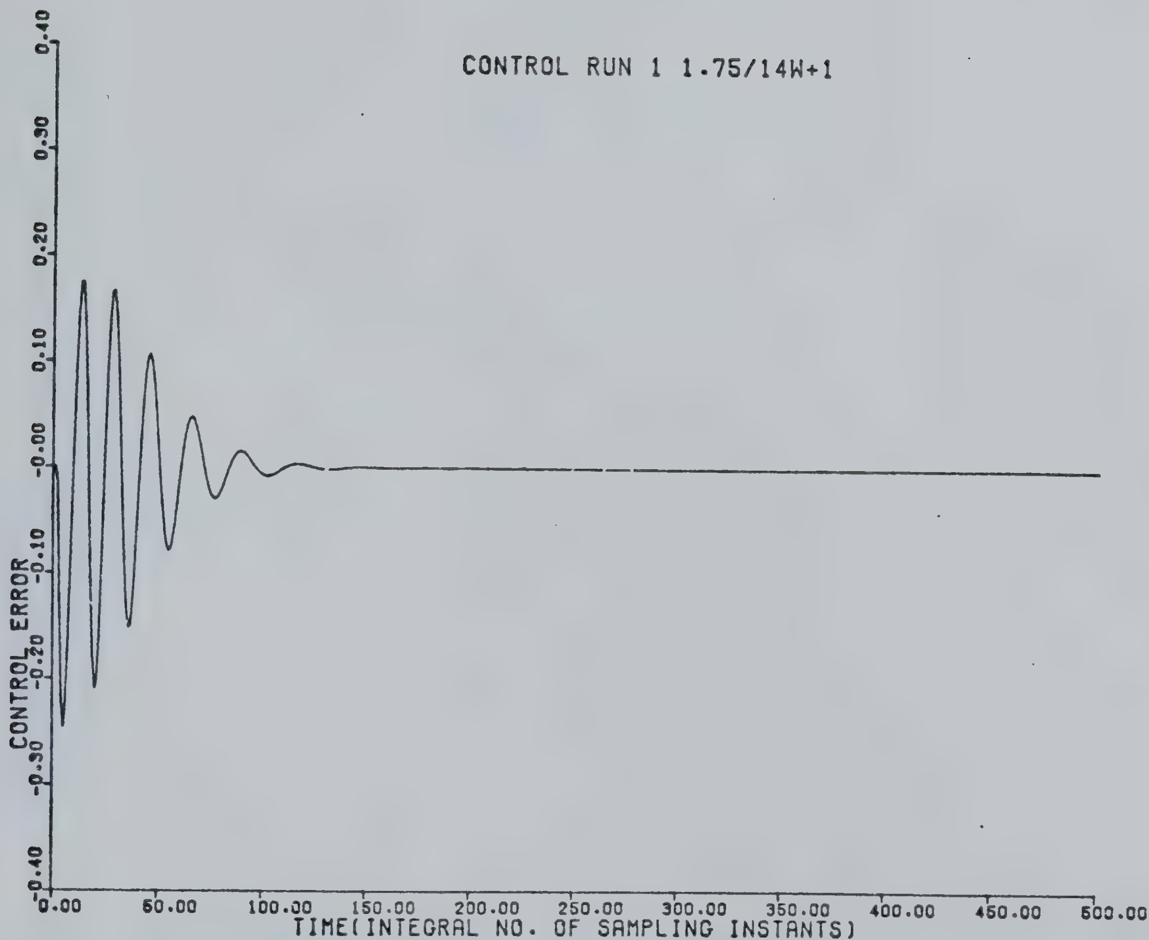


FIGURE 6.10(c): SISO SYSTEM 2 1.75/(14w + 1)
CONTROL ERROR VS TIME

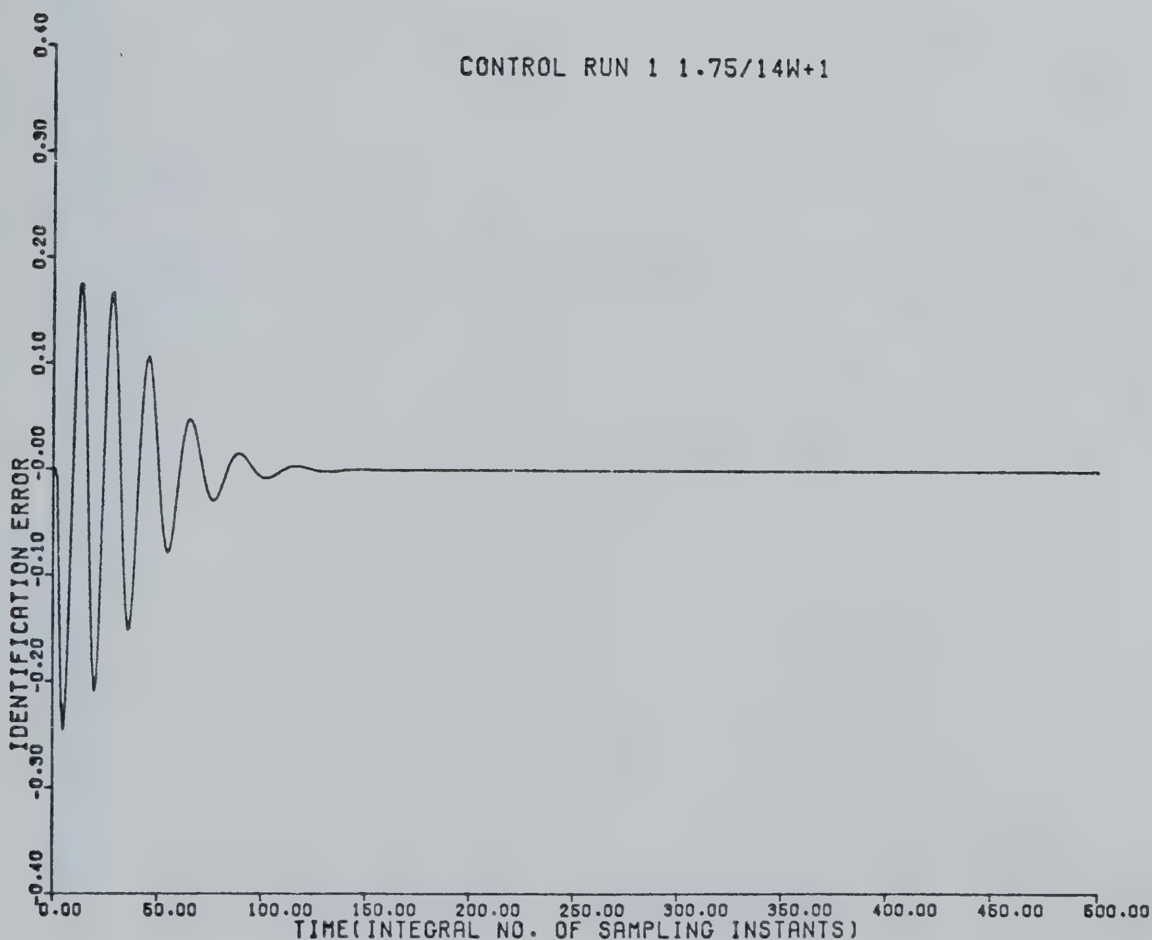


FIGURE 6.10(d): SISO SYSTEM 2 1.75/(14w + 1)
IDENTIFICATION ERROR VS TIME

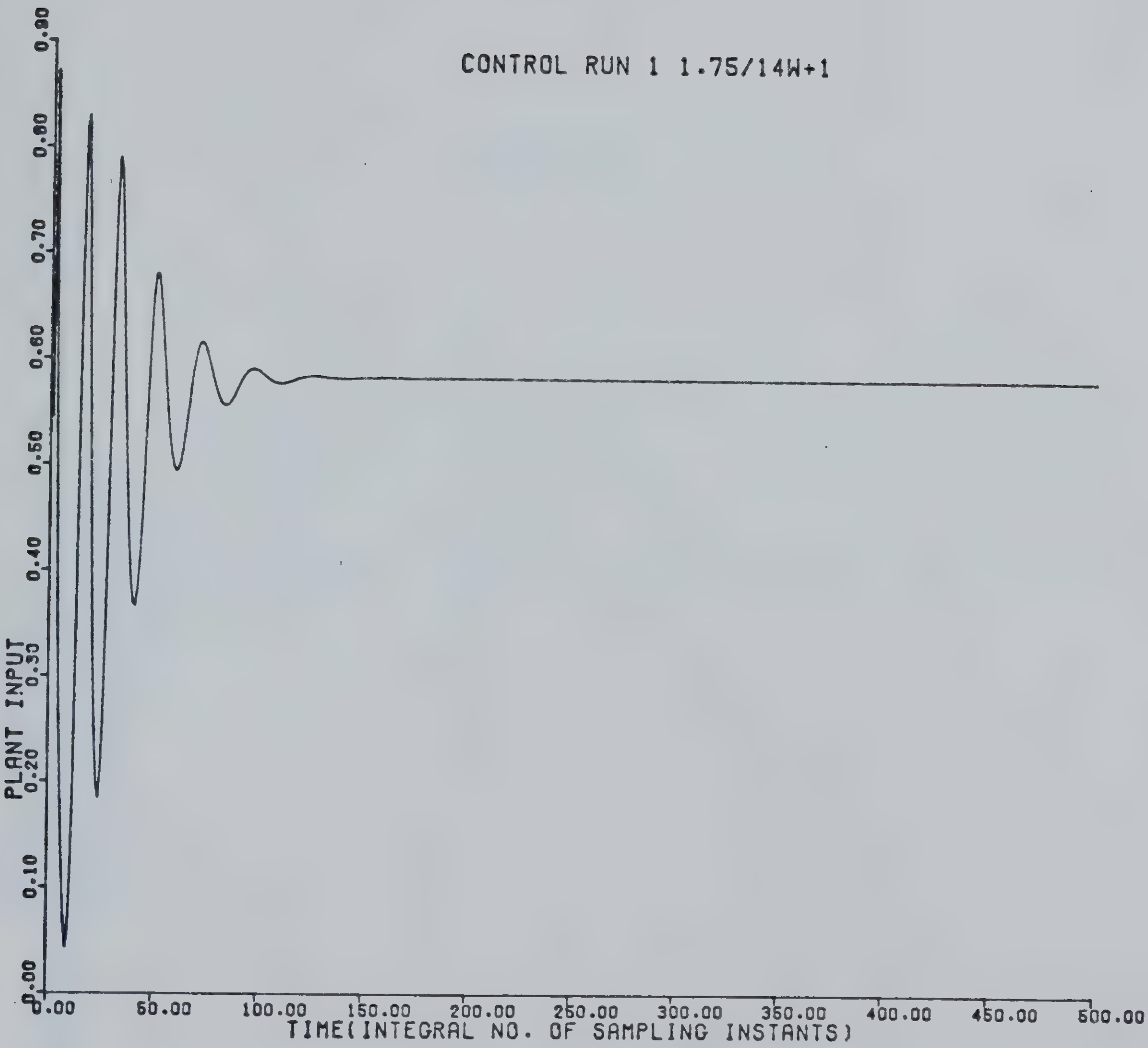


FIGURE 6.10(e): SISO SYSTEM 2 1.75/(14w + 1)
PLANT INPUT VS TIME

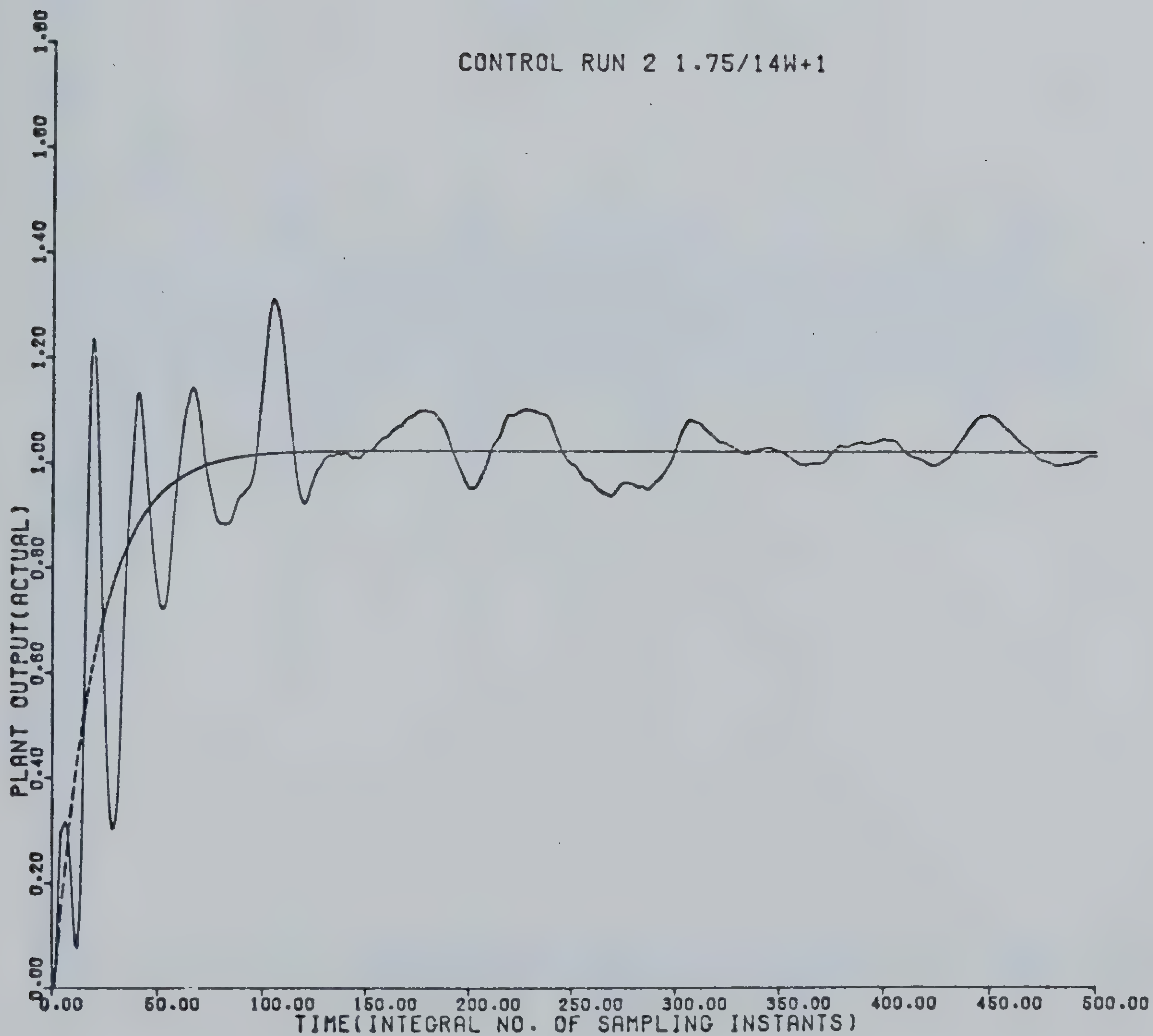


FIGURE 6.11(a): SISO SYSTEM 2 1.75/(14w + 1)
PLANT OUTPUT (ACTUAL) VS TIME

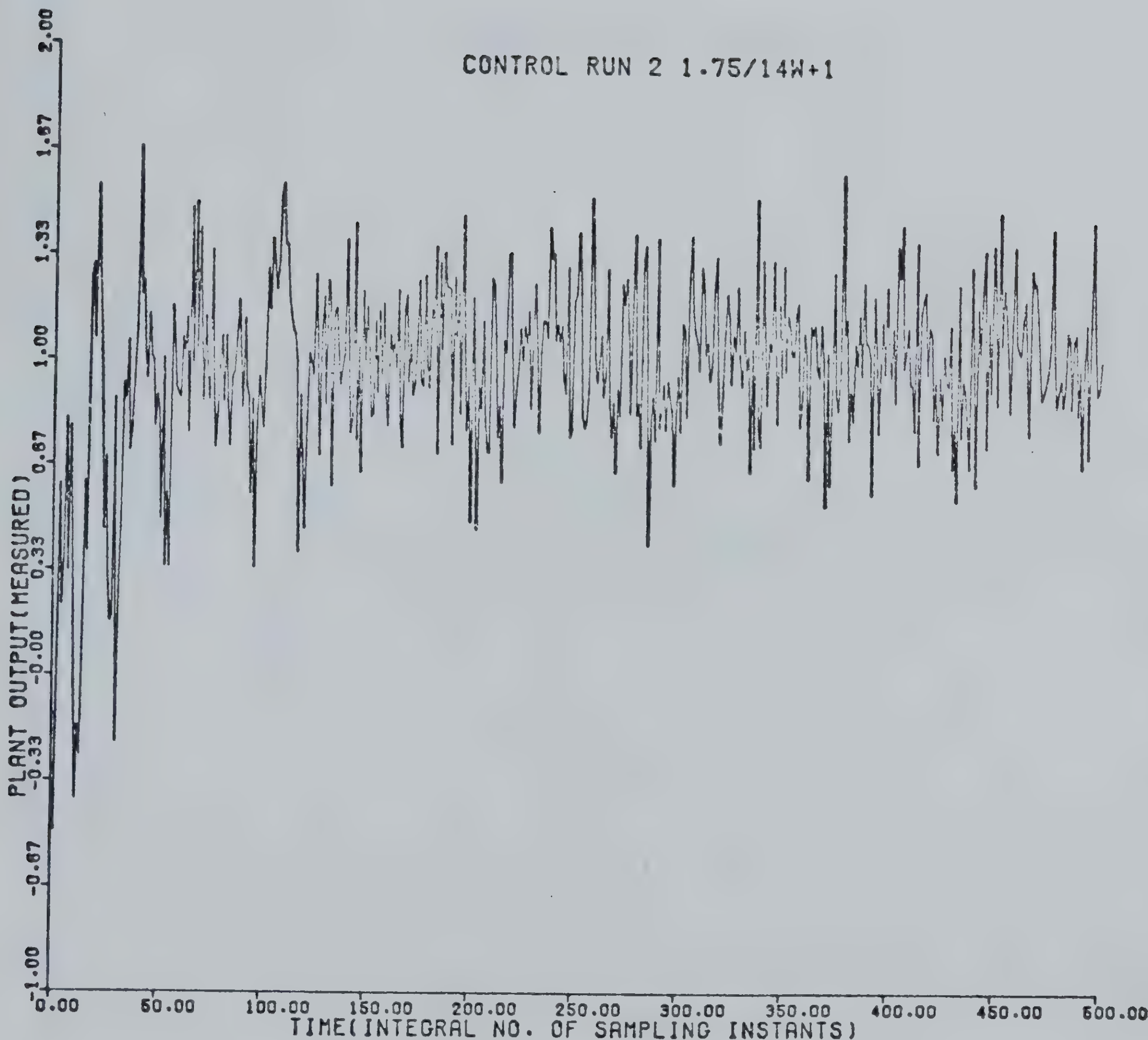


FIGURE 6.11(b): SISO SYSTEM 2 1.75/(14w + 1)
PLANT OUTPUT (MEASURED) VS TIME

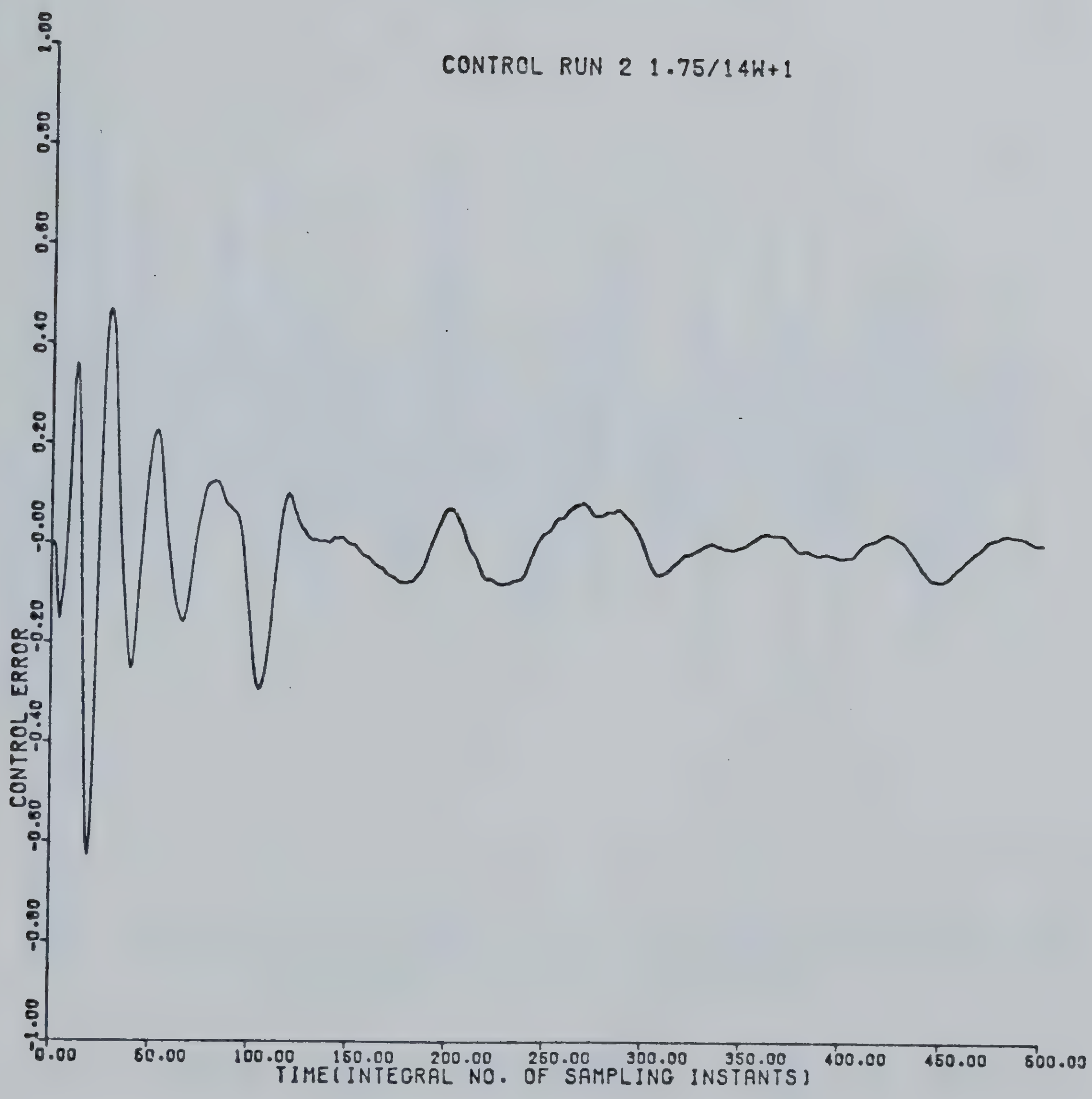


FIGURE 6.11(c): SISO SYSTEM 2 1.75/(14w + 1)
CONTROL ERROR VS TIME

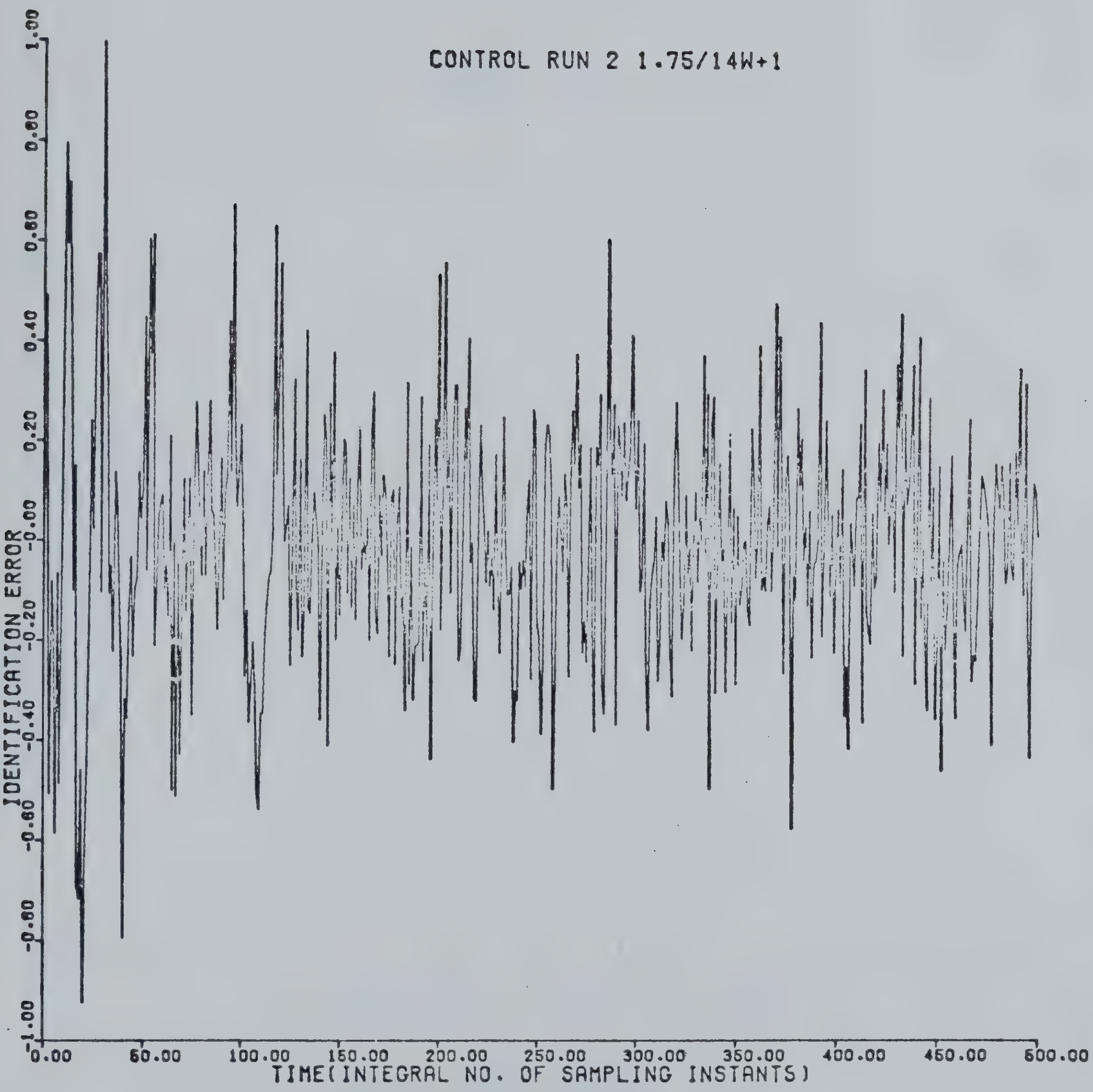


FIGURE 6.11(d): SISO SYSTEM 2 1.75/(14w + 1)
IDENTIFICATION ERROR VS TIME

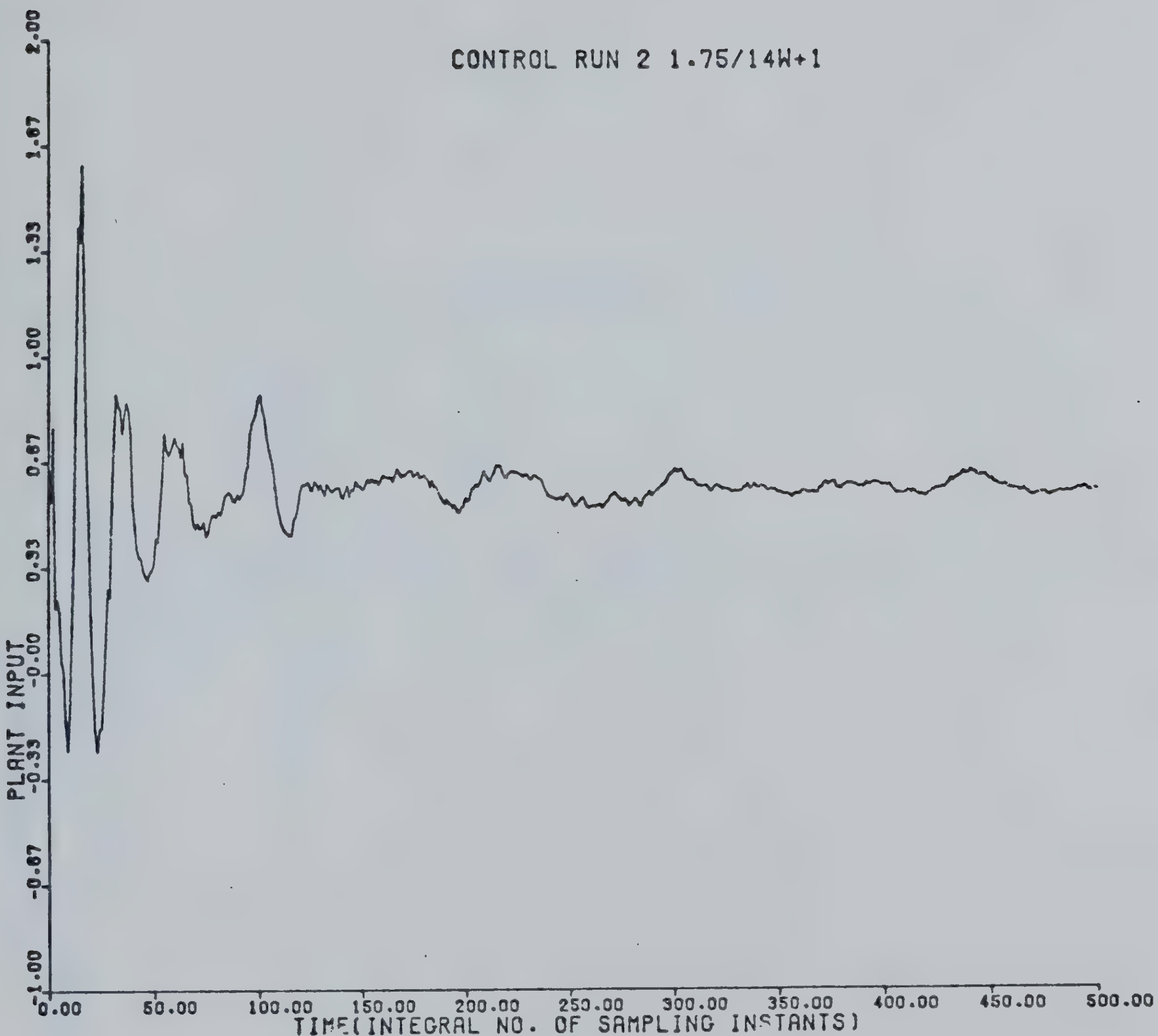


FIGURE 6.11(e): SISO SYSTEM 2 1.75/(14w + 1)
PLANT INPUT VS TIME

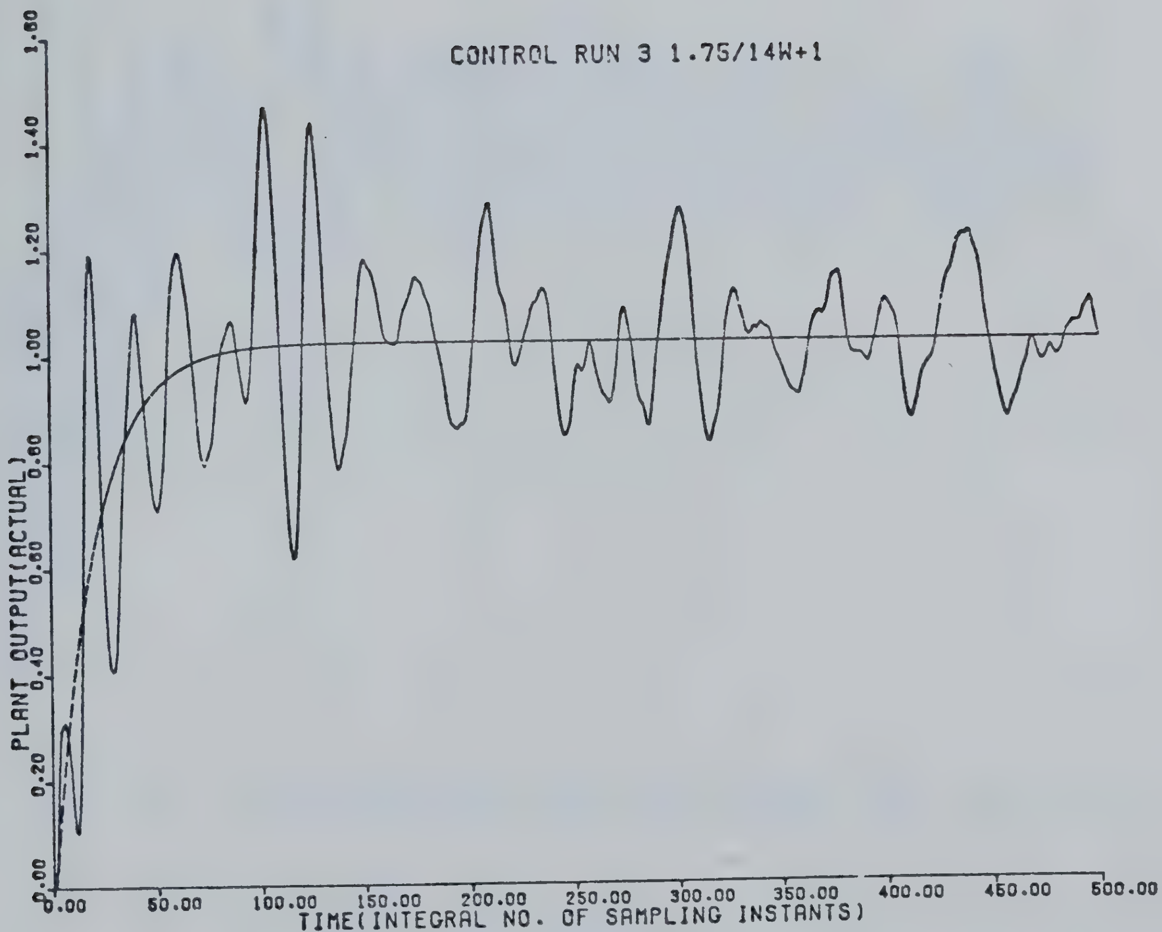


FIGURE 6.12(a): SISO SYSTEM 2 1.75/(14w + 1)
PLANT OUTPUT (ACTUAL) VS TIME

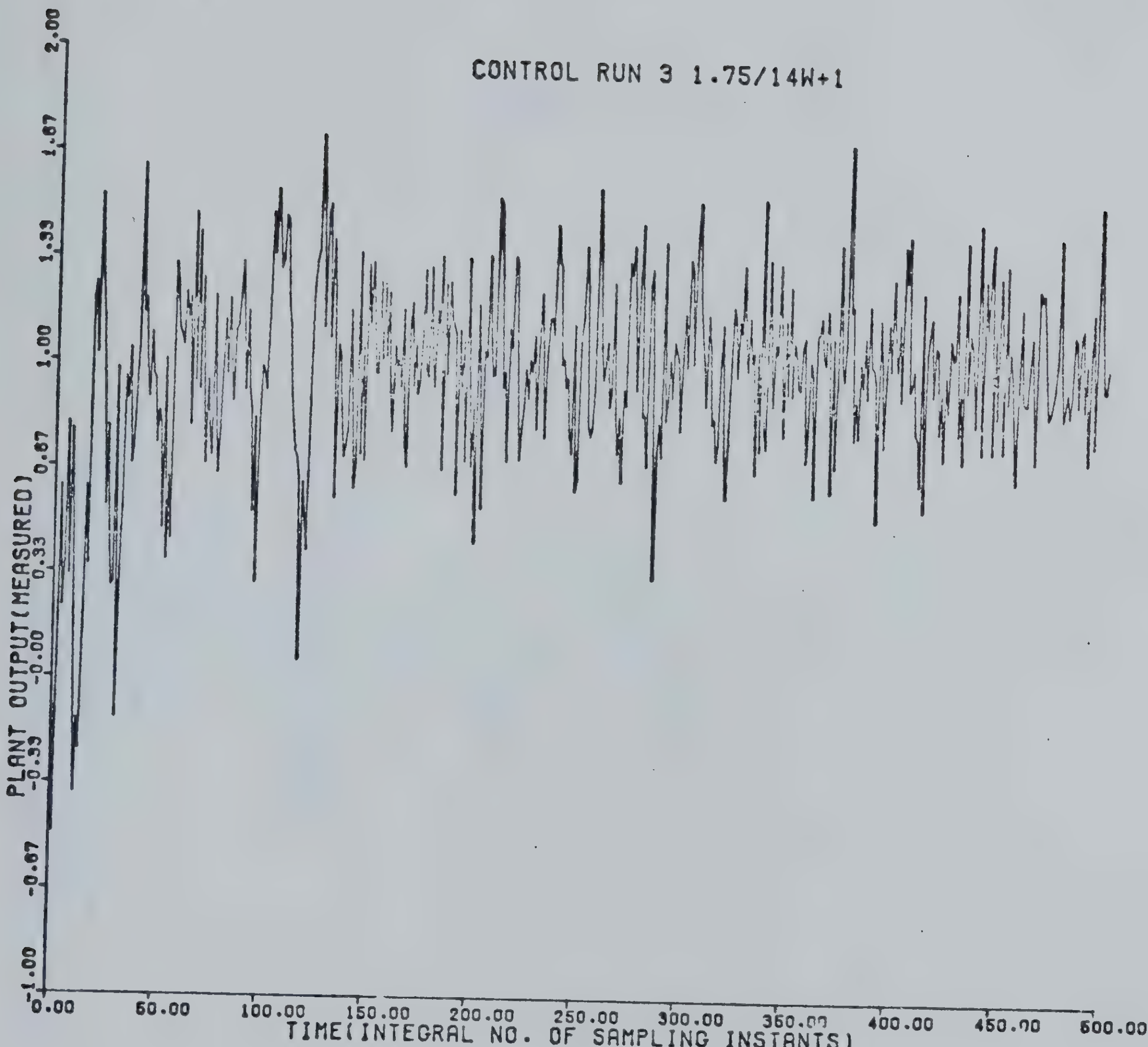


FIGURE 6.12(b): SISO SYSTEM 2 1.75/(14w + 1)
PLANT OUTPUT (MEASURED) VS TIME

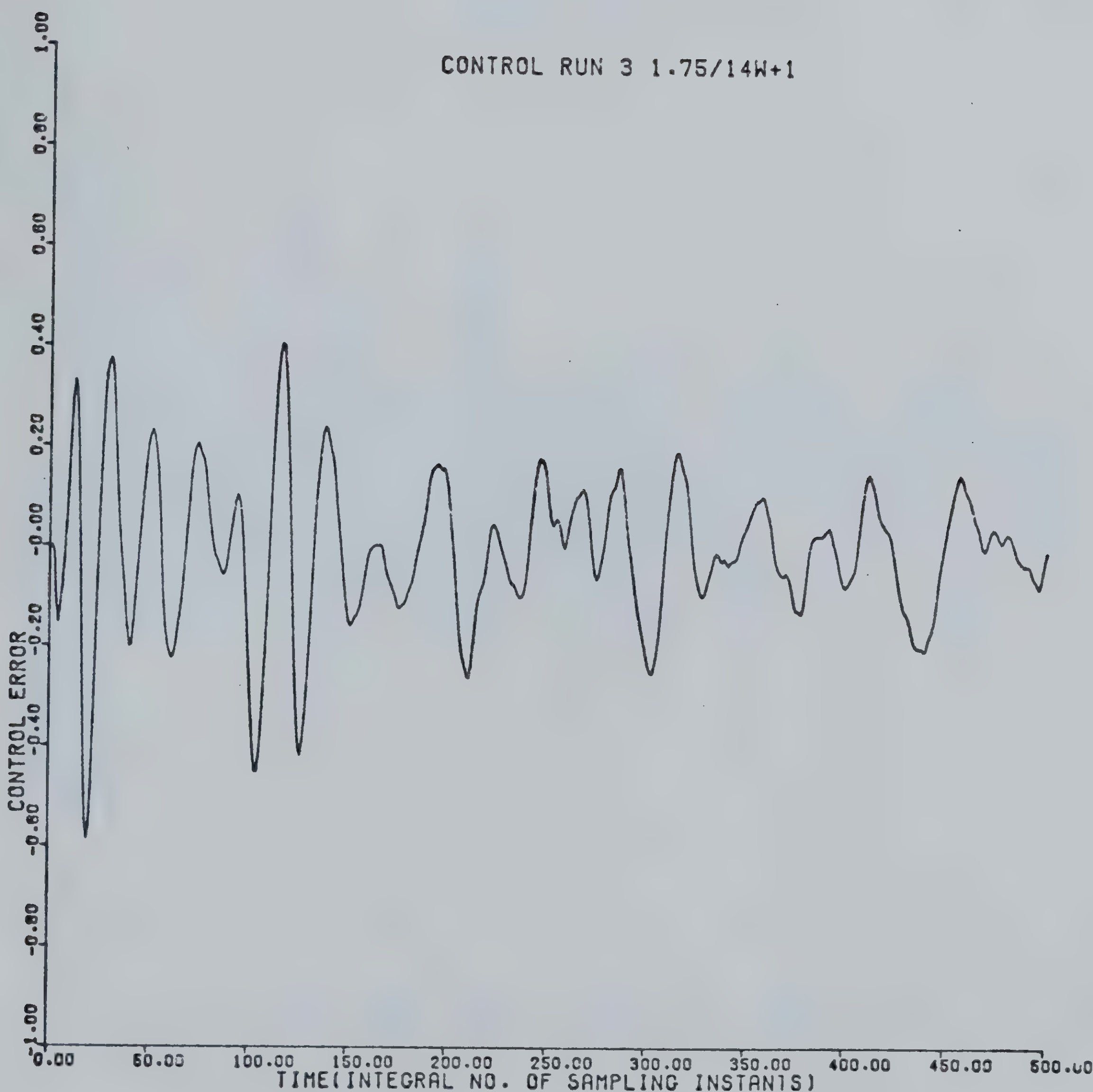


FIGURE 6.12(c): SISO SYSTEM 2 1.75/(14w + 1)
CONTROL ERROR VS TIME

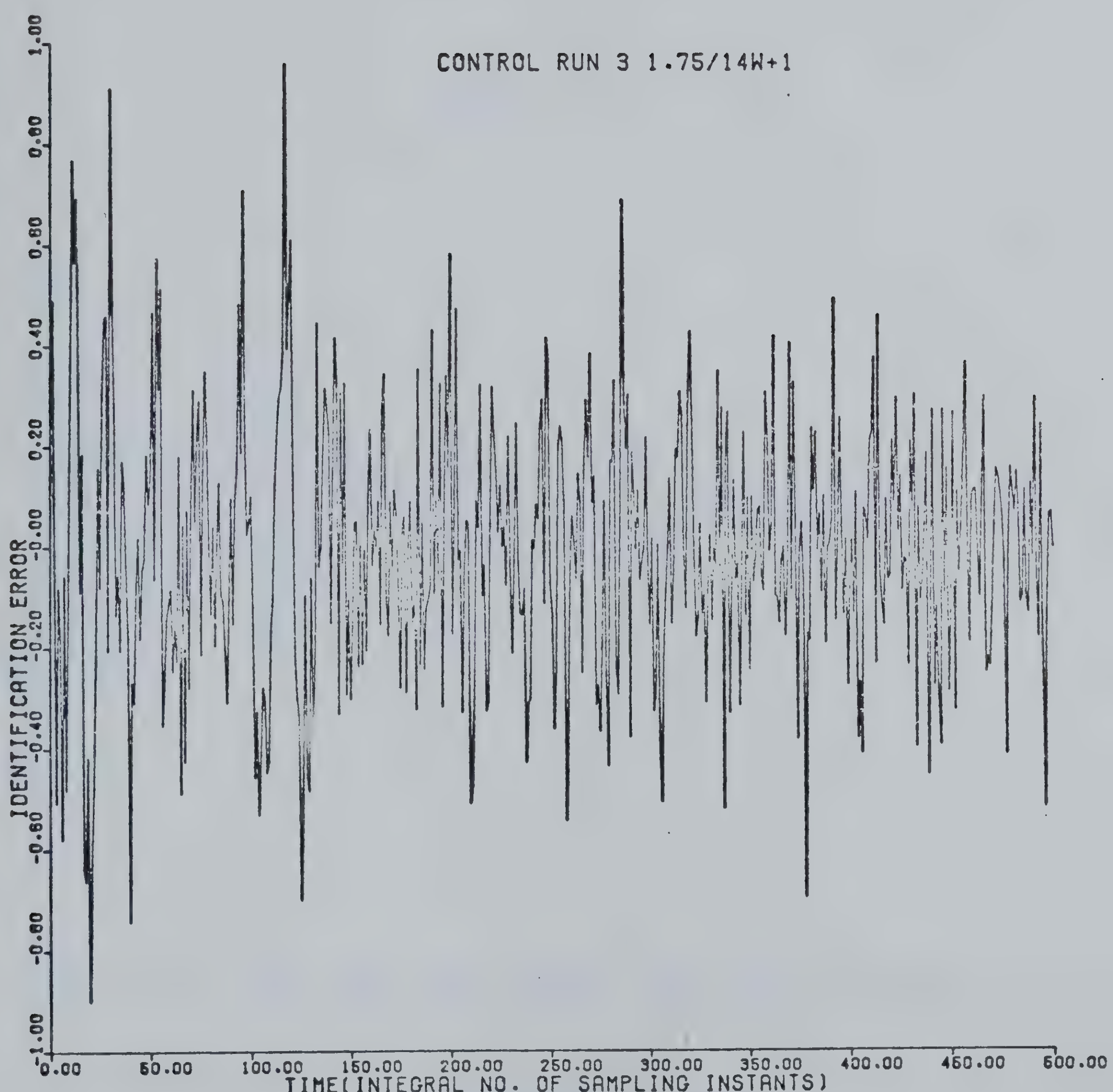


FIGURE 6.12(d): SISO SYSTEM 2 1.75/(14w + 1)
IDENTIFICATION ERROR VS TIME

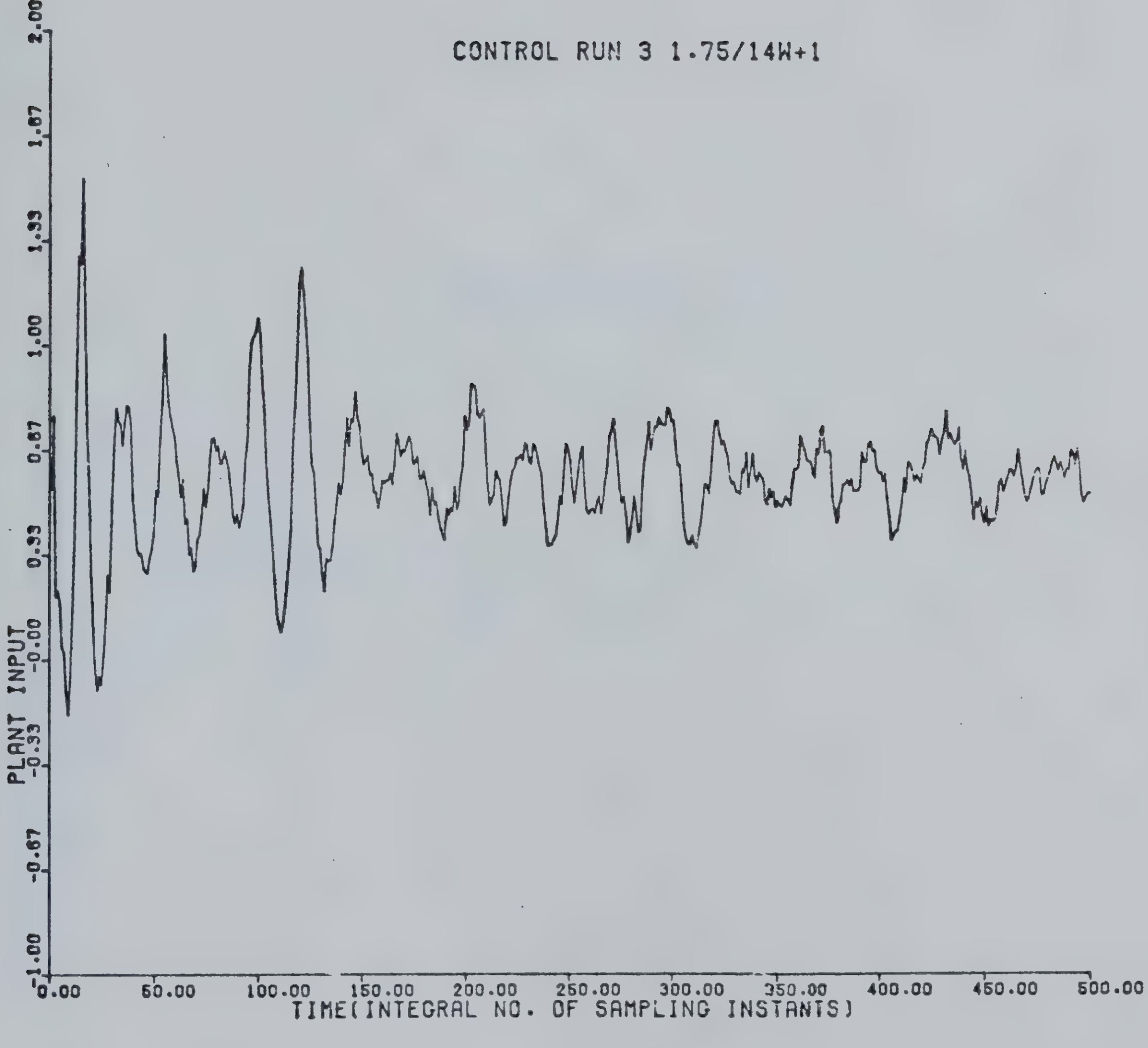


FIGURE 6.12(e): SISO SYSTEM 2 1.75/(14w + 1)
PLANT INPUT VS TIME

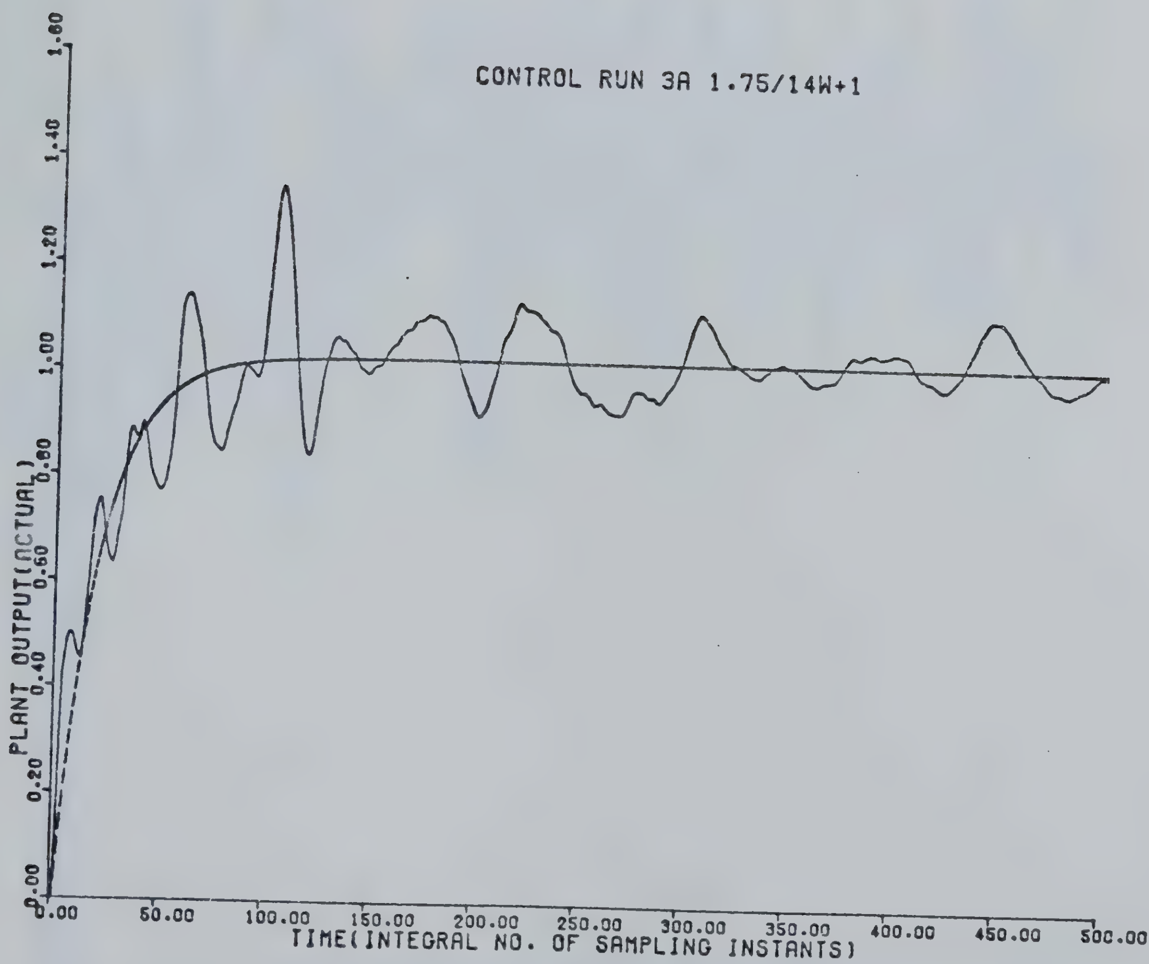


FIGURE 6.13(a): SISO SYSTEM 2 1.75/(14w + 1)
PLANT OUTPUT (ACTUAL) VS TIME

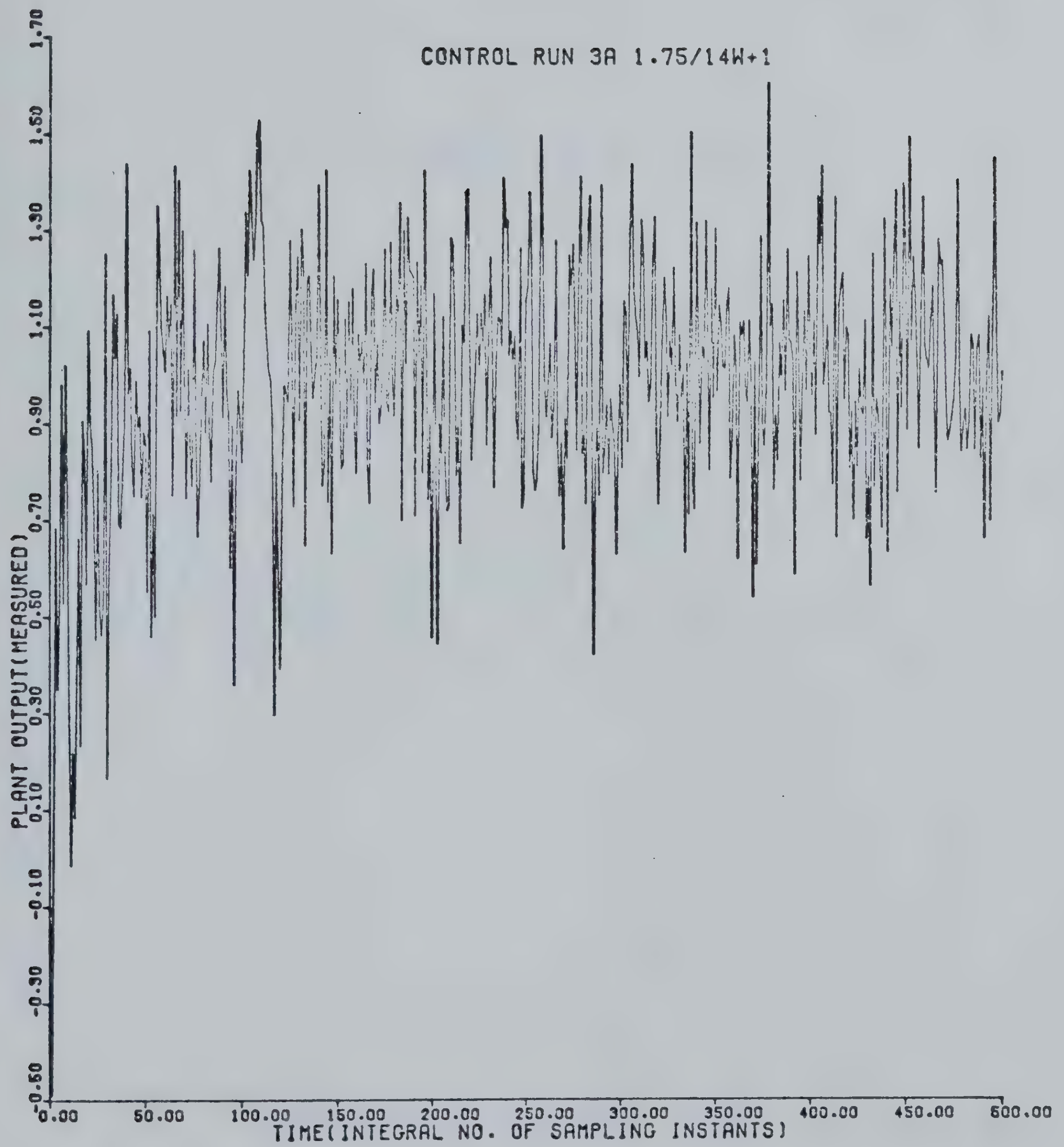


FIGURE 6.13(b): SISO SYSTEM 2 1.75/(14w + 1)
PLANT OUTPUT (MEASURED) VS TIME

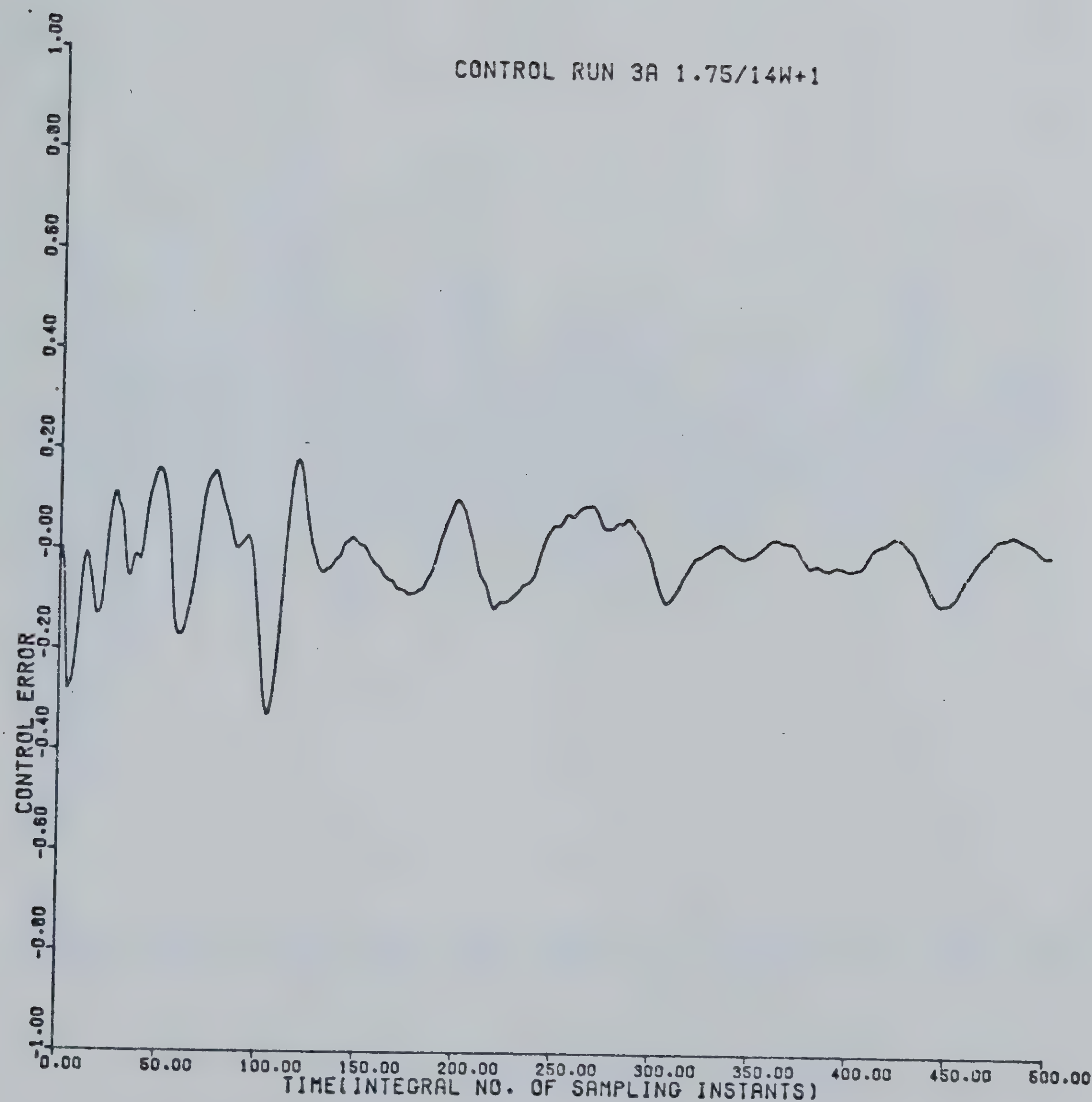


FIGURE 6.13(c): SISO SYSTEM 2 1.75/(14w + 1)
CONTROL ERROR VS TIME

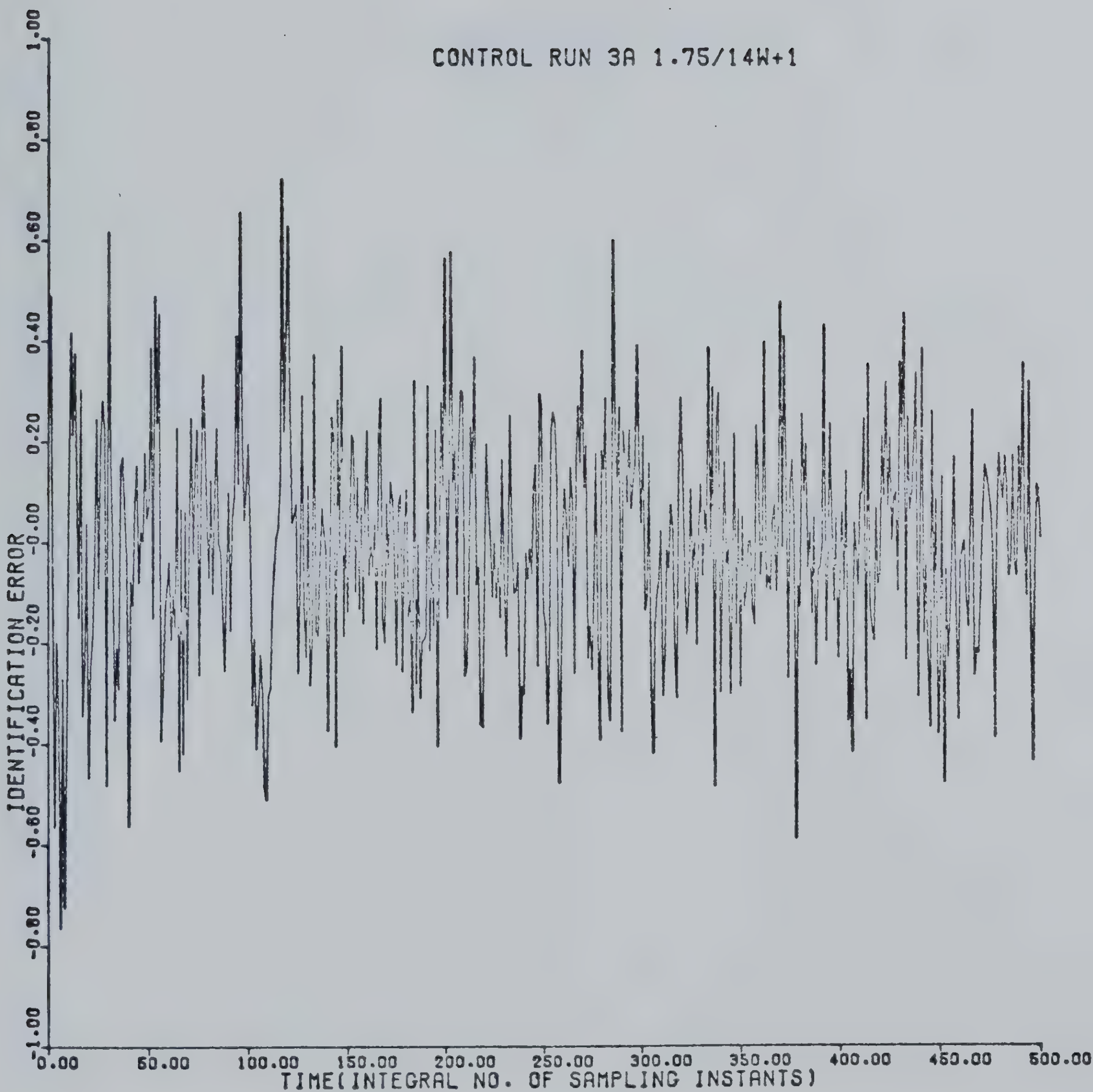


FIGURE 6.13(d): SISO SYSTEM 2 1.75/(14w + 1)
IDENTIFICATION ERROR VS TIME

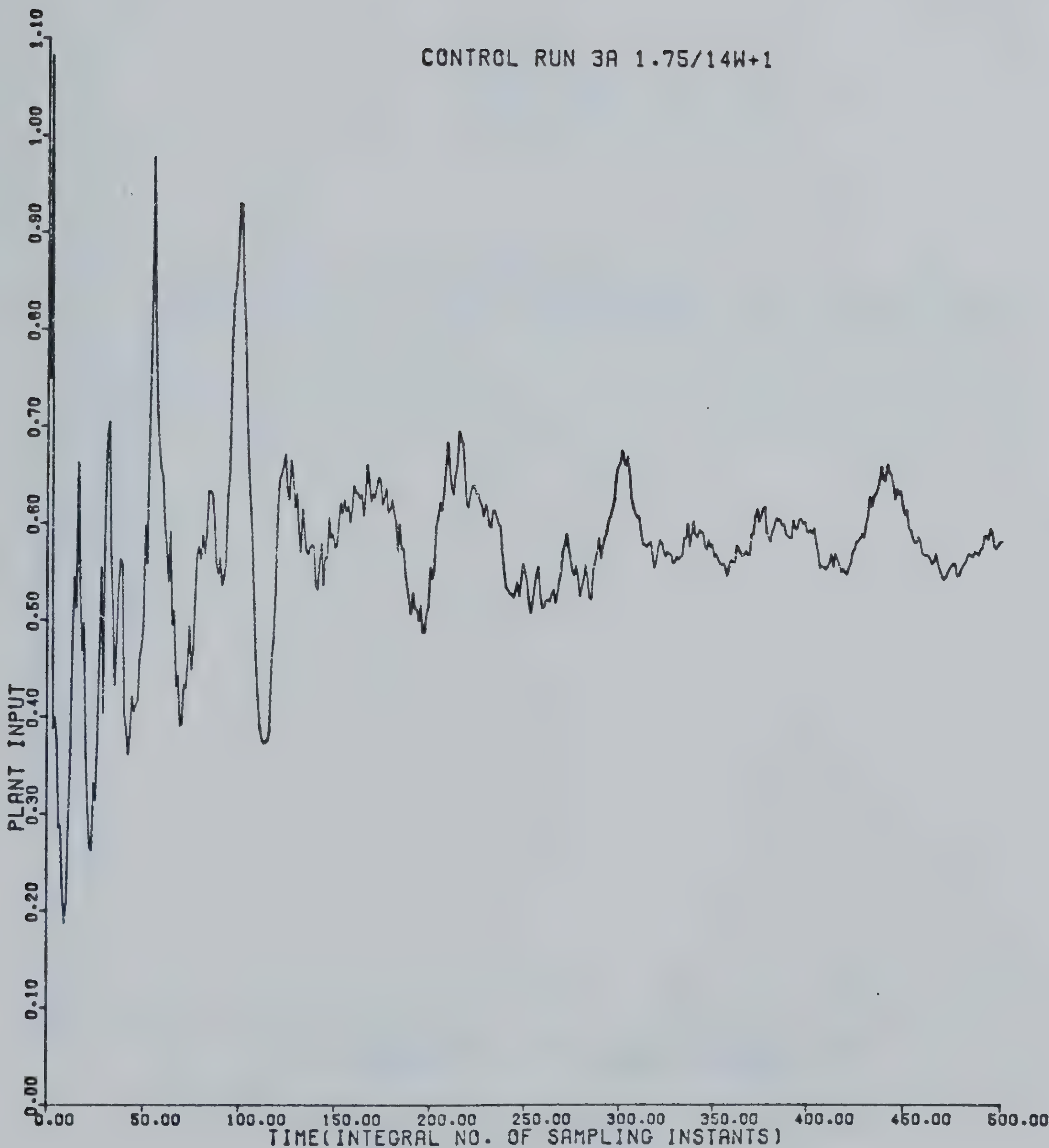


FIGURE 6.13(e): SISO SYSTEM 2 1.75/(14w + 1)
PLANT INPUT VS TIME

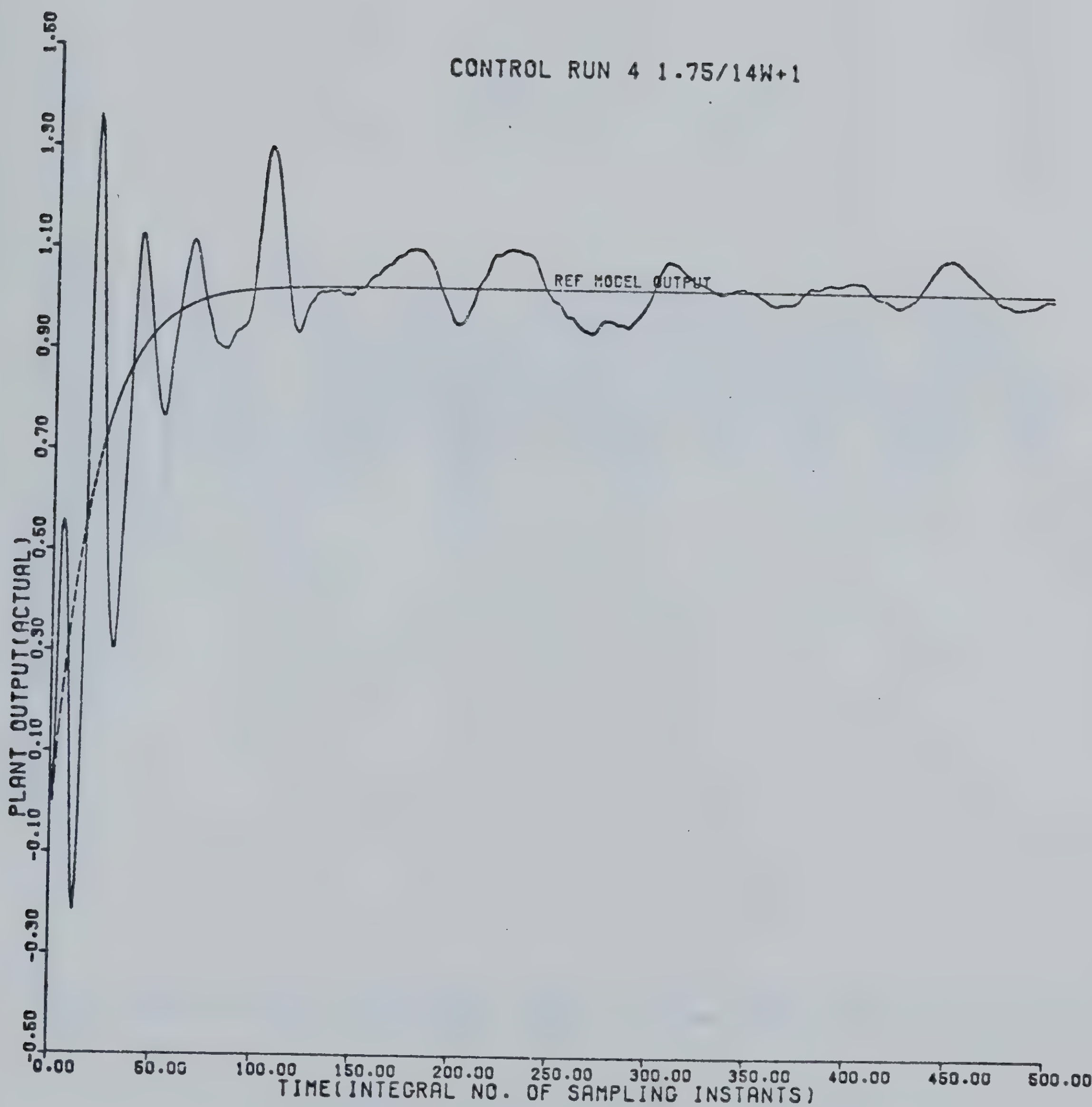


FIGURE 6.14(a): SISO SYSTEM 2 1.75/(14w + 1)
PLANT OUTPUT (ACTUAL) VS TIME

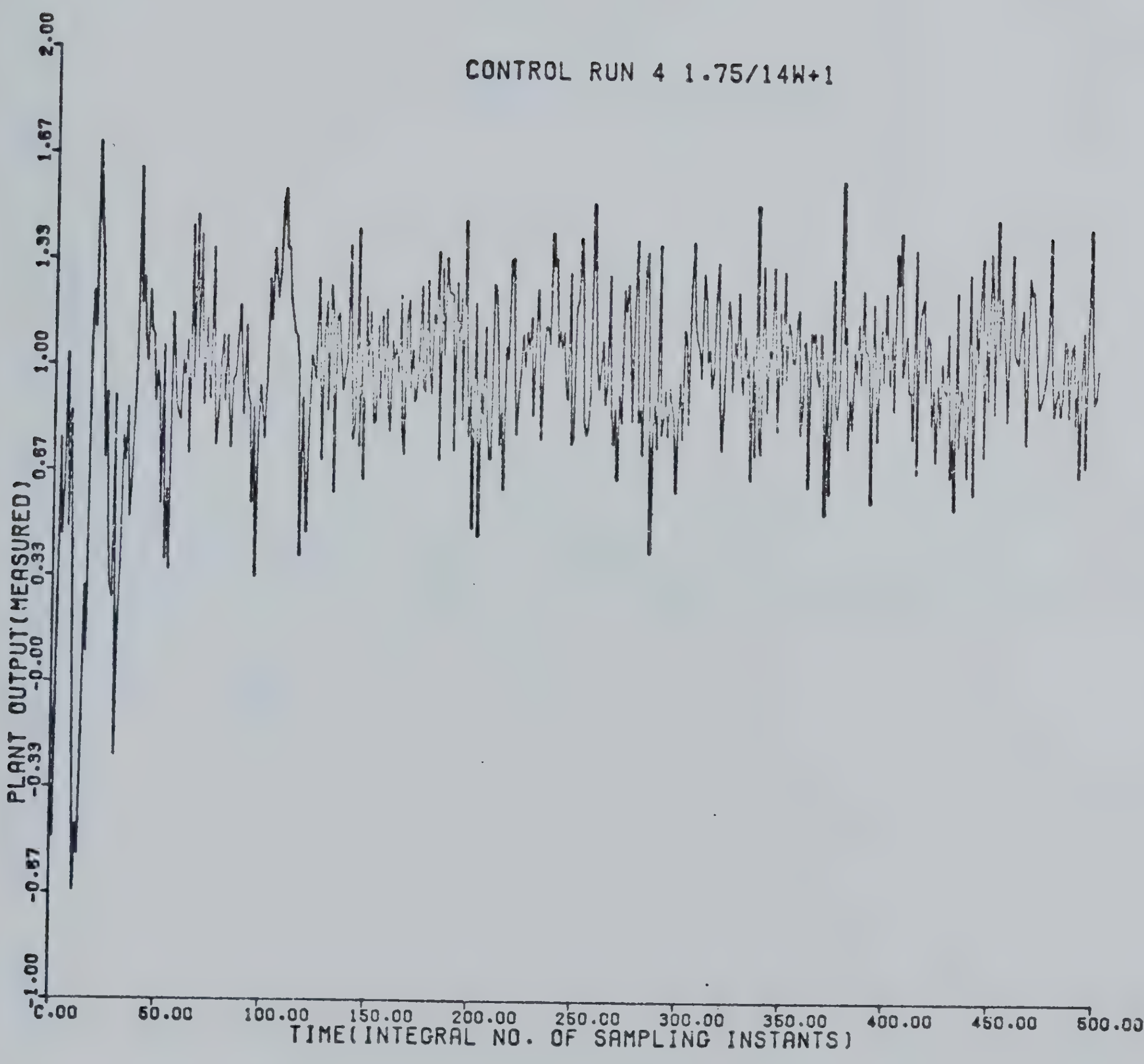


FIGURE 6.14(b): SISO SYSTEM 2 1.75/(14w + 1)
PLANT OUTPUT (MEASURED) VS TIME

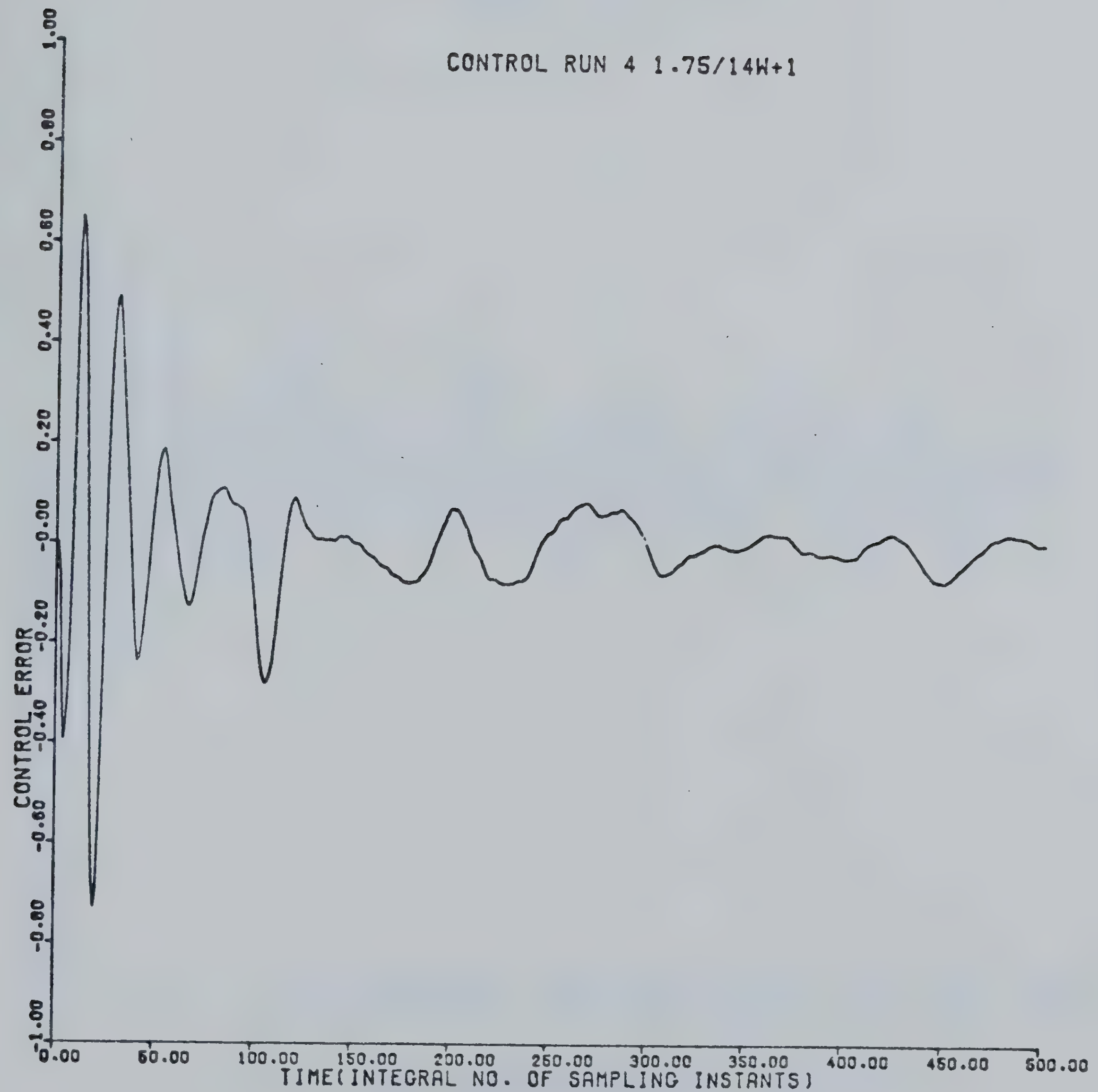


FIGURE 6.14(c): SISO SYSTEM 2 1.75/(14w + 1)
CONTROL ERROR VS TIME

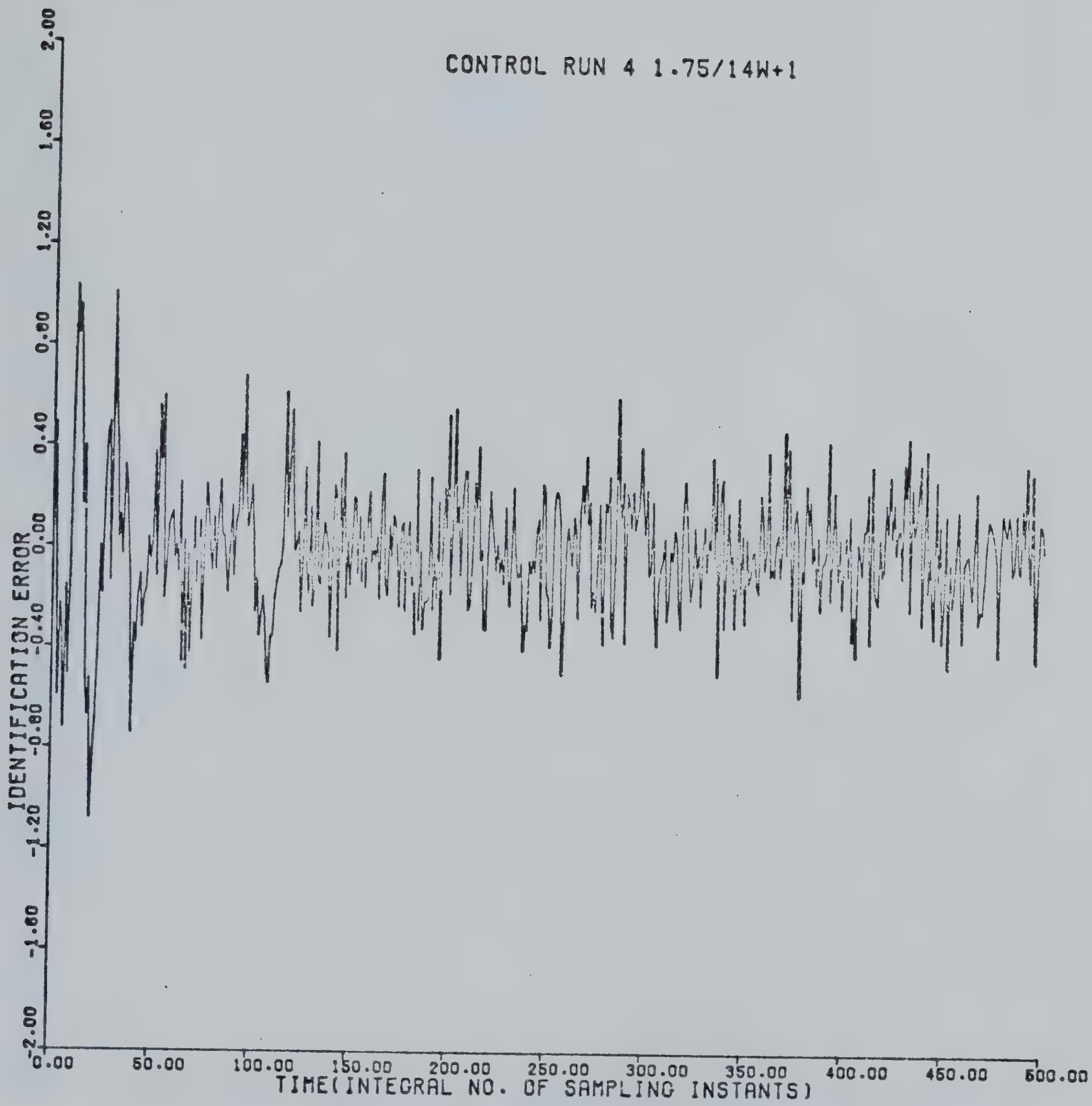


FIGURE 6.14(d): SISO SYSTEM 2 1.75/(14w + 1)
IDENTIFICATION ERROR VS TIME

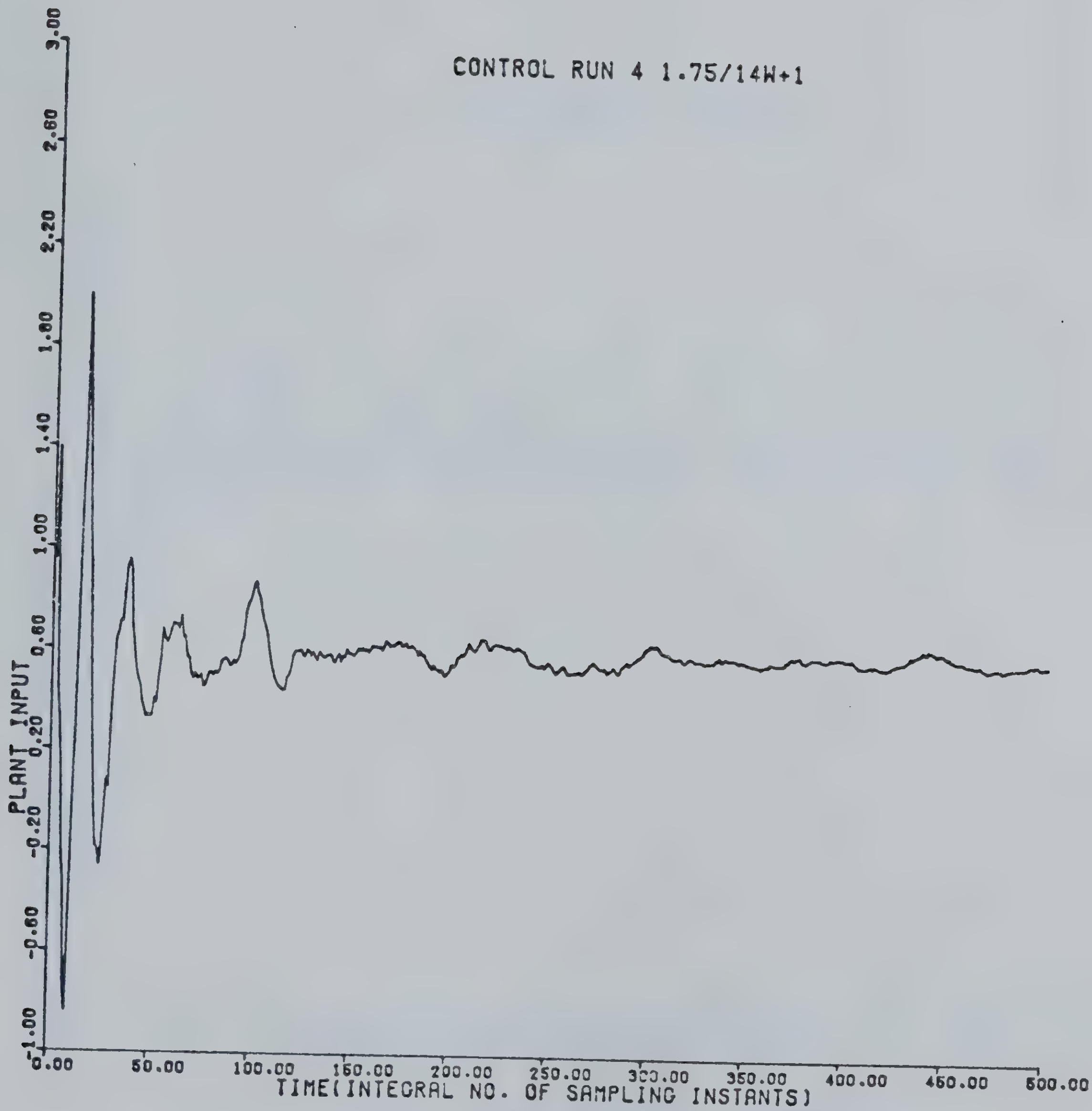


FIGURE 6.14(e): SISO SYSTEM 2 1.75/(14w + 1)
PLANT INPUT VS TIME

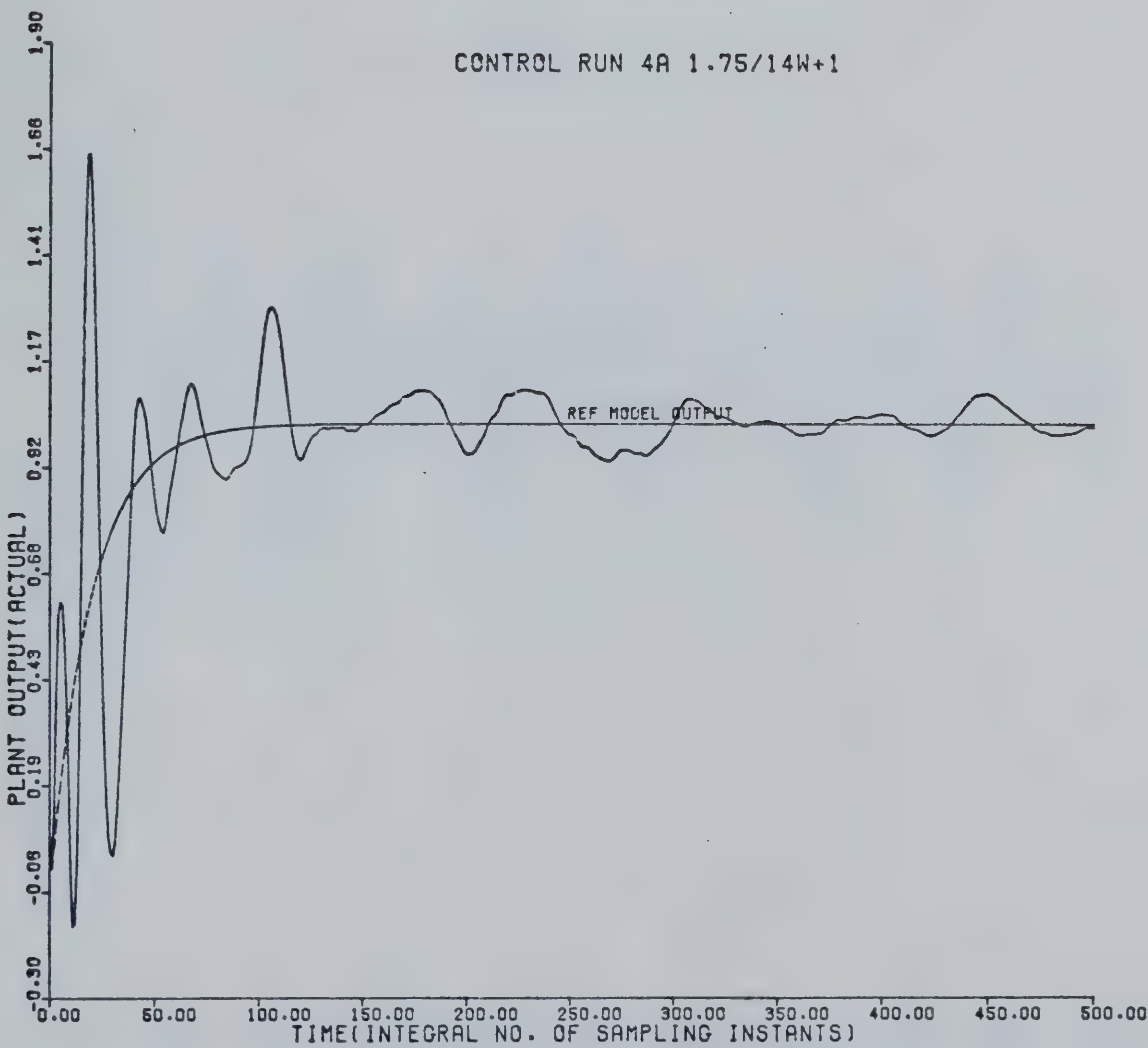


FIGURE 6.15(a): SISO SYSTEM 2 1.75/(14w + 1)
PLANT OUTPUT (ACTUAL) VS TIME

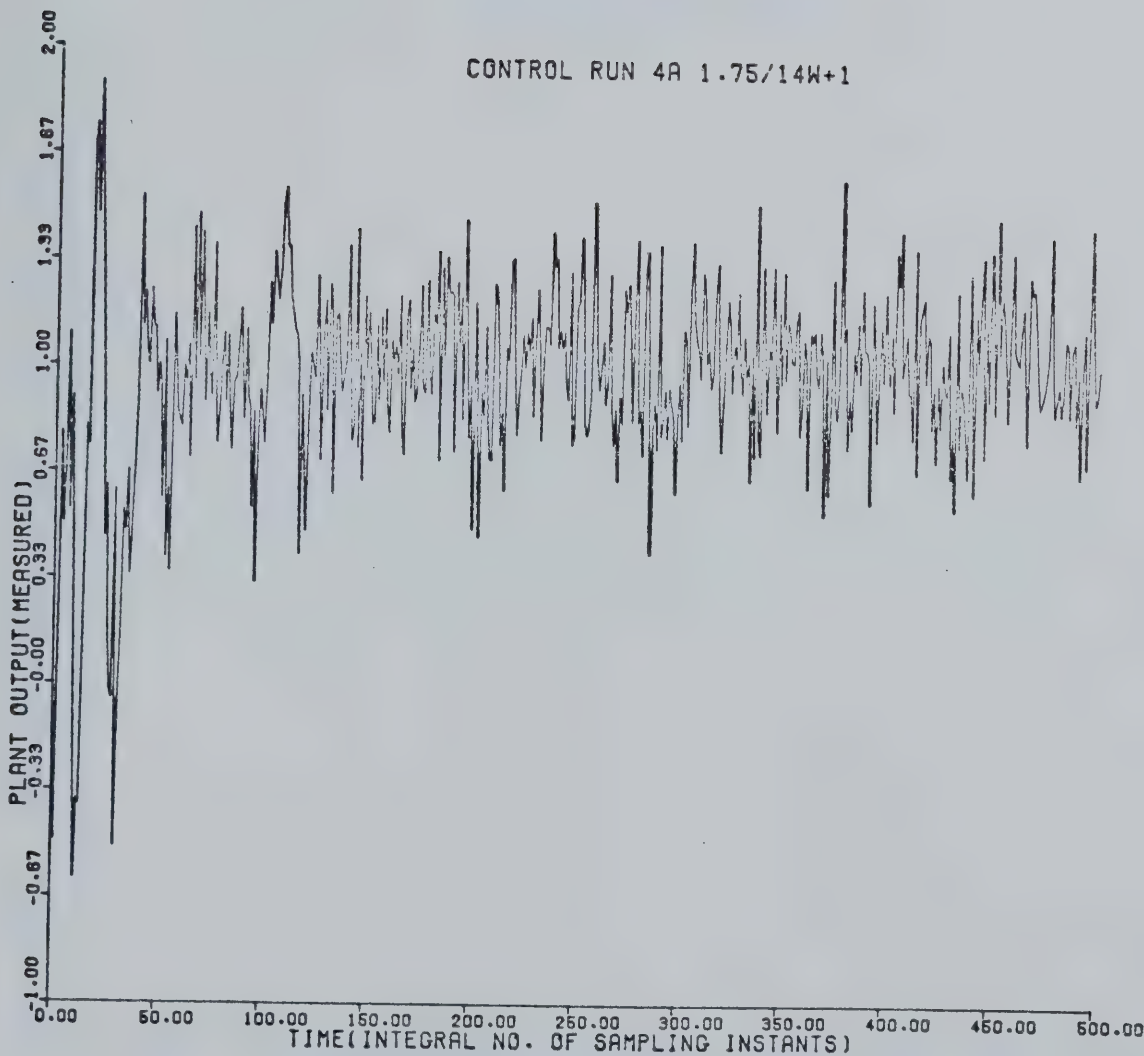


FIGURE 6.15(b): SISO SYSTEM 2 1.75/(14w + 1)
PLANT OUTPUT (MEASURED) VS TIME

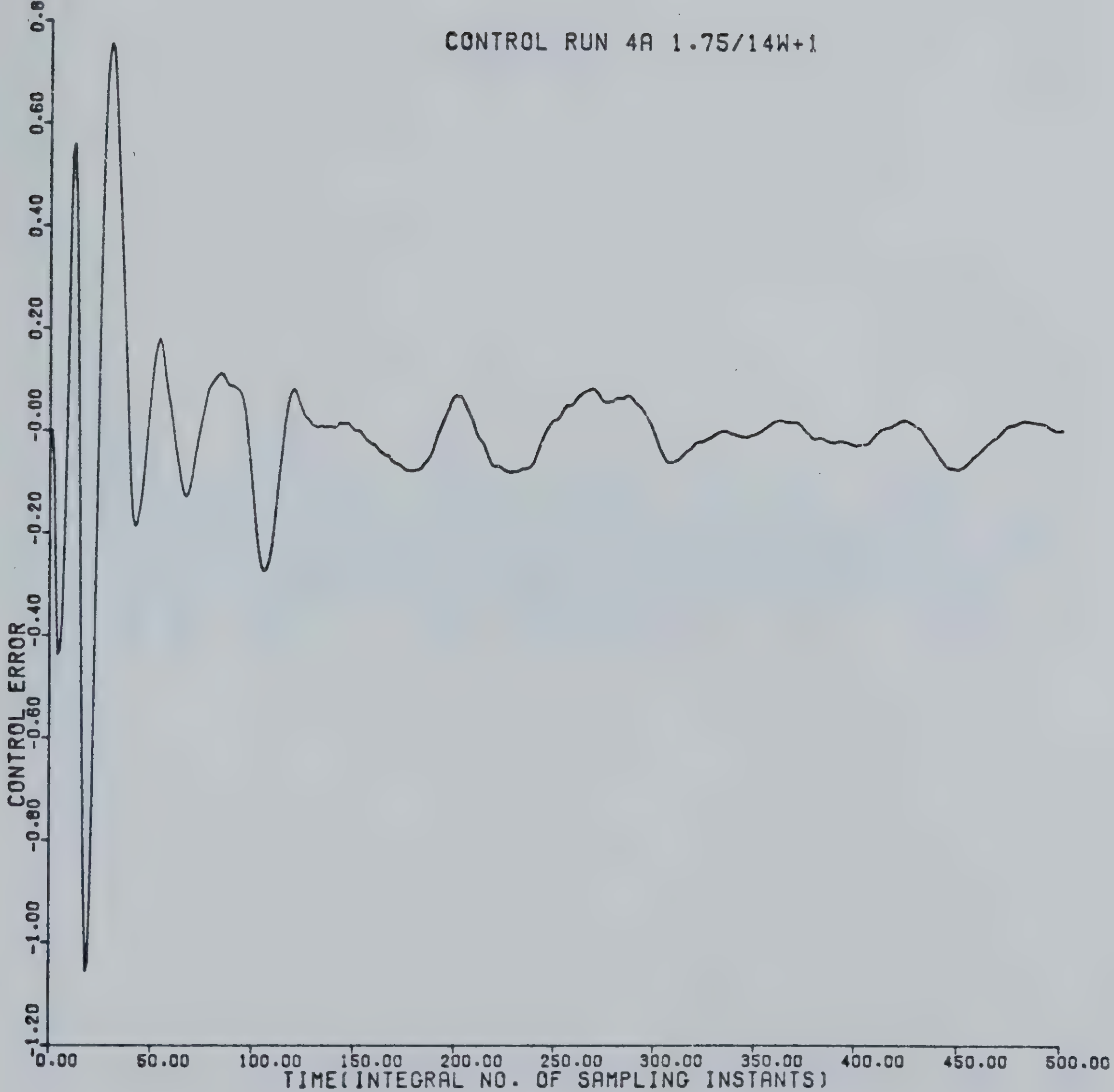


FIGURE 6.15(c): SISO SYSTEM 2 1.75/(14w + 1)
CONTROL ERROR VS TIME

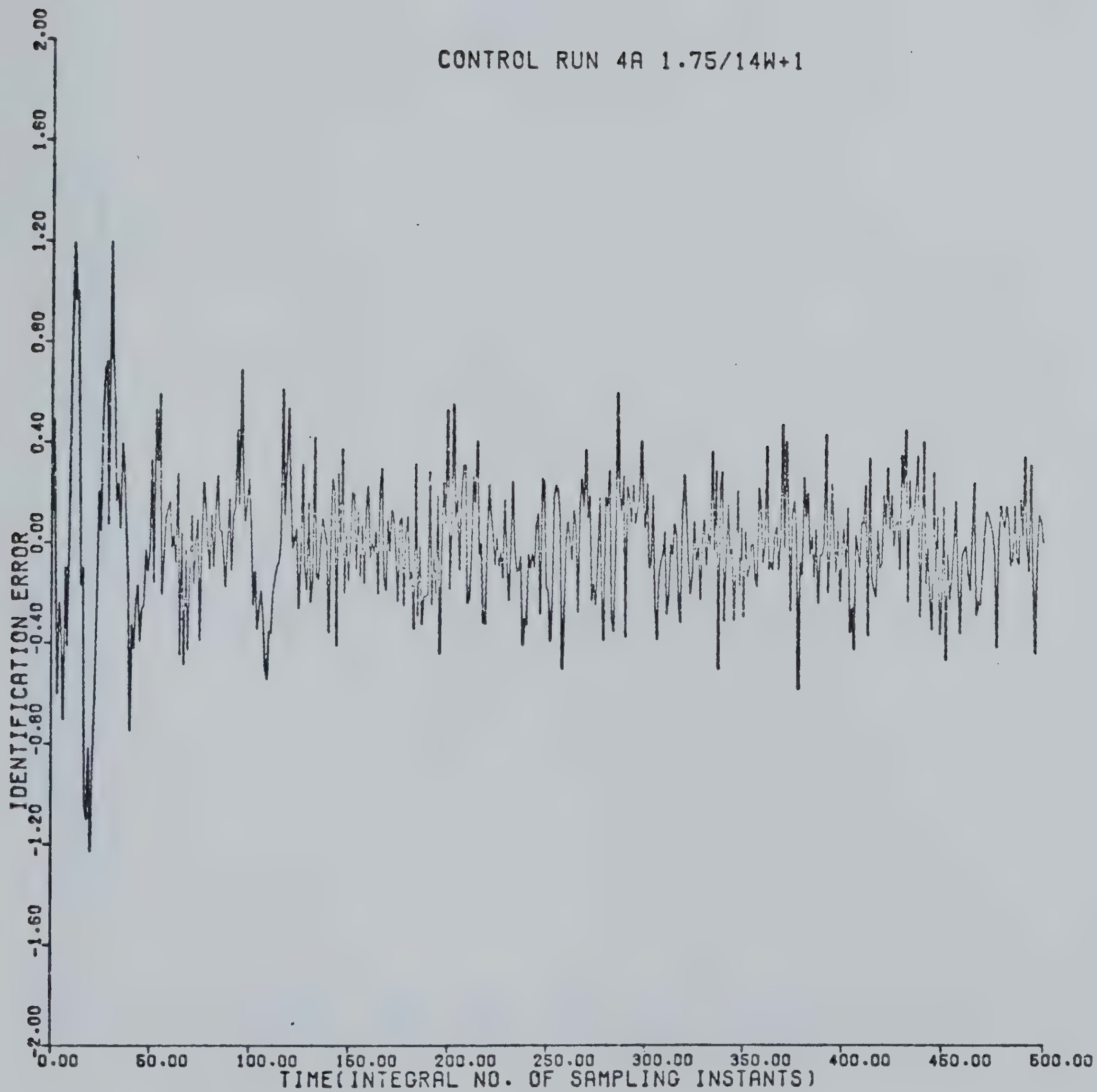


FIGURE 6.15(d): SISO SYSTEM 2 1.75/(14w + 1)
IDENTIFICATION ERROR VS TIME

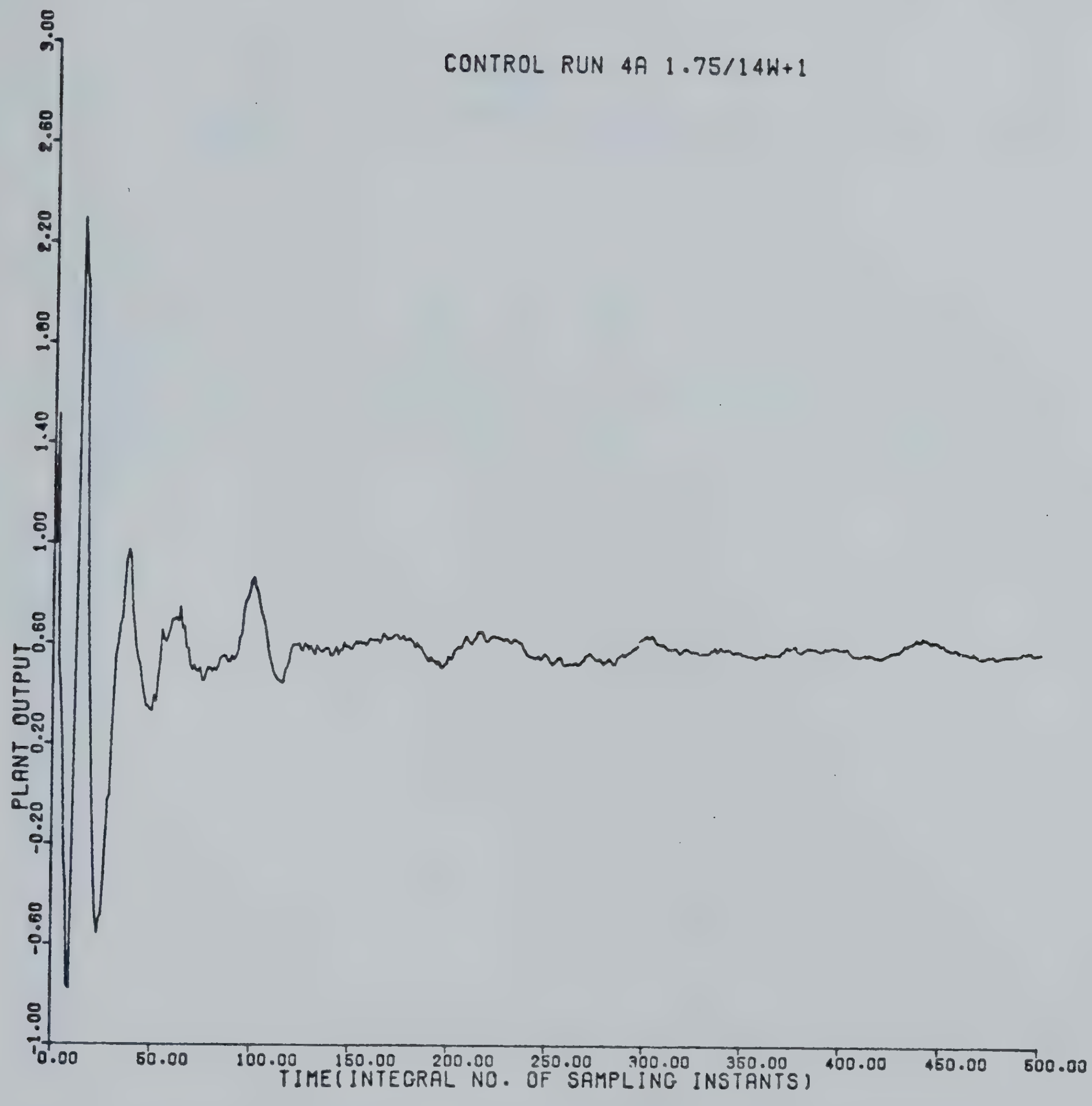


FIGURE 6.15(e): SISO SYSTEM 2 1.75/(14w + 1)
PLANT INPUT VS TIME

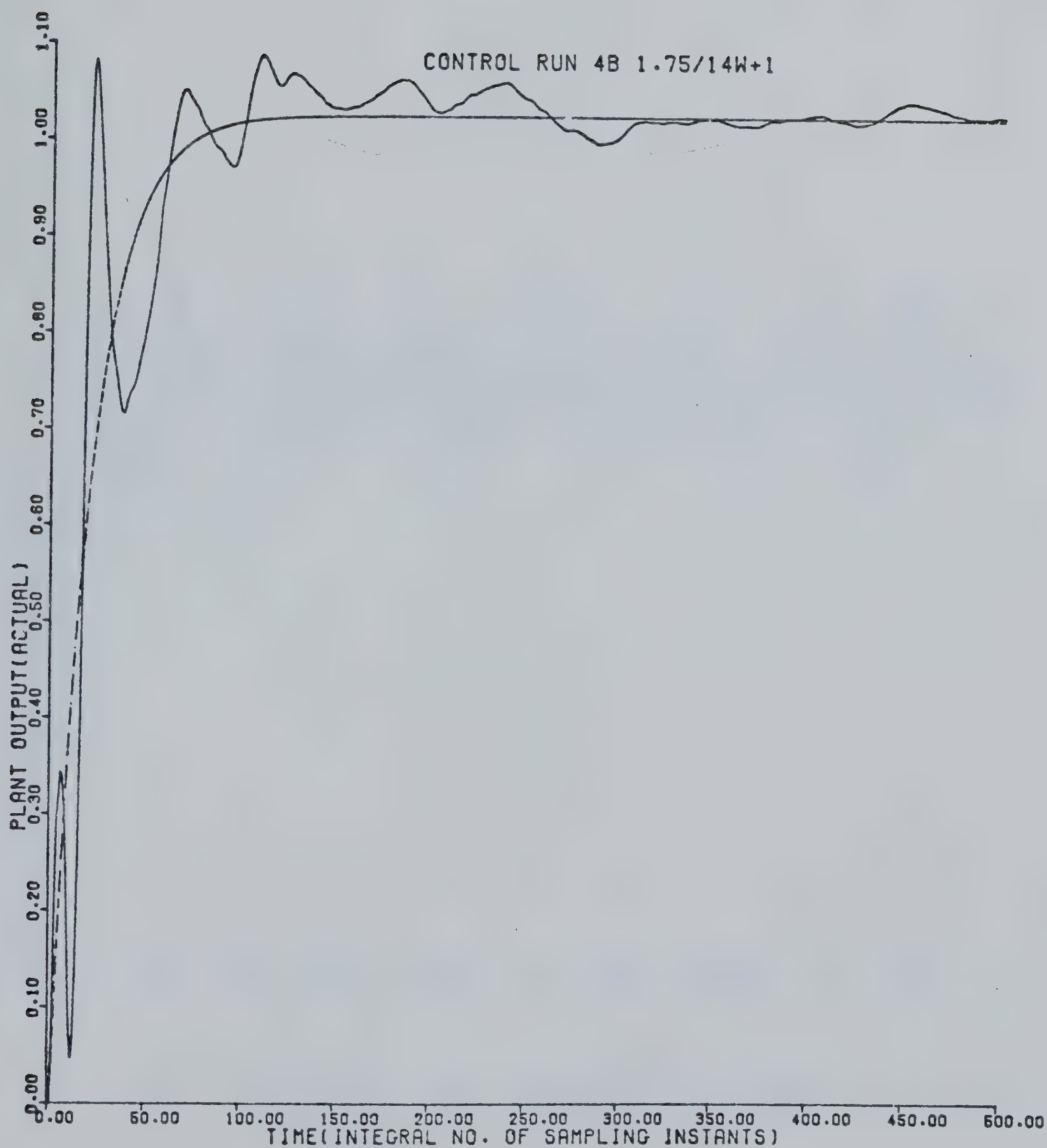


FIGURE 6.16(a): SISO SYSTEM 2 1.75/(14w + 1)
PLANT OUTPUT (ACTUAL) VS TIME

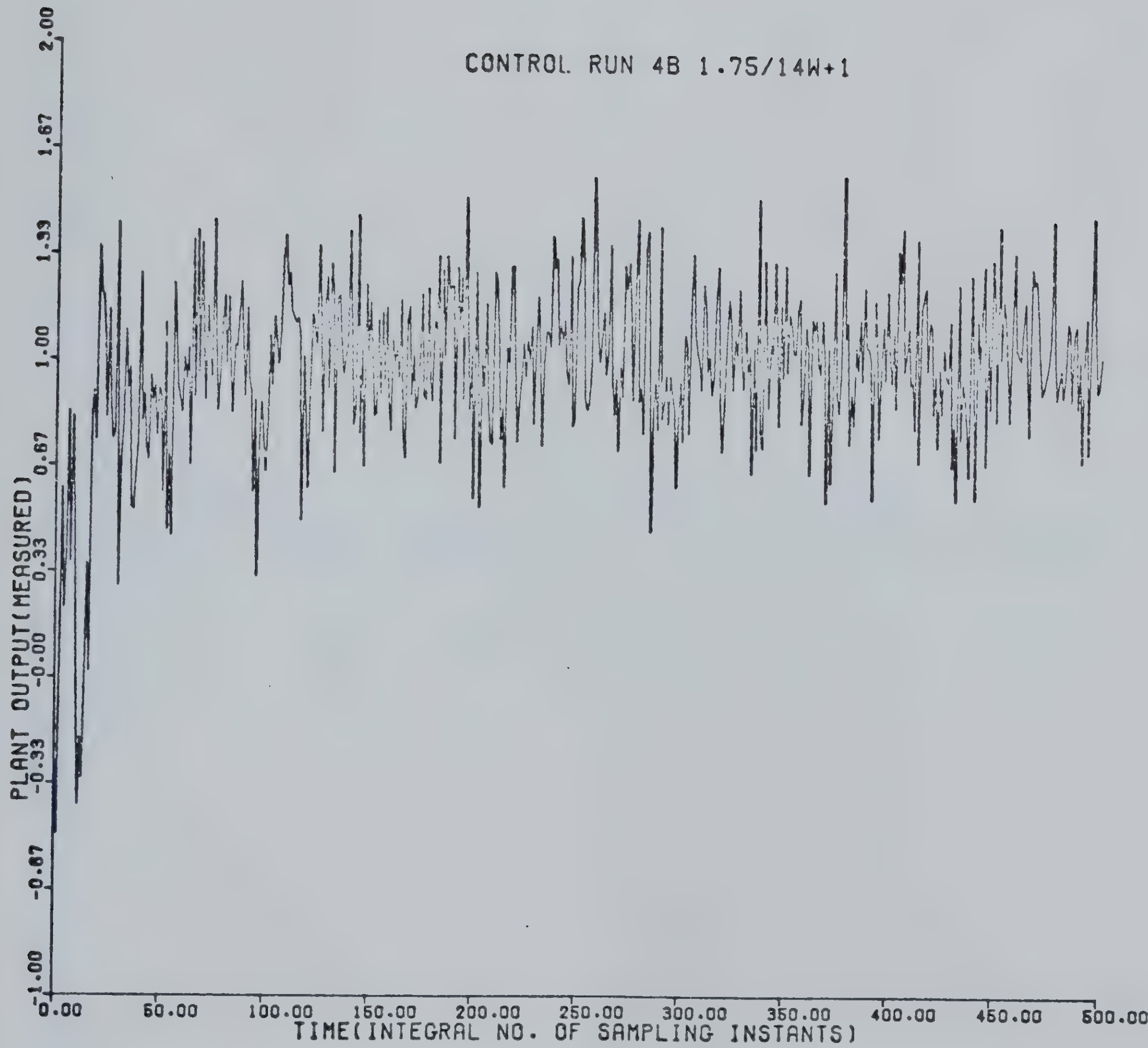


FIGURE 6.16(b): SISO SYSTEM 2 1.75/(14w + 1)
PLANT OUTPUT (MEASURED) VS TIME

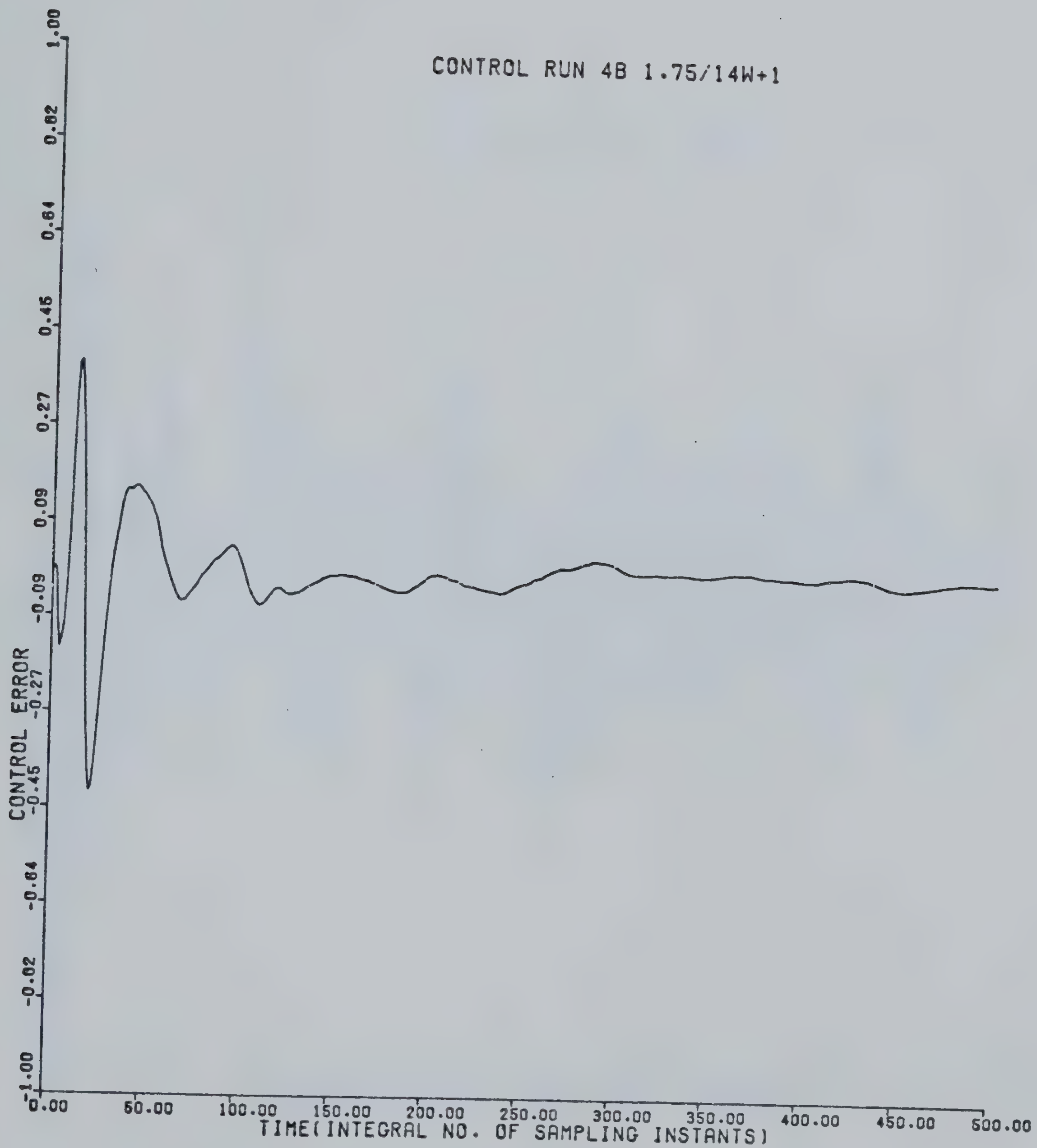


FIGURE 6.16(c): SISO SYSTEM 2 1.75/(14w + 1)
CONTROL ERROR VS TIME

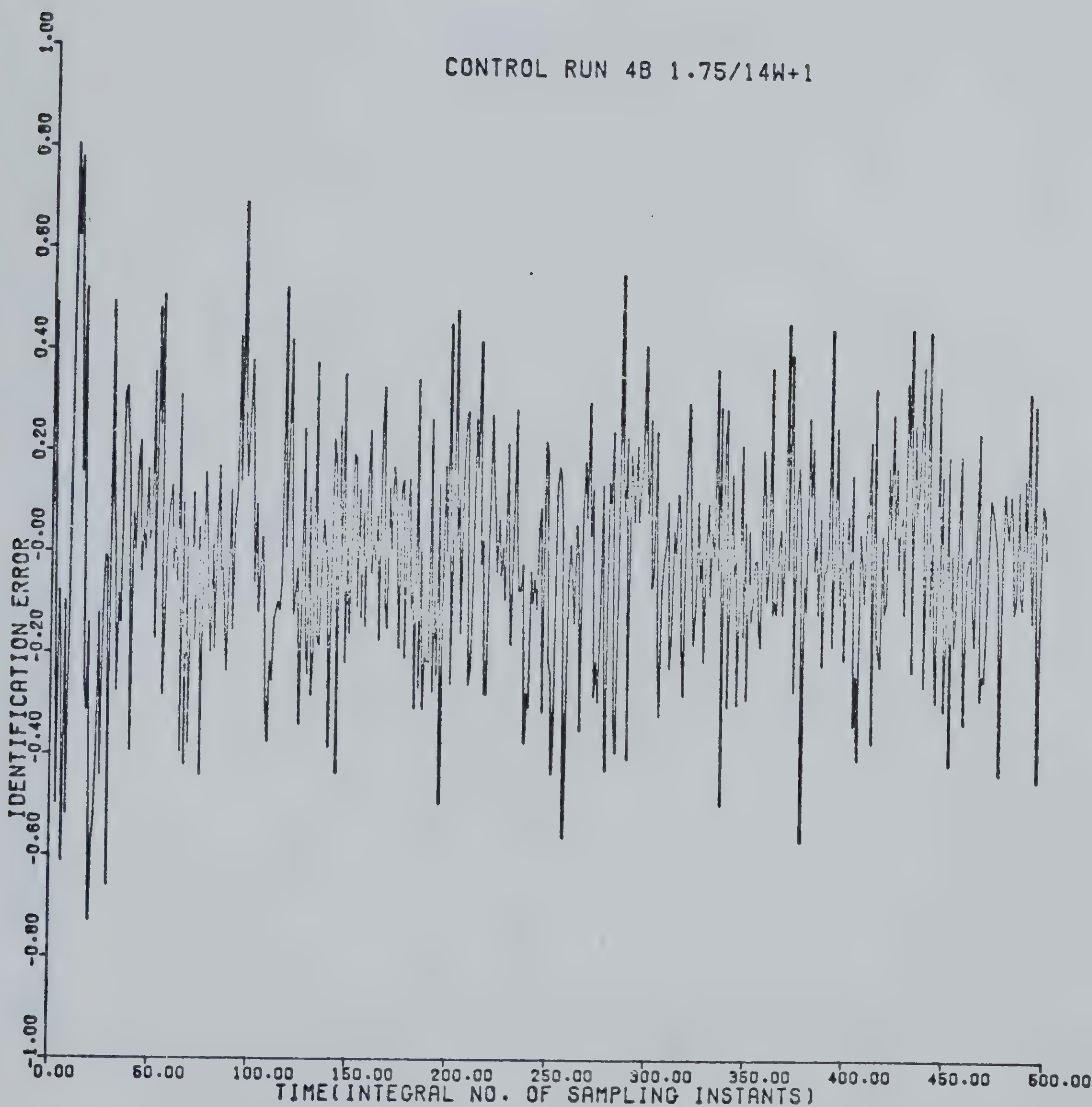


FIGURE 6.16(d): SISO SYSTEM 2 1.75/(14w + 1)
IDENTIFICATION ERROR VS TIME

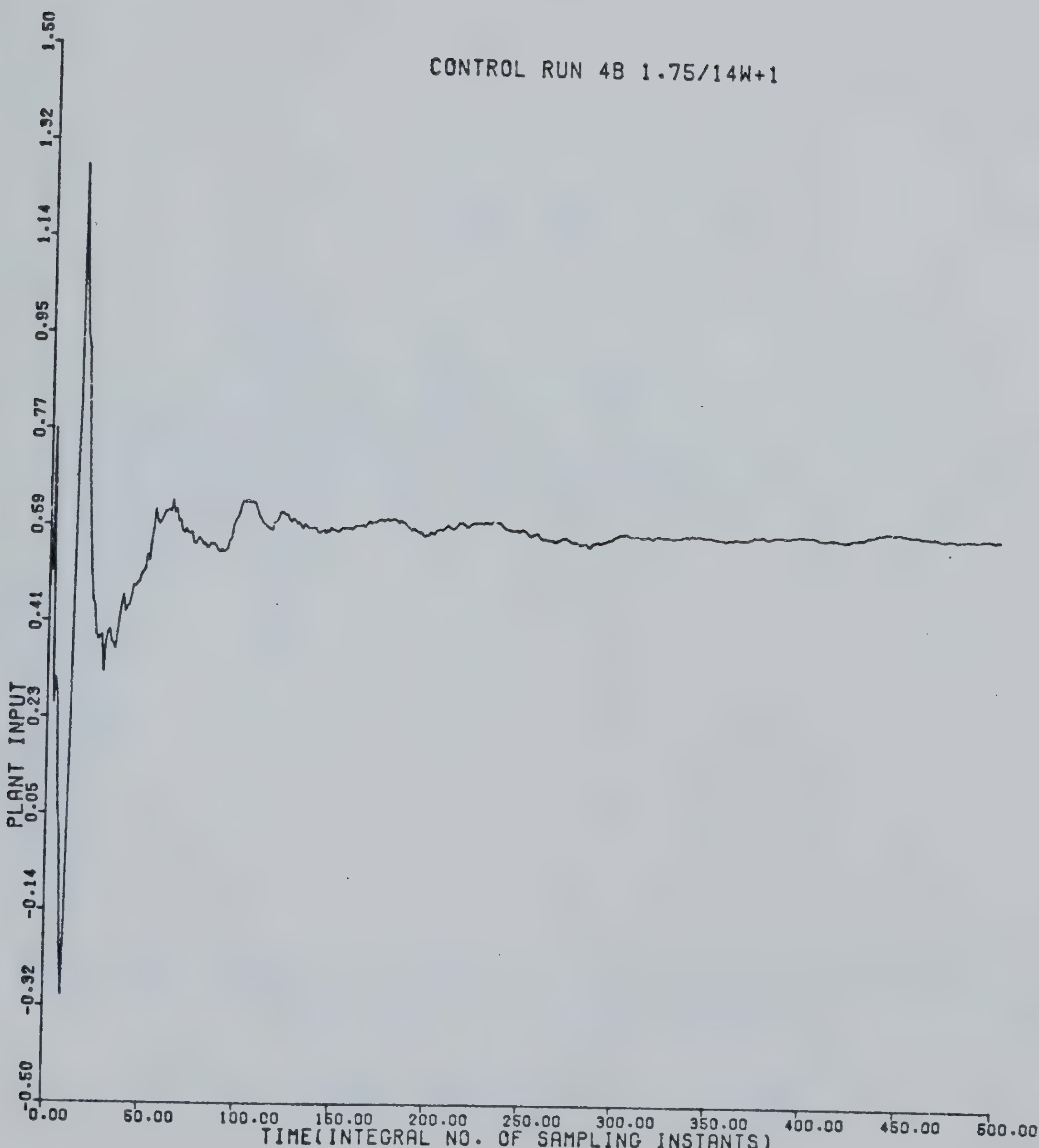


FIGURE 6.16(e): SISO SYSTEM 2 1.75/(14w + 1)
PLANT INPUT VS TIME

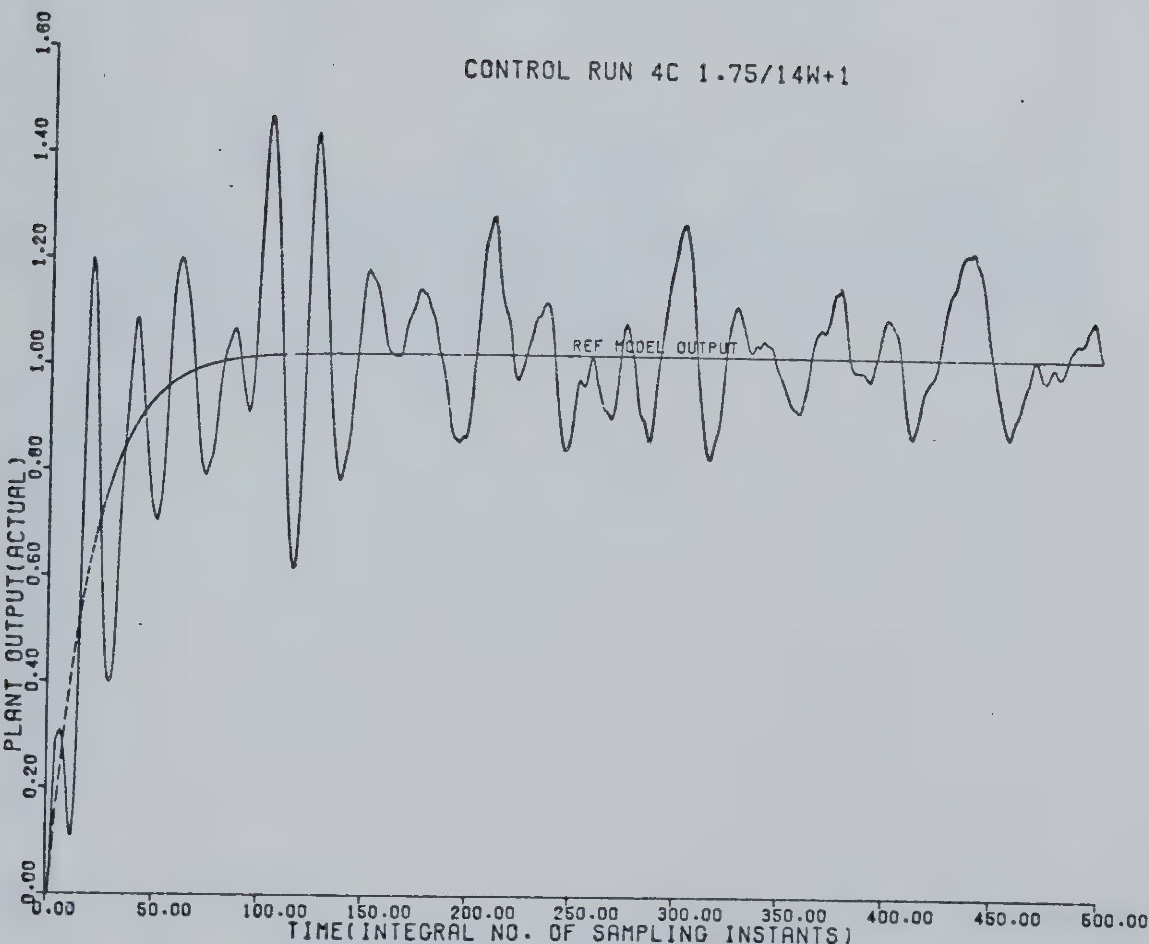


FIGURE 6.17(a): SISO SYSTEM 2 1.75/(14w + 1)
PLANT OUTPUT (ACTUAL) VS TIME

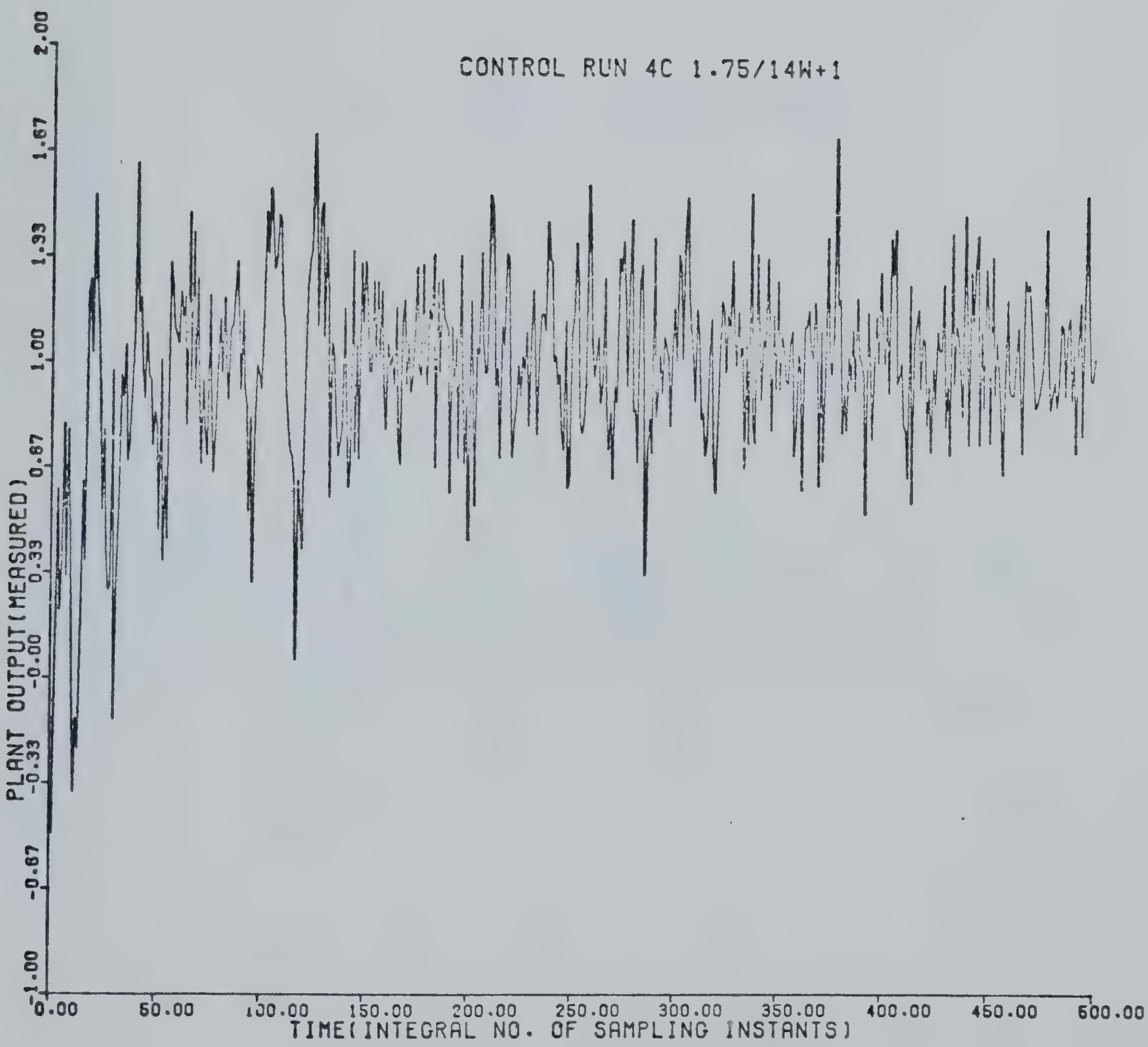


FIGURE 6.17(b): SISO SYSTEM 2 1.75/(14w + 1)
PLANT OUTPUT (MEASURED) VS TIME

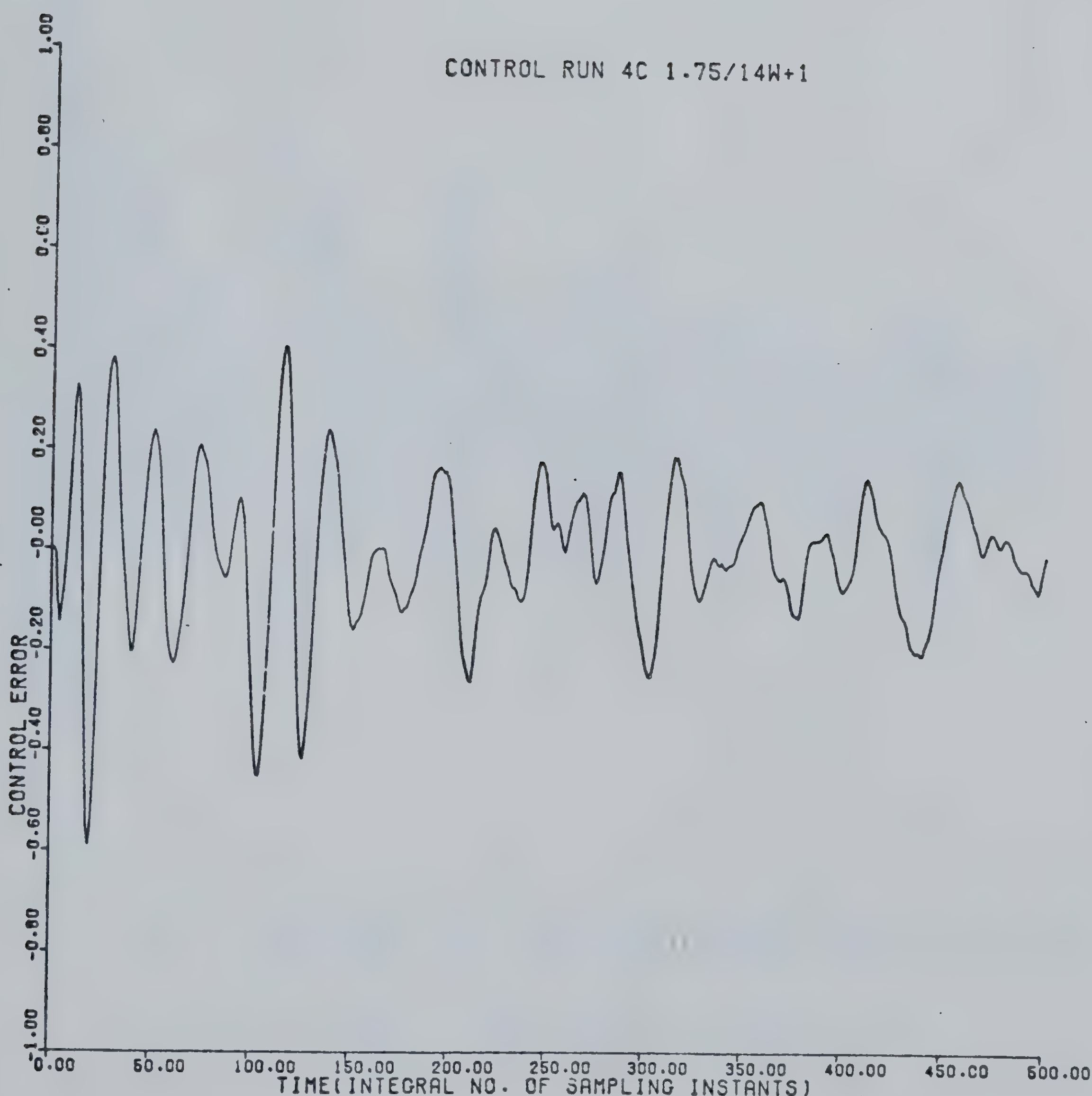


FIGURE 6.17(c): SISO SYSTEM 2 1.75/(14w + 1)
CONTROL ERROR VS TIME

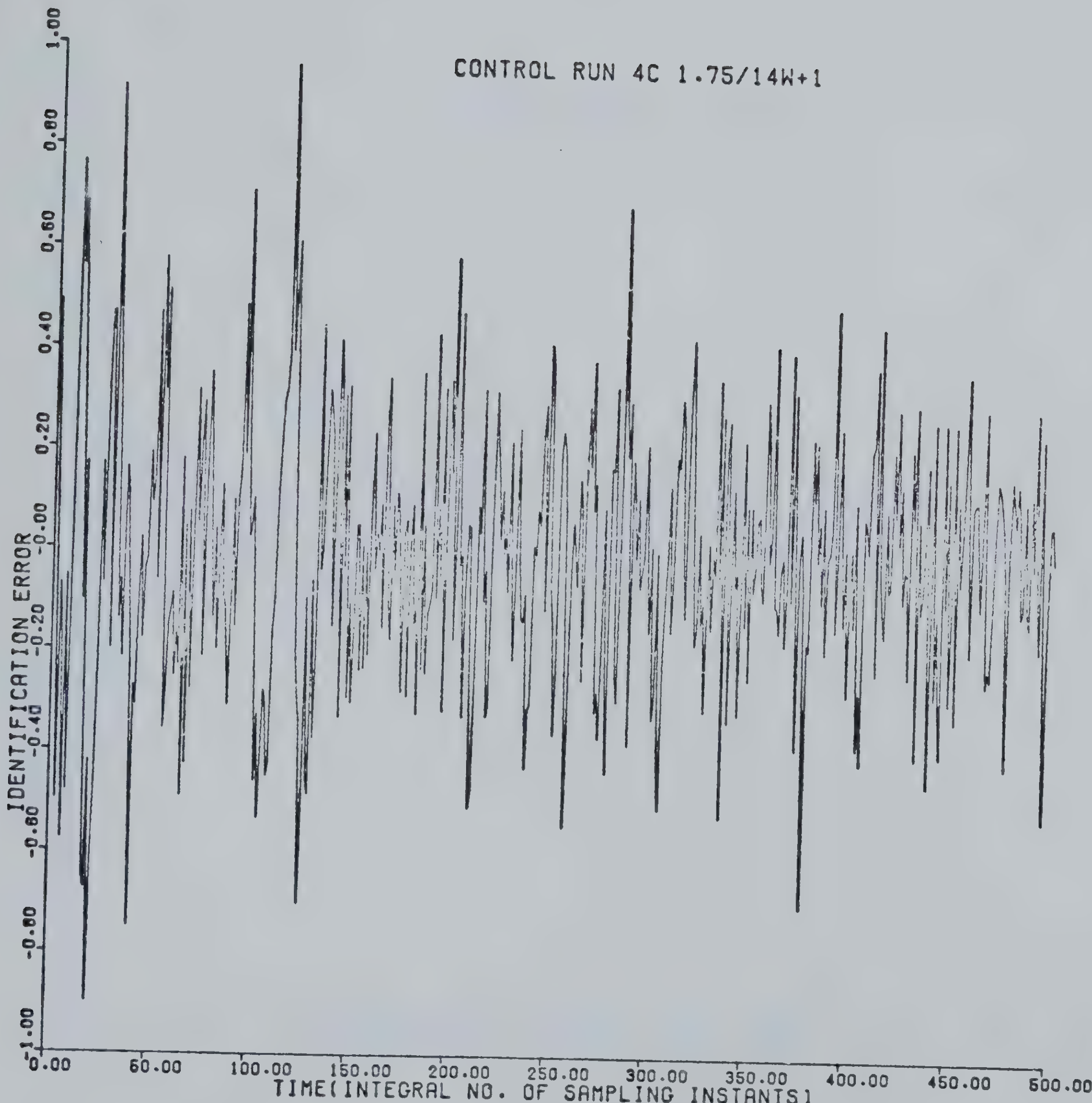


FIGURE 6.17(d): SISO SYSTEM 2 1.75/(14w + 1)
IDENTIFICATION ERROR VS TIME

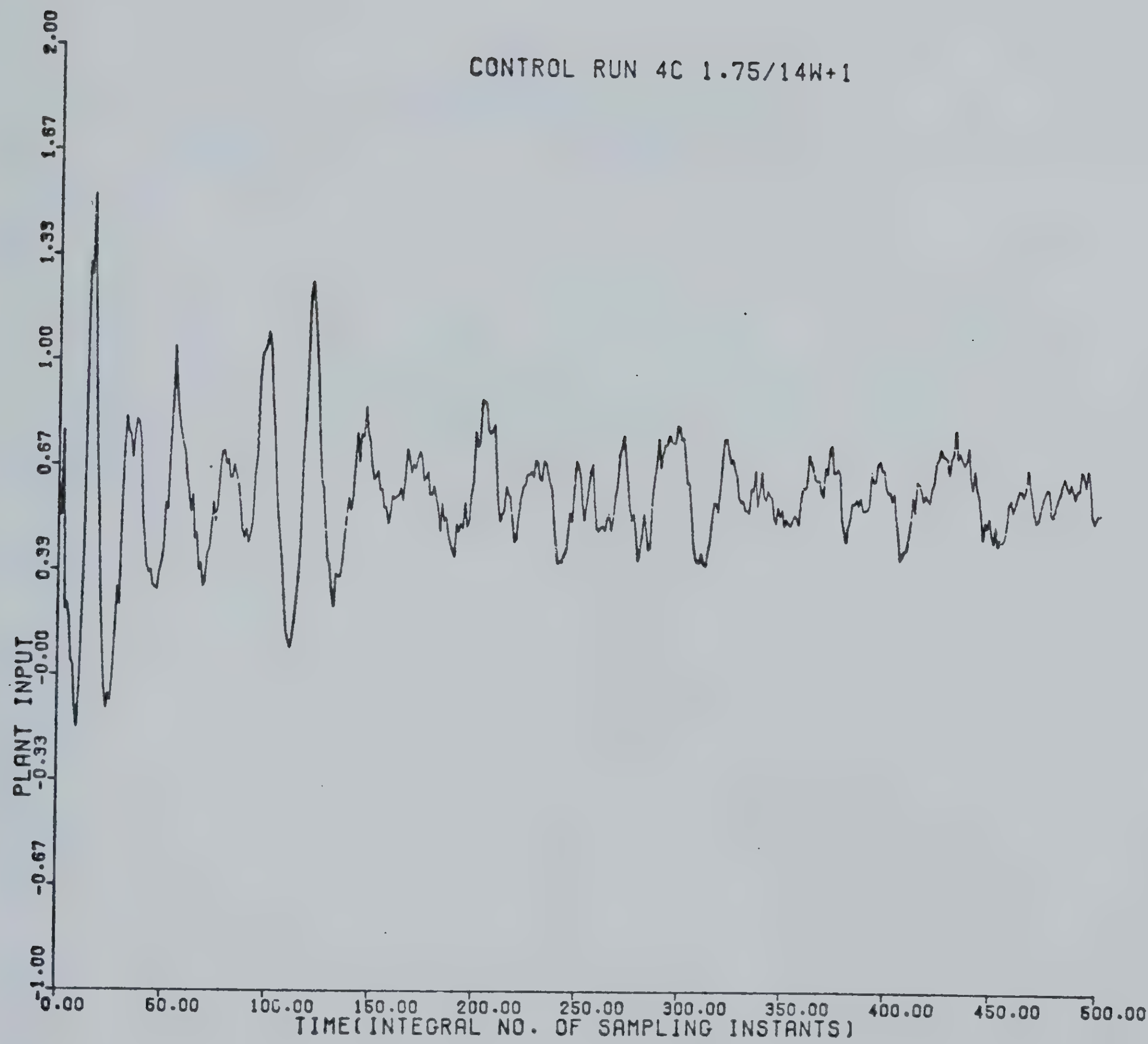


FIGURE 6.17(e): SISO SYSTEM 2 1.75/(14w + 1)
PLANT INPUT VS TIME

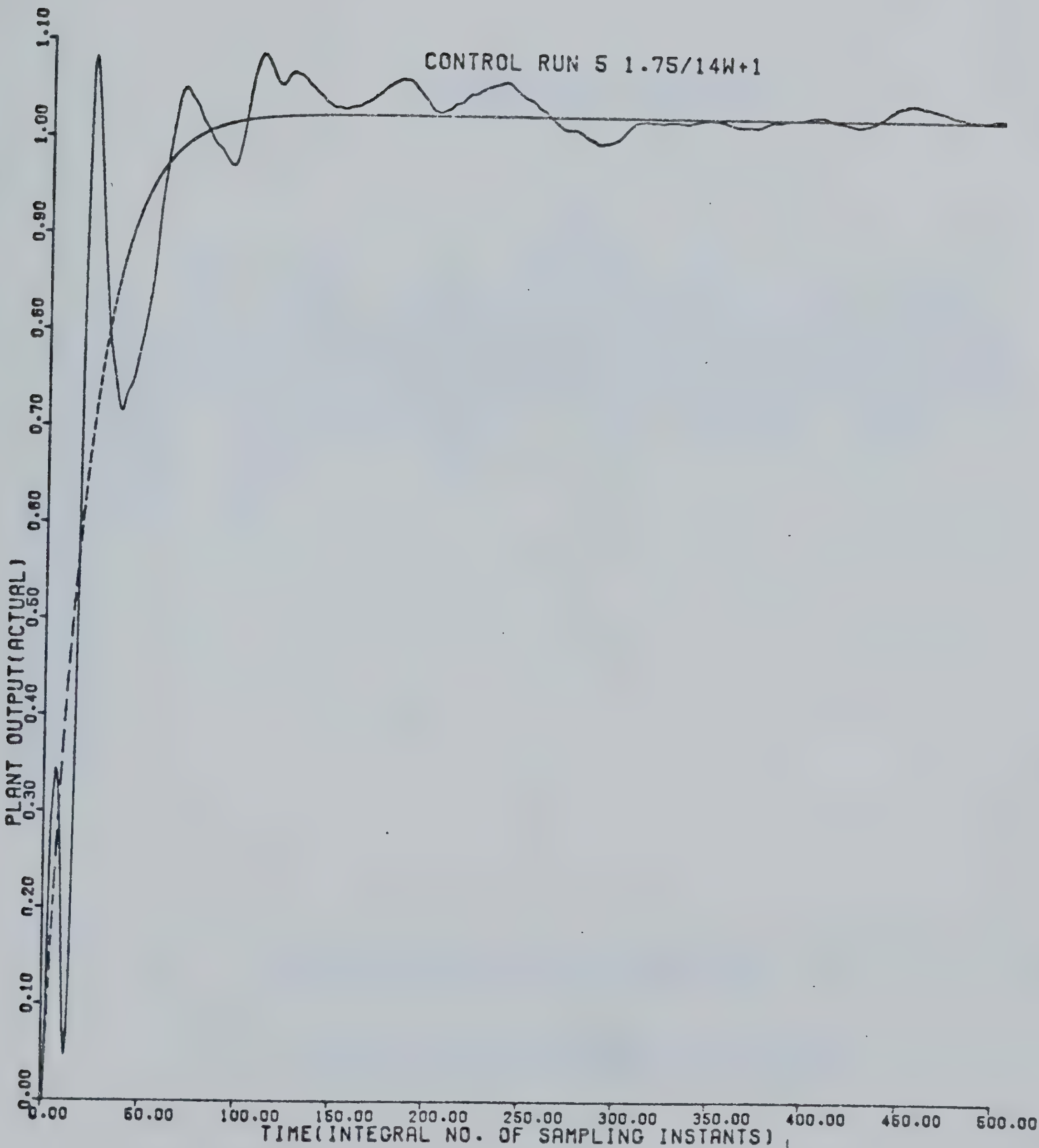


FIGURE 6.18(a): SISO SYSTEM 2 1.75/(14w + 1)
PLANT OUTPUT (ACTUAL) VS TIME

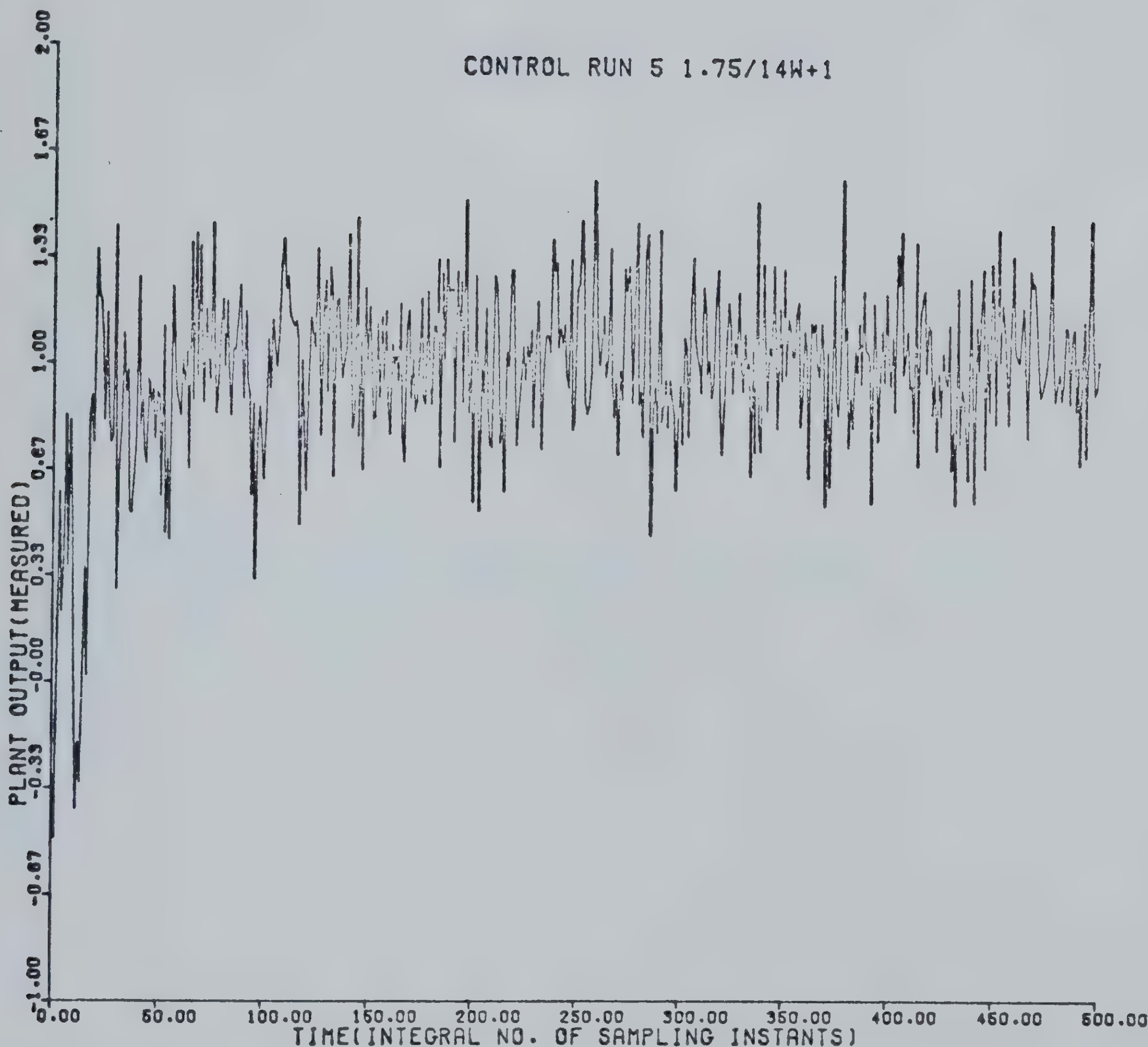


FIGURE 6.18(b): SISO SYSTEM 2 1.75/(14w + 1)
PLANT OUTPUT (MEASURED) VS TIME

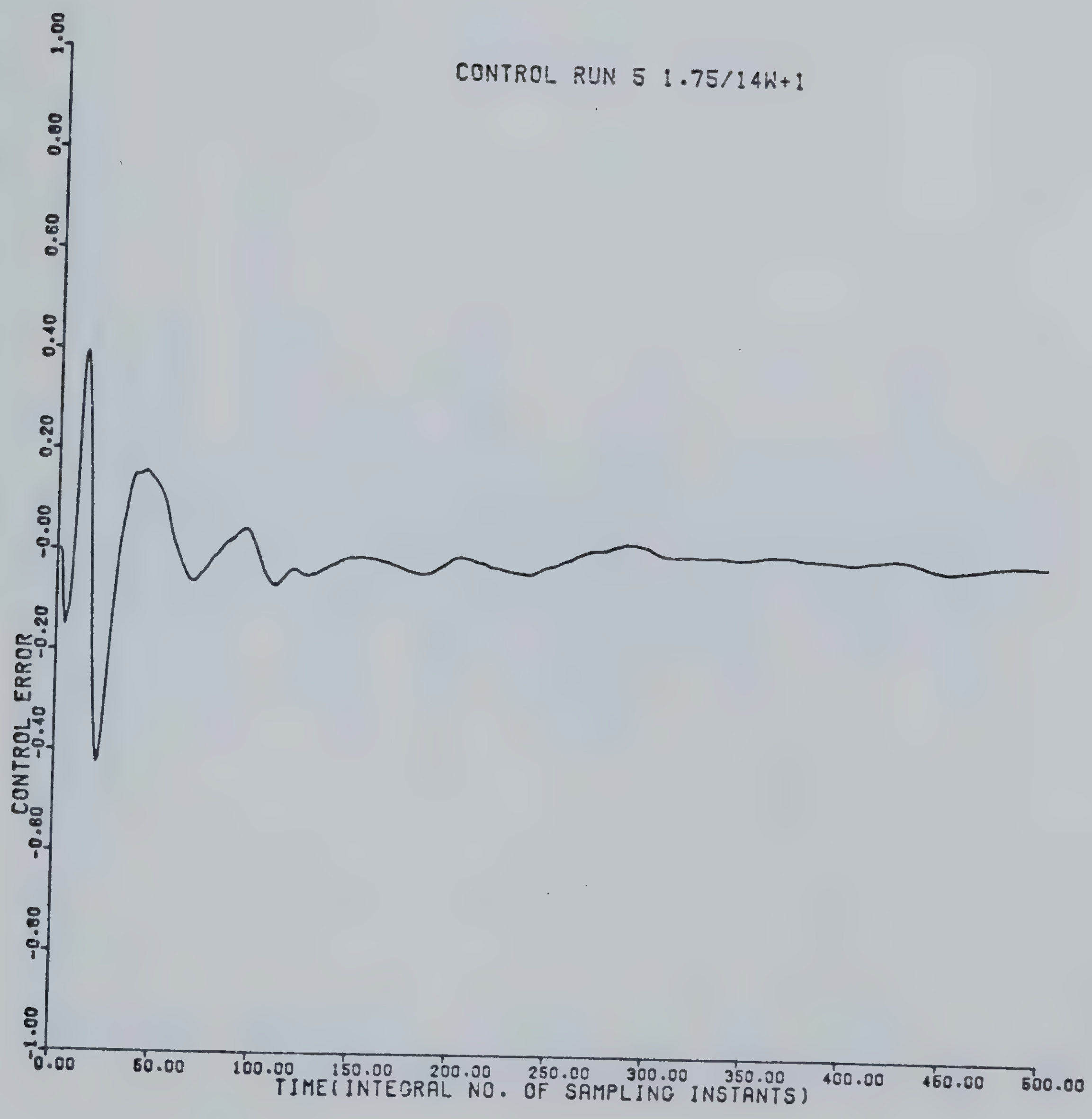


FIGURE 6.18(c): SISO SYSTEM 2 1.75/(14w + 1)
CONTROL ERROR VS TIME

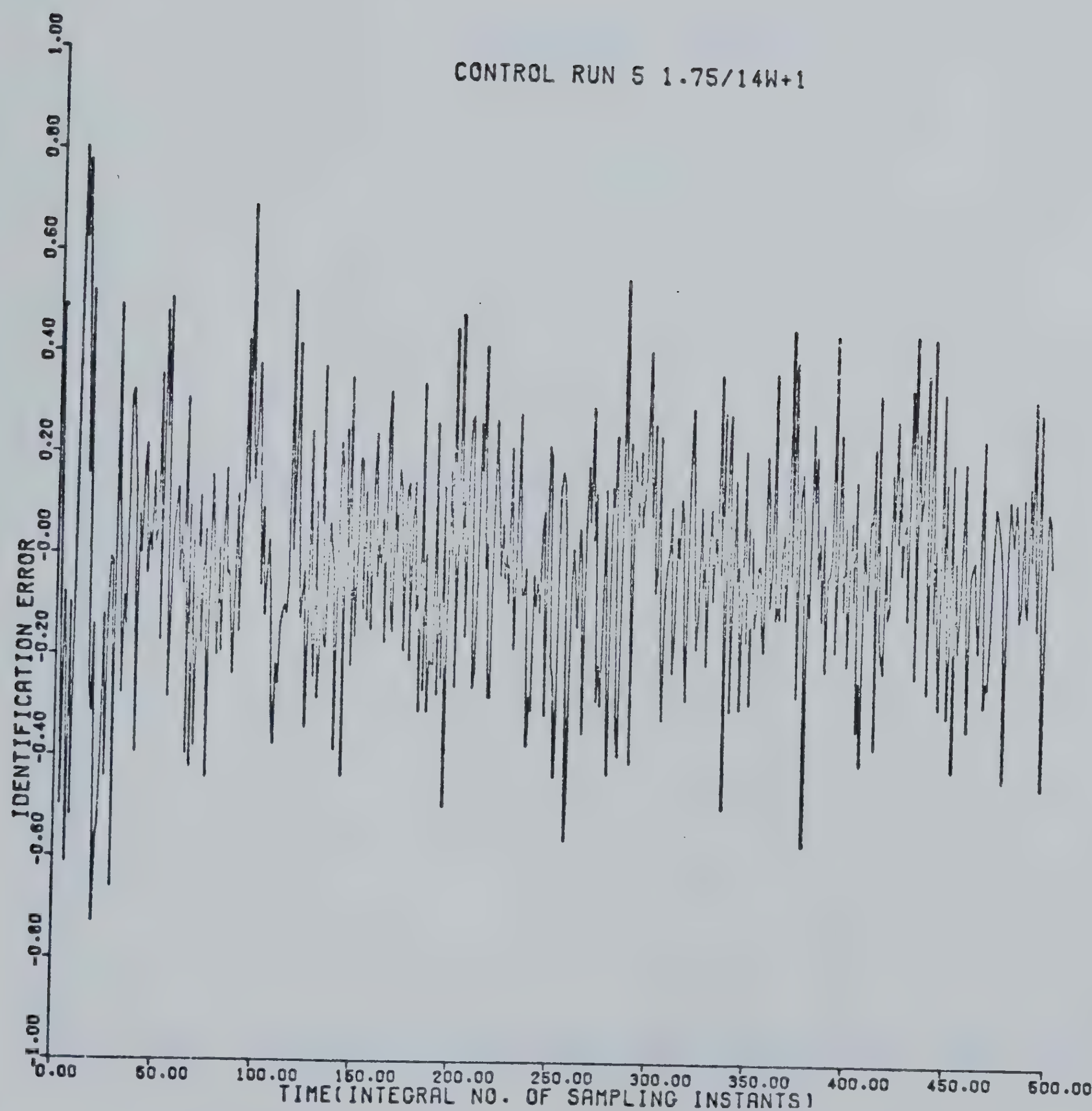


FIGURE 6.18(d): SISO SYSTEM 2 1.75/(14w + 1)
IDENTIFICATION ERROR VS TIME

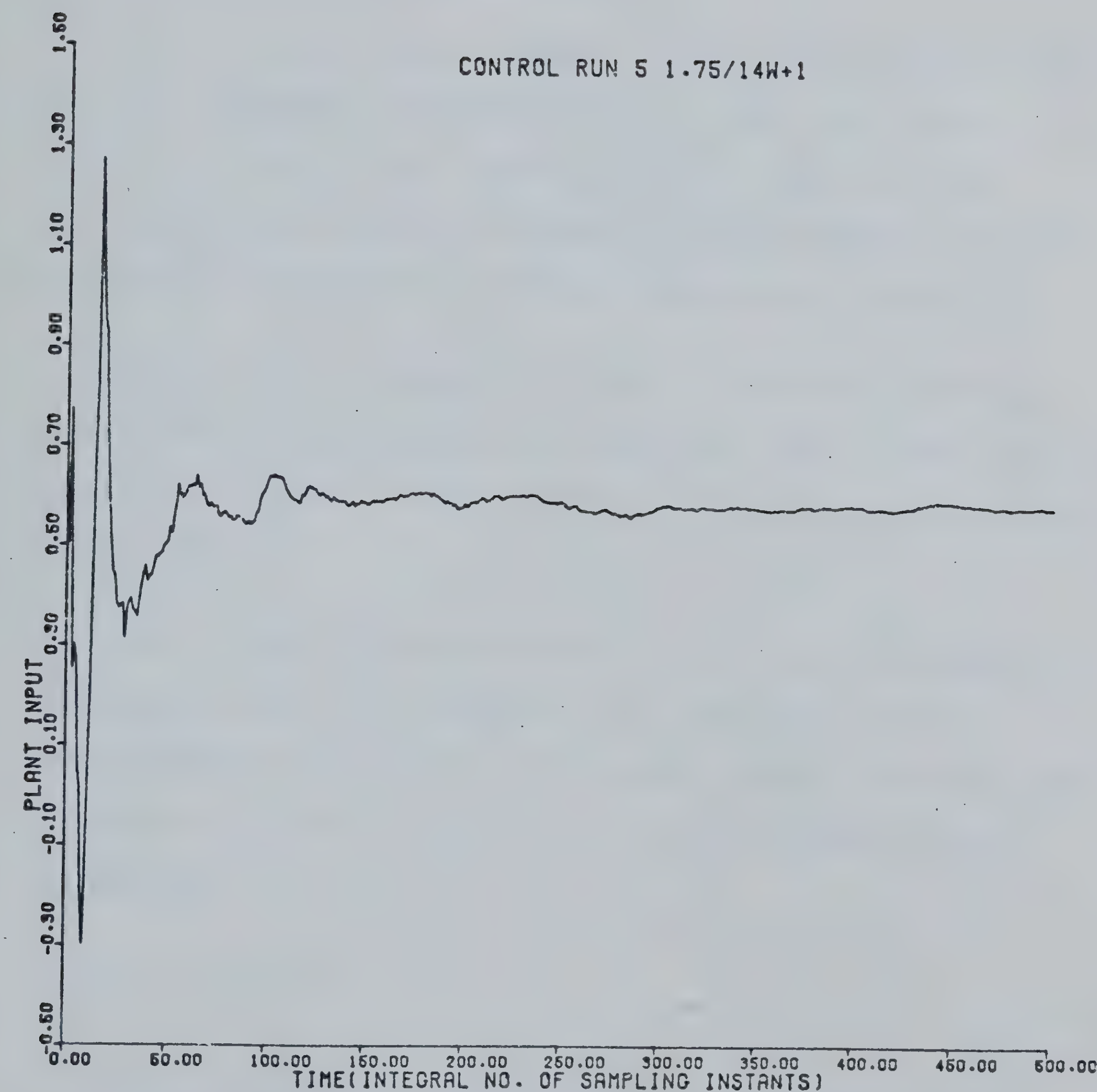


FIGURE 6.18(e): SISO SYSTEM 2 1.75/(14w + 1)
PLANT INPUT VS TIME

initial gains give faster identification and closer subsequent convergence of the plant output to the desired value.

6.7 Multivariable Simulation Runs

Table 6.3 provides a summary of the multivariable runs that are presented in this section[1]. These runs were included to illustrate, primarily, the output driving function of the proposed adaptive control scheme. As such, they do not represent a definitive simulation study.

Runs 1 and 5 (Figures 6.23 and 6.25) employ constant identification loop gains; the second of the set showing the results of control in the face of constraints imposed on the plant inputs.

Runs 2 and 3 (Figures 6.20 and 6.21) show the effect of changing the rate of decrease of the magnitude of the identification loop gains. A tenfold increase does not seem to represent a significant change in the response time of the system.

1

For relevant run data not included in Table 6.3 the reader is referred to Appendix 6.1.

RUN NO.	SYSTEM	MEASUREMENT NOISE (S.D.,MEAN)	DESCREASING MAGNITUDE I.D. LOOP GAINS (λ)	CONTROL CONSTRAINTS (MIN.,MAX.)
1	I	NO	NO	NO
2	II	NO	YES ($\lambda = 1$)	NO
3	II	NO	YES ($\lambda = 10$)	NO
4	II	YES $\begin{bmatrix} 0.05,0 \\ 0.04,0 \end{bmatrix}$	YES ($\lambda = 10$)	NO
5	II	NO	NO	YES $\begin{bmatrix} -1,3 \\ -1,3 \end{bmatrix}$
6	III	YES $\begin{bmatrix} 0.6,0 \\ 0.6,0 \end{bmatrix}$	NO	NO
7	III	YES $\begin{bmatrix} 0.1,0 \\ 0.1,0 \end{bmatrix}$	YES ($\lambda = 1$)	NO

TABLE 6.3 MULTIVARIABLE SIMULATION RUNS

Finally, runs 4 and 7 (Figures 6.22 and 6.25) depict the response in the face of noise corrupted measurements. The addition of decreasing magnitude identification loop gains can be seen to provide an improvement in control with respect to that noted in run 6 (Figures 6.24). Run 6 further demonstrates the dependence of the system on the output convergence of the estimation scheme; the oscillatory response of the plant output (Figure 6.24(a)) being principally due to the identification error variation (Figure 6.24(d)).

General Remarks

1. The systems investigated have included a state-space formulation (System I) as well as two general input-output configurations. Thus, the simulations have included the incomplete state information case.

2. The plant models chosen exhibit very large interactions and yet it has been possible to drive the outputs towards arbitrarily chosen values.

3. It has been noted that for large noise levels, the systems are apt to produce unstable outputs. This is thought to be due to the identification model displaying non-minimum phase characteristics. A projection feature, such as that suggested by Ljung [3], is needed, especially

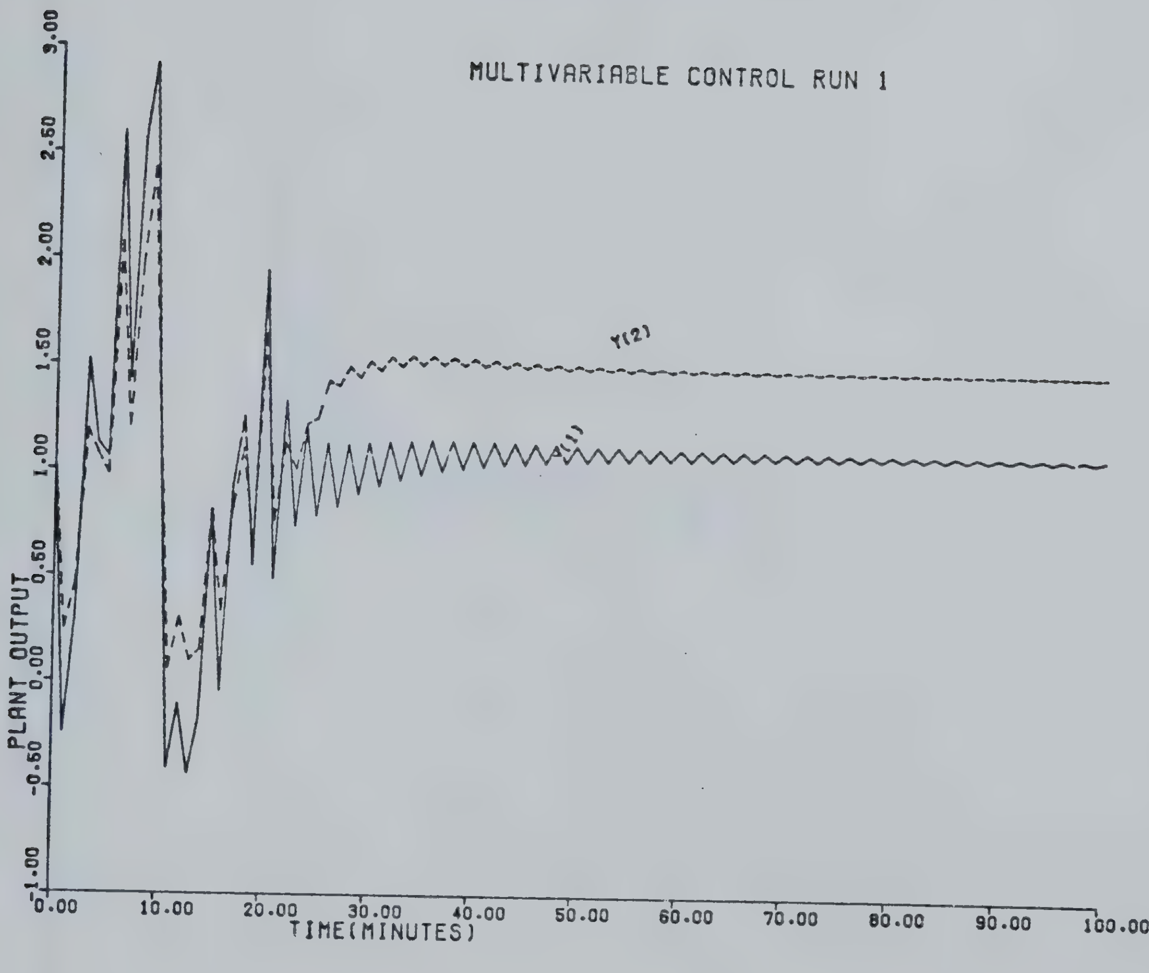


FIGURE 6.19(a): MIMO SYSTEM I
PLANT OUTPUT VS TIME

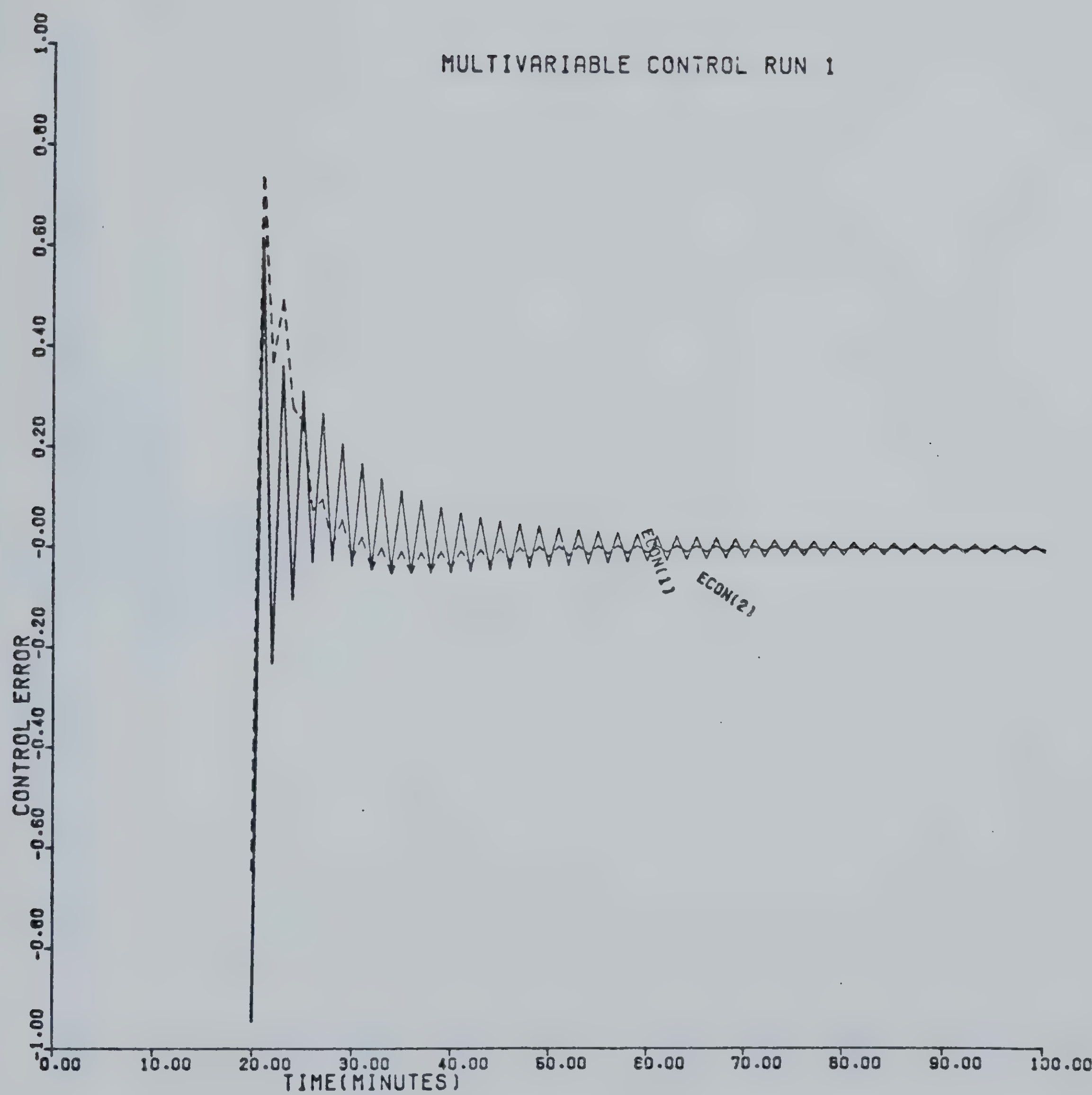


FIGURE 6.19(b): MIMO SYSTEM I
CONTROL ERROR VS TIME

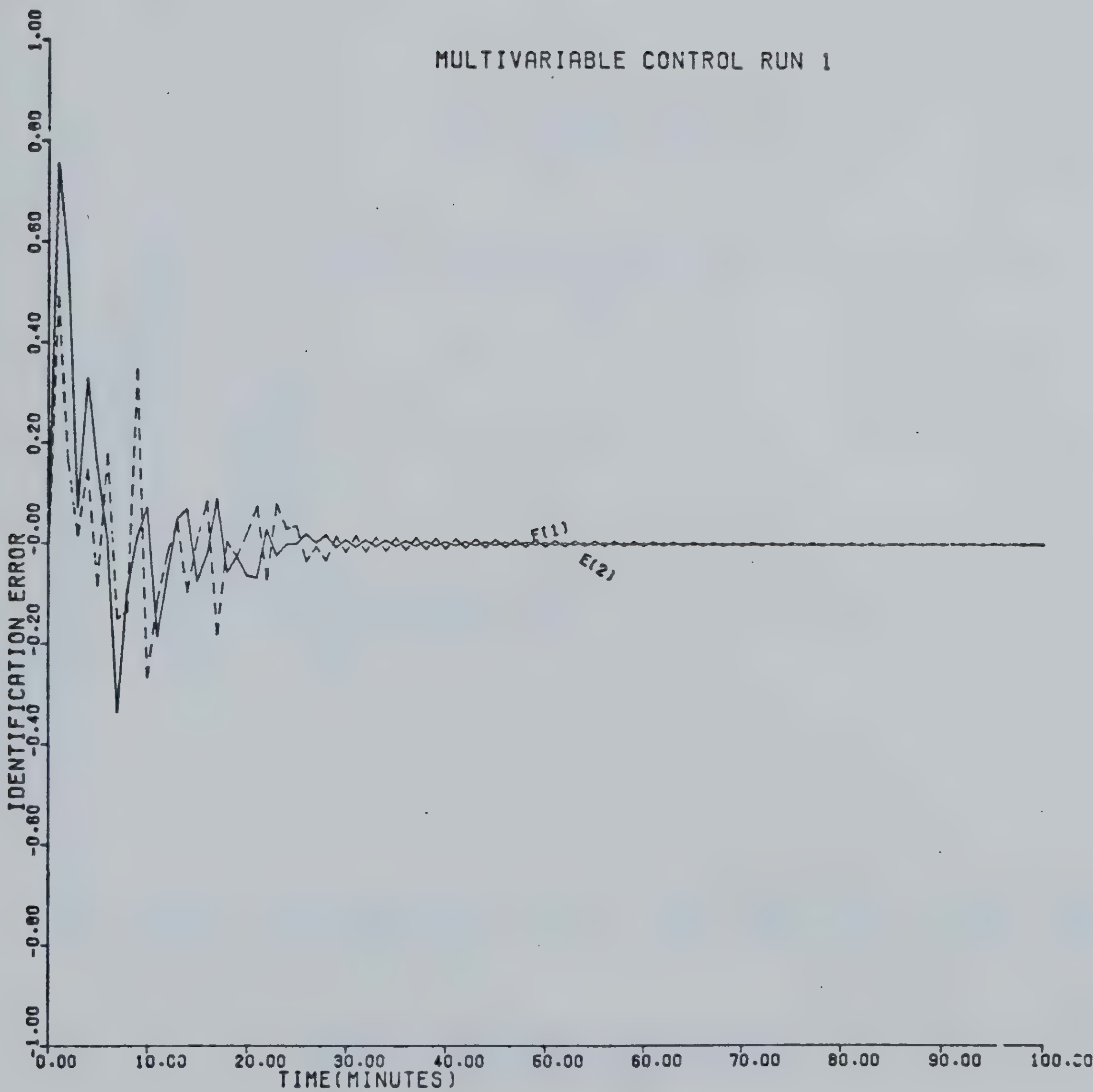


FIGURE 6.19(c): MIMO SYSTEM I
IDENTIFICATION ERROR VS TIME

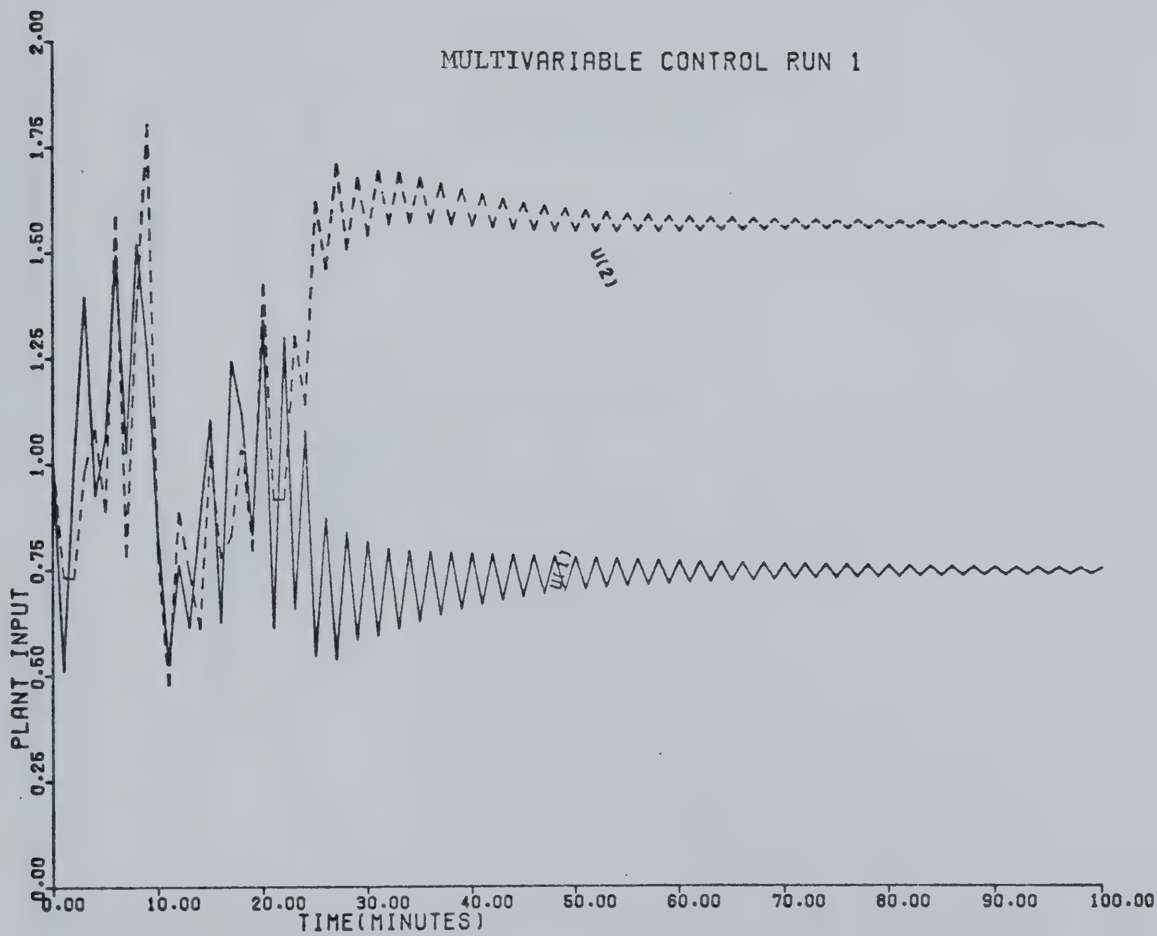


FIGURE 6.19(d): MIMO SYSTEM I
PLANT INPUT VS TIME

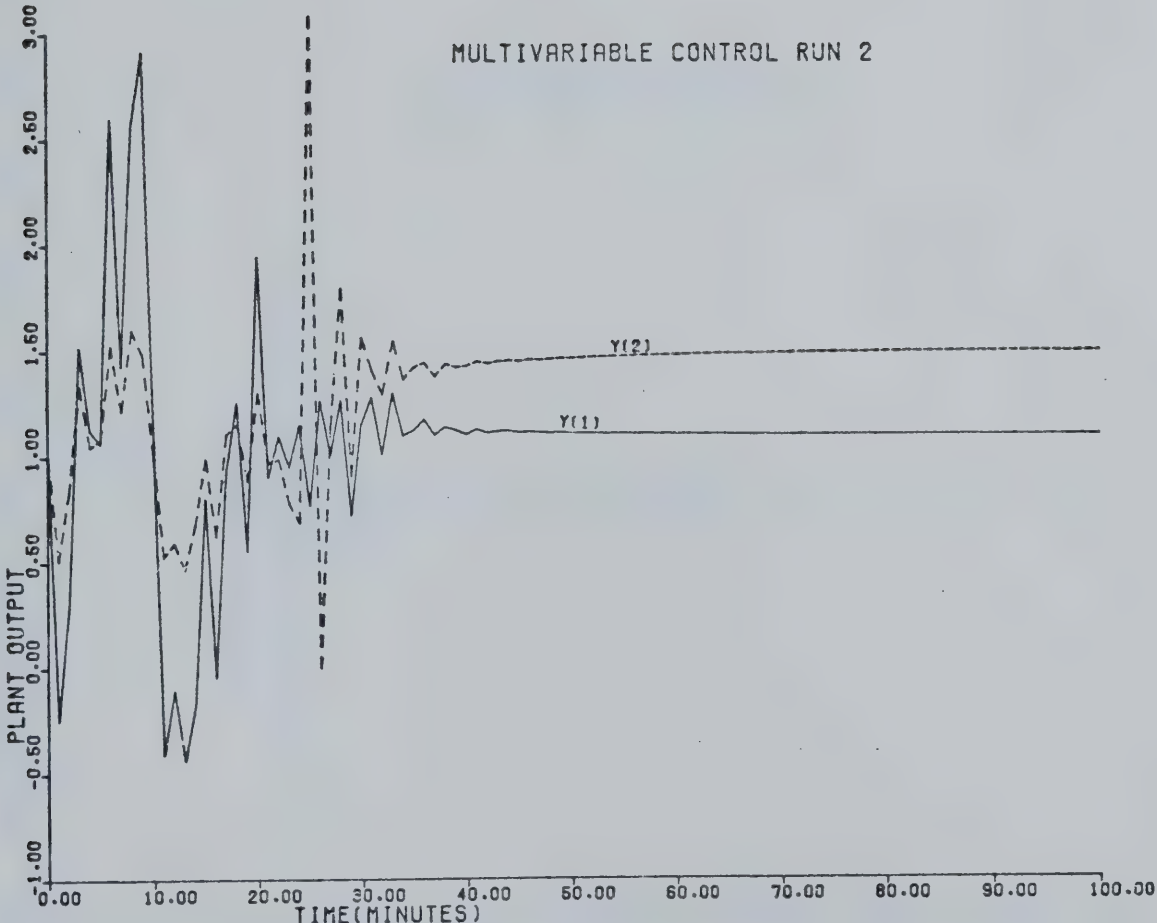


FIGURE 6.20(a): MIMO SYSTEM II
PLANT OUTPUT VS TIME

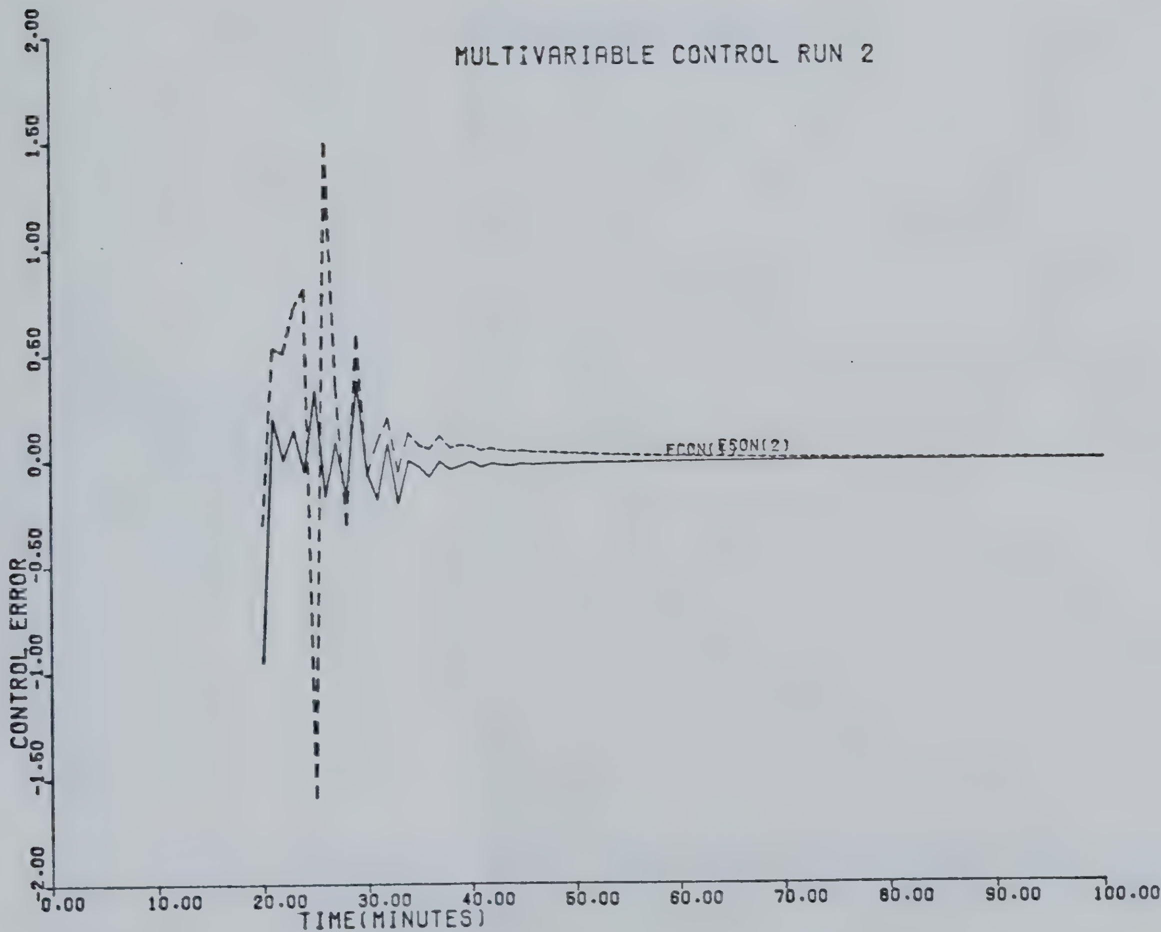


FIGURE 6.20(b): MIMO SYSTEM II
CONTROL ERROR VS TIME

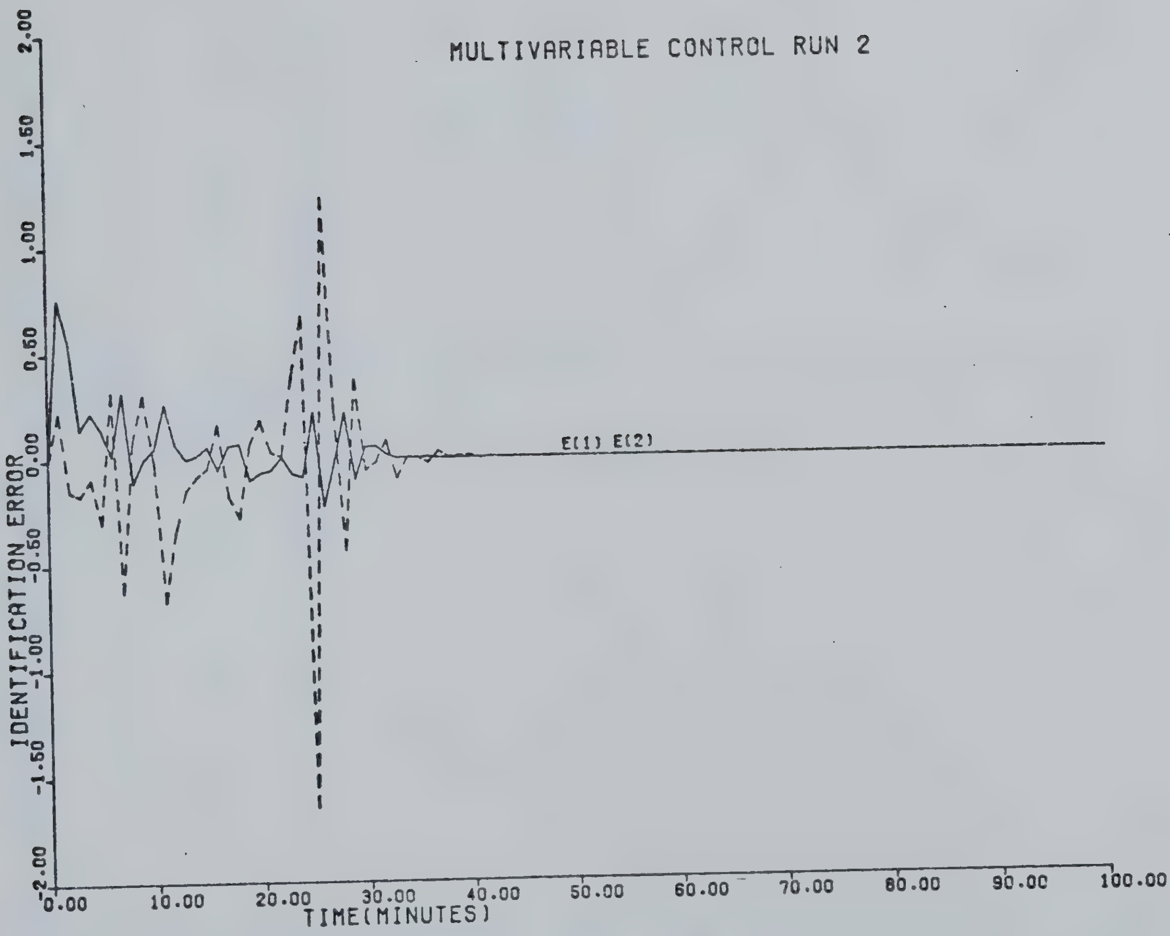


FIGURE 6.20(c): MIMO SYSTEM II
IDENTIFICATION ERROR VS TIME

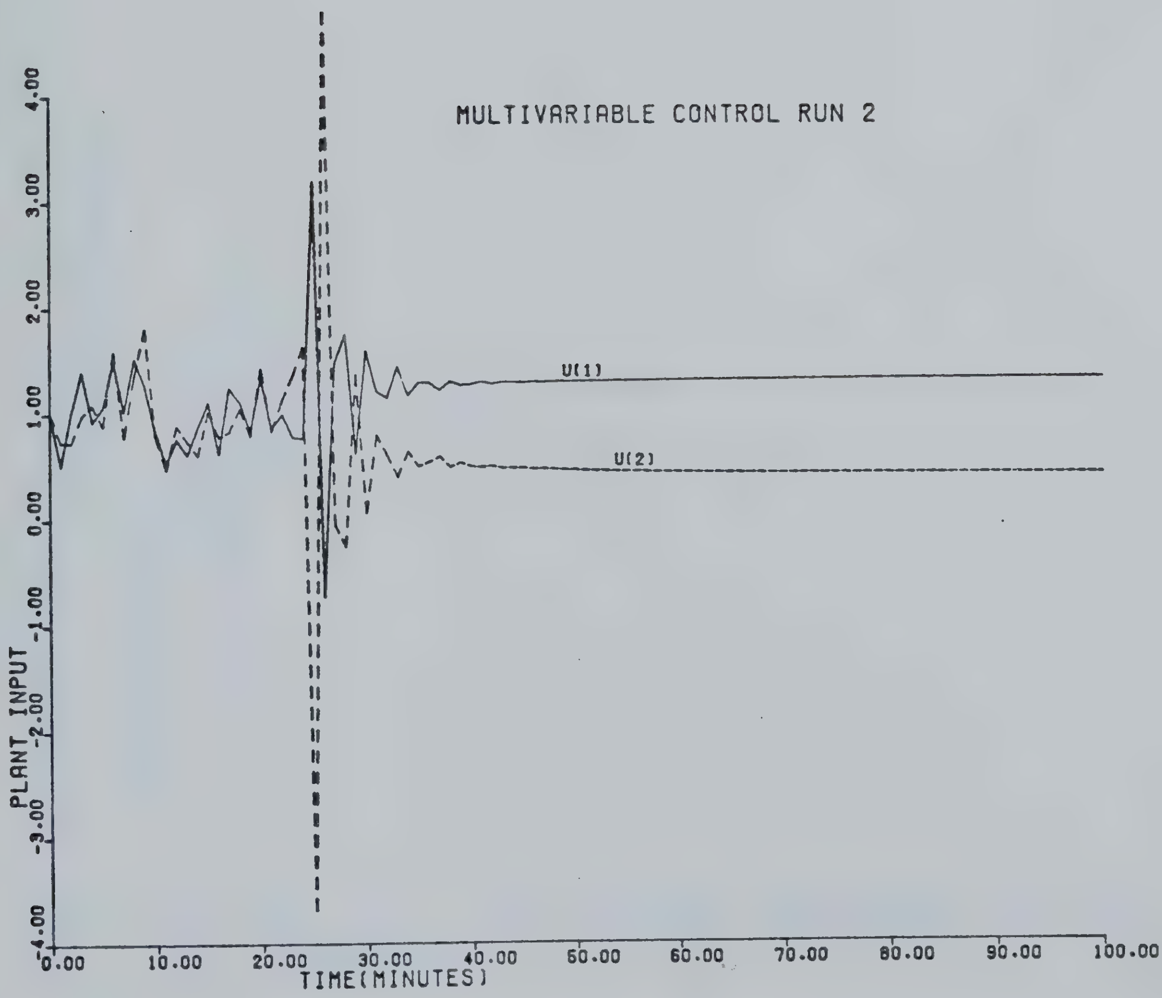


FIGURE 6.20(d): MIMO SYSTEM II
PLANT INPUT VS TIME

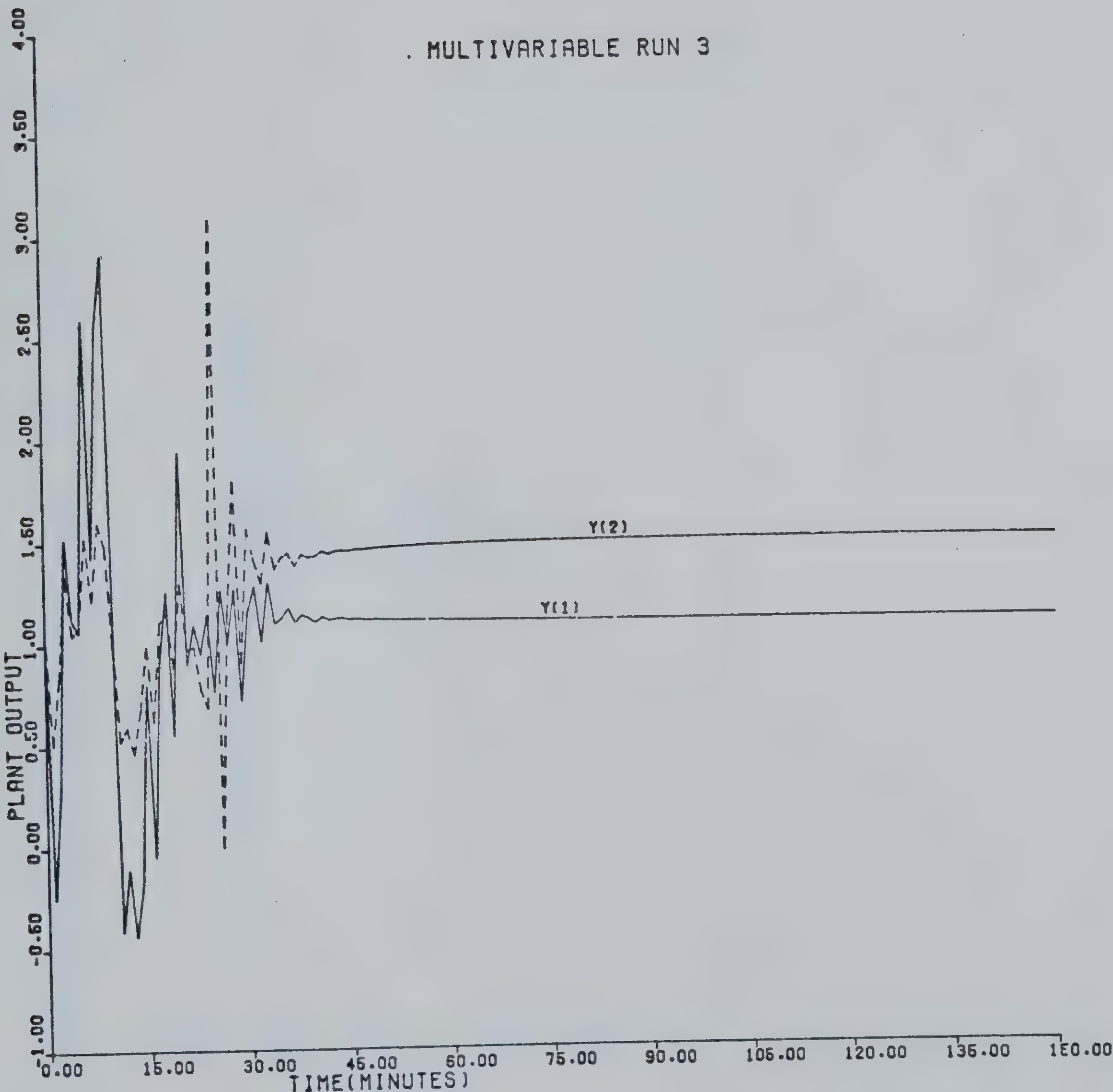


FIGURE 6.21(a): MIMO SYSTEM II
PLANT OUTPUT VS TIME

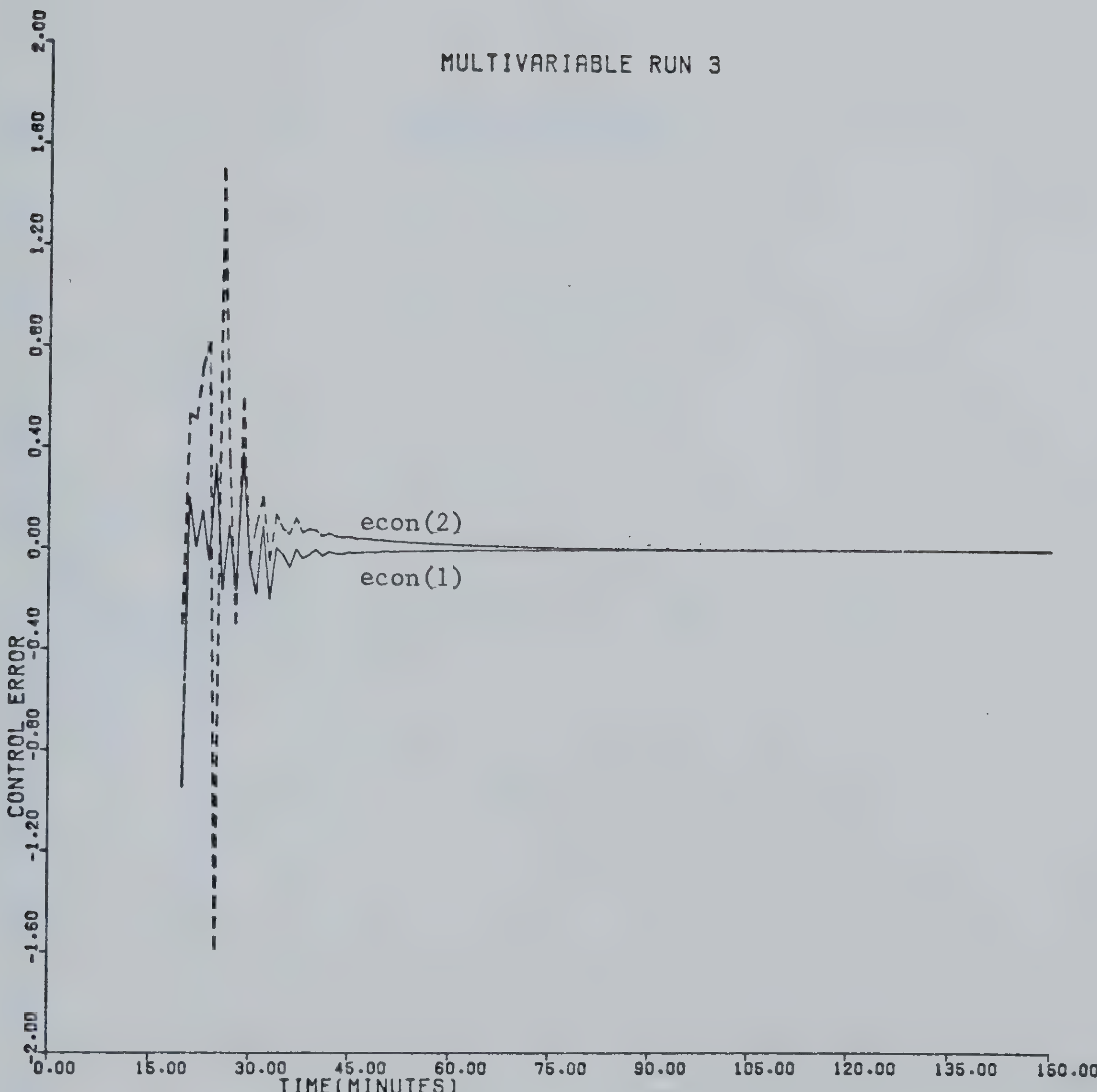


FIGURE 6.21(b): MIMO SYSTEM II
CONTROL ERROR VS TIME

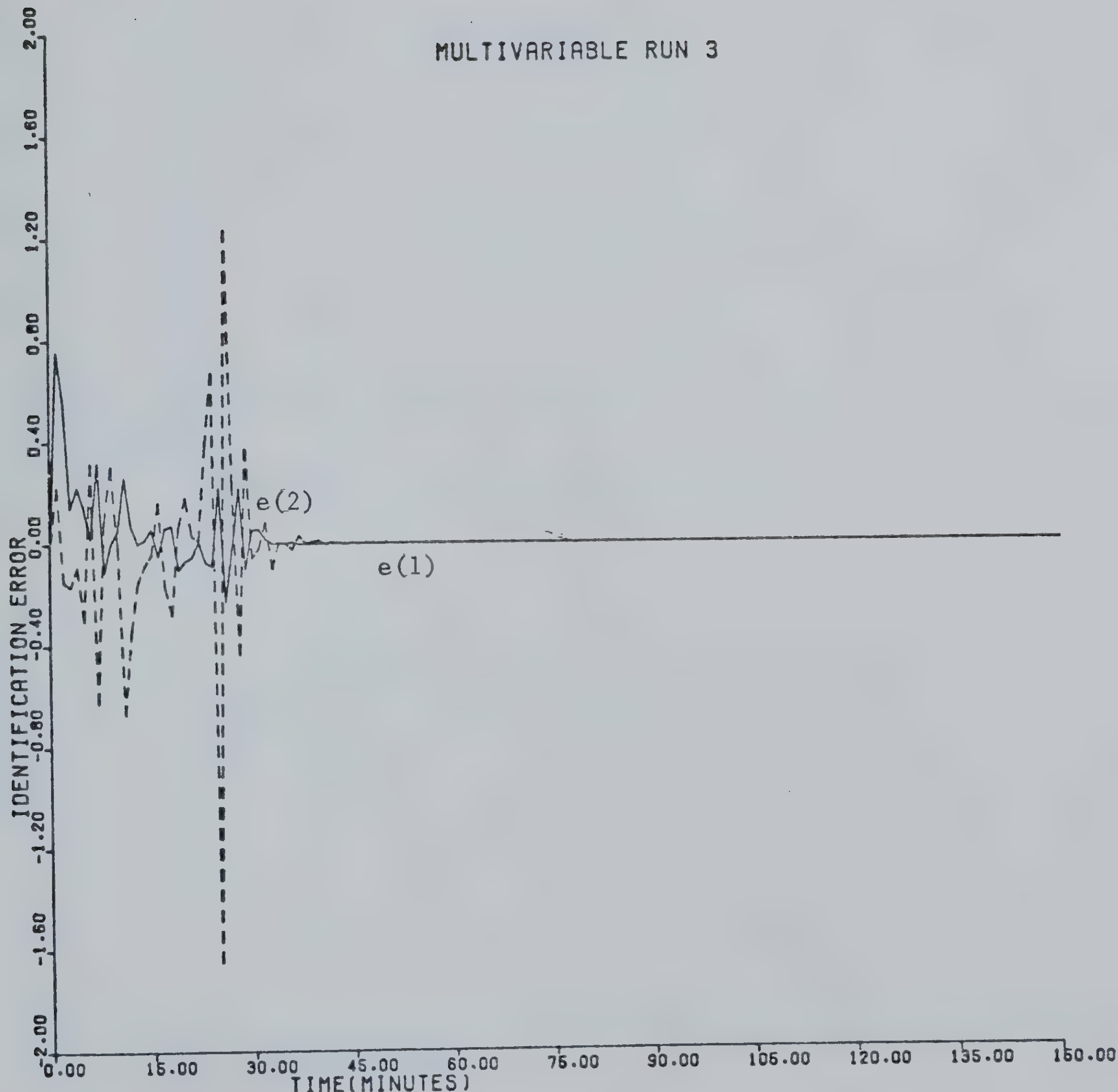


FIGURE 6.21(c): MIMO SYSTEM II
IDENTIFICATION ERROR VS TIME

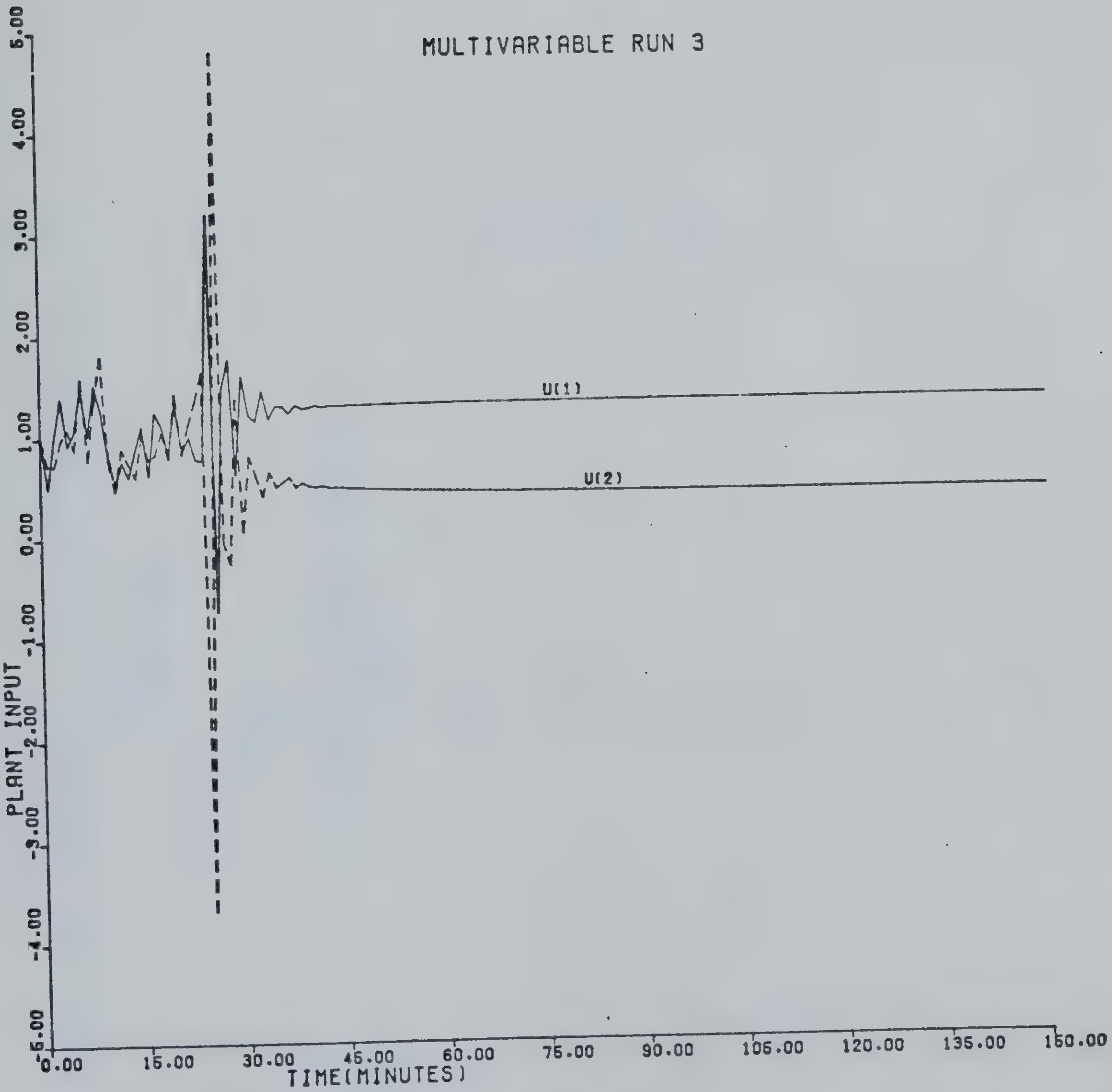


FIGURE 6.21(d): MIMO SYSTEM II
PLANT INPUT VS TIME

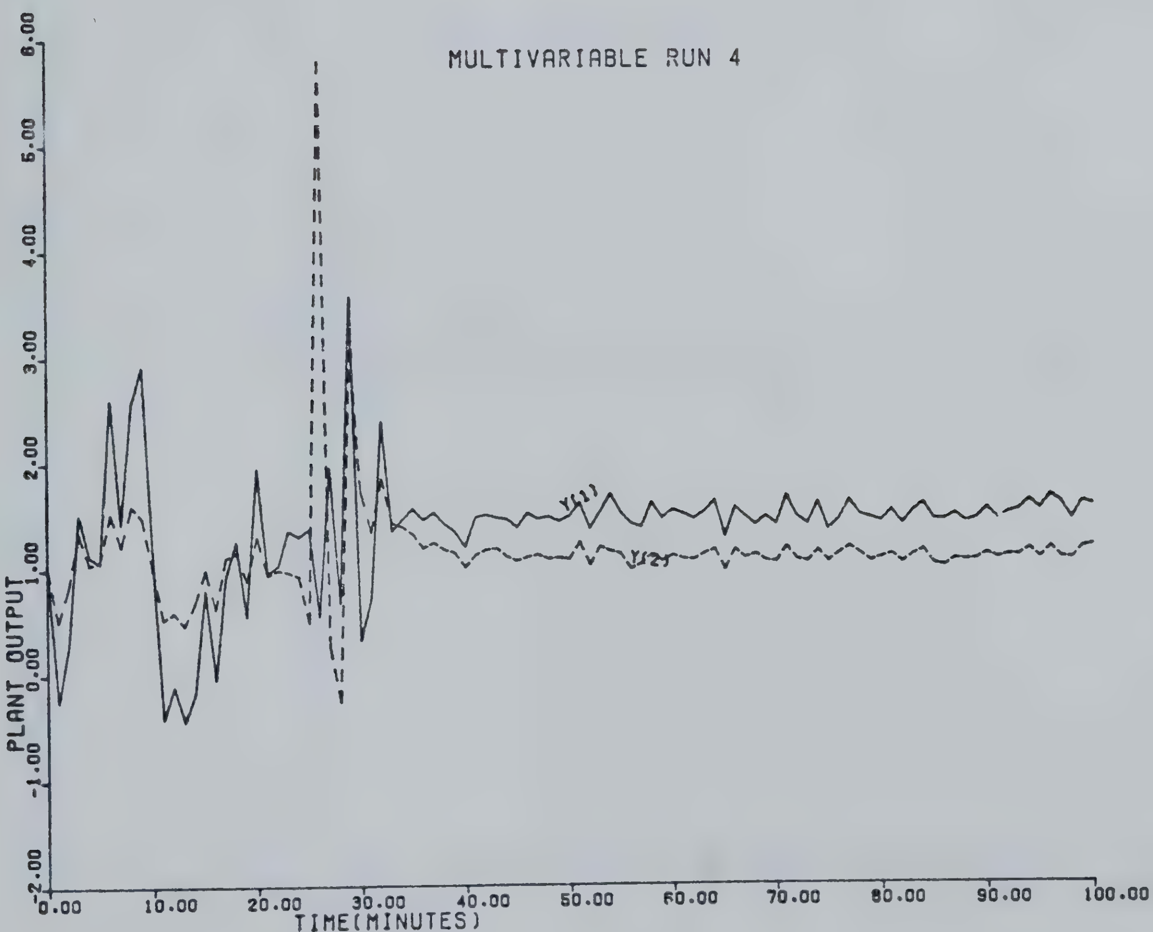


FIGURE 6.22(a): MIMO SYSTEM II
PLANT OUTPUT VS TIME

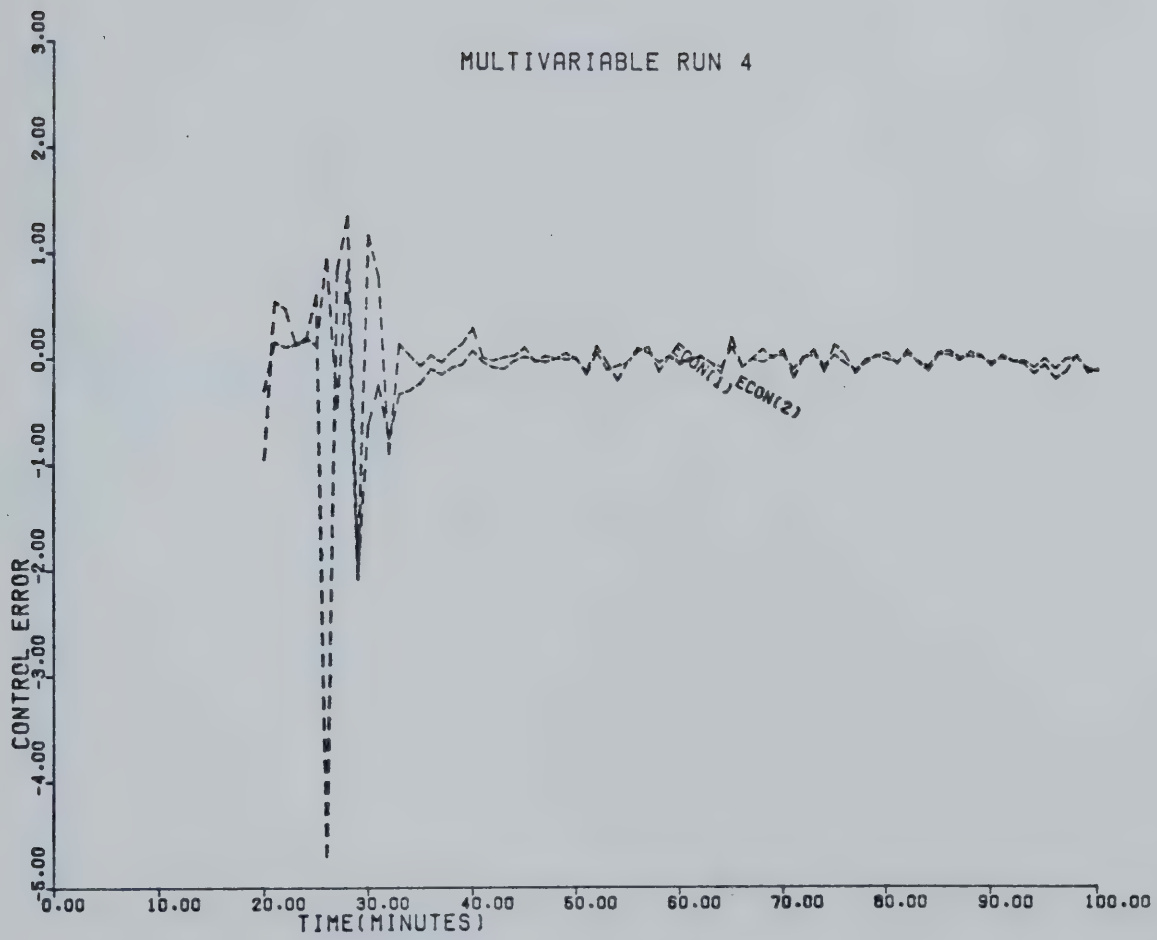


FIGURE 6.22(b): MIMO SYSTEM II
CONTROL ERROR VS TIME

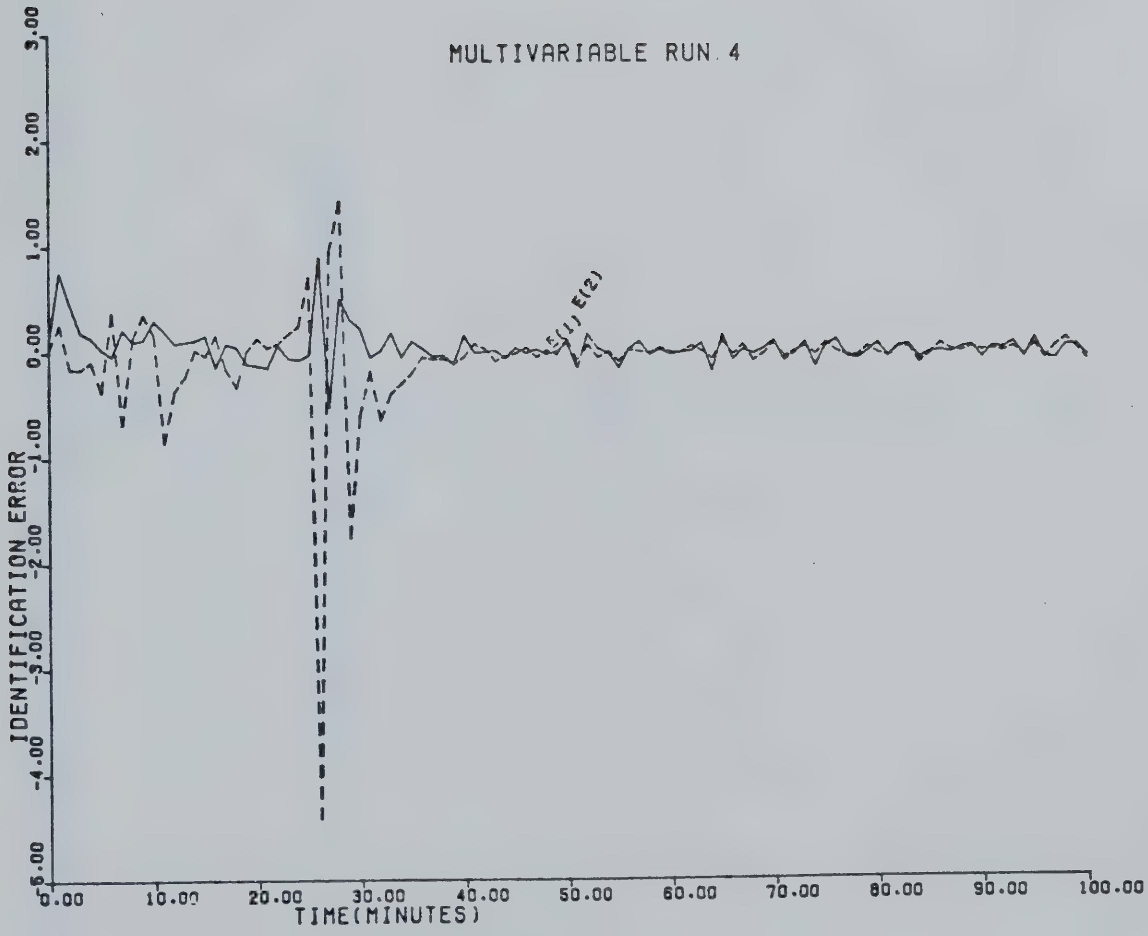


FIGURE 6.22(c): MIMO SYSTEM II
IDENTIFICATION ERROR VS TIME

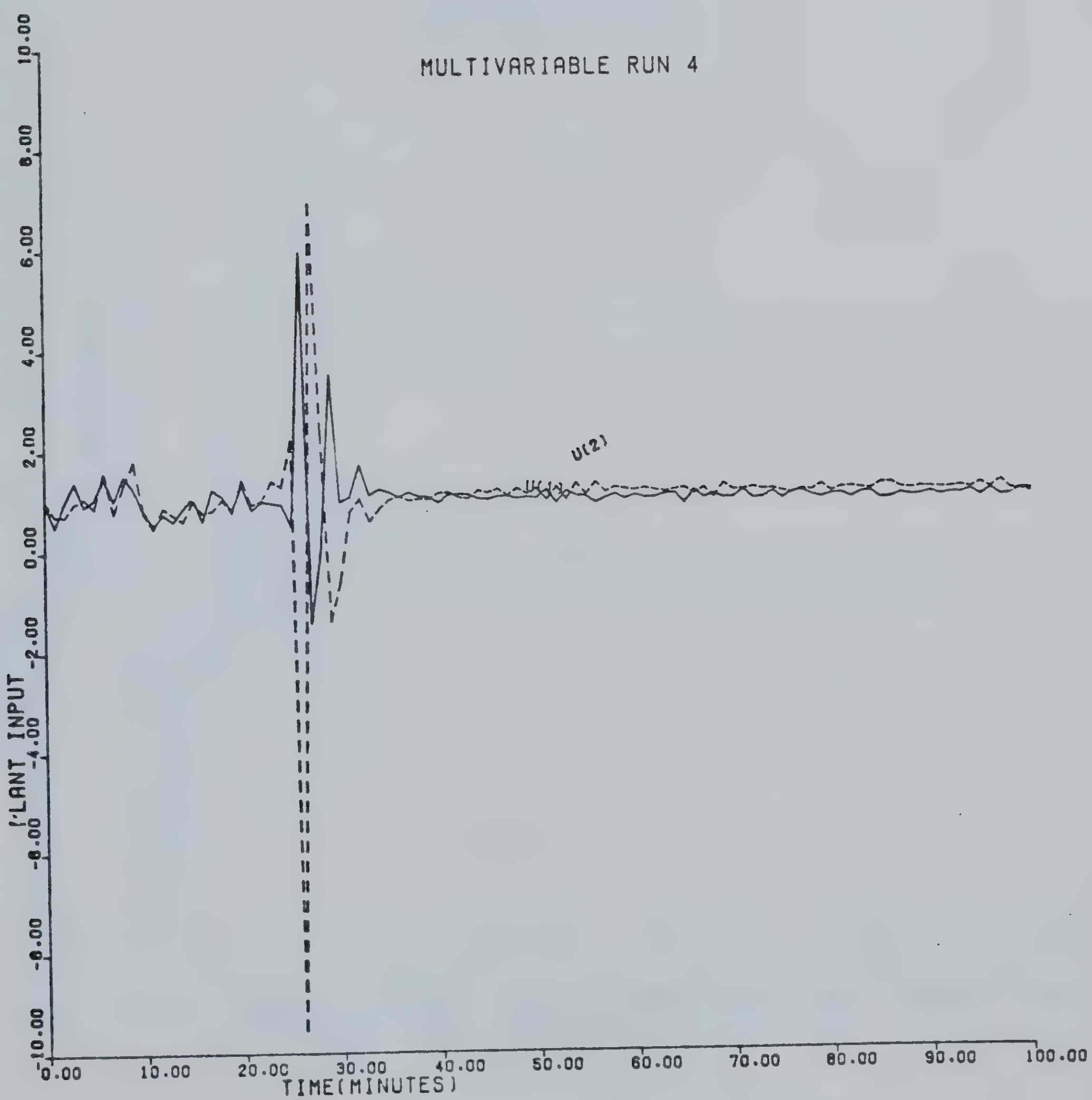


FIGURE 6.22(d): MIMO SYSTEM II
PLANT INPUT VS TIME

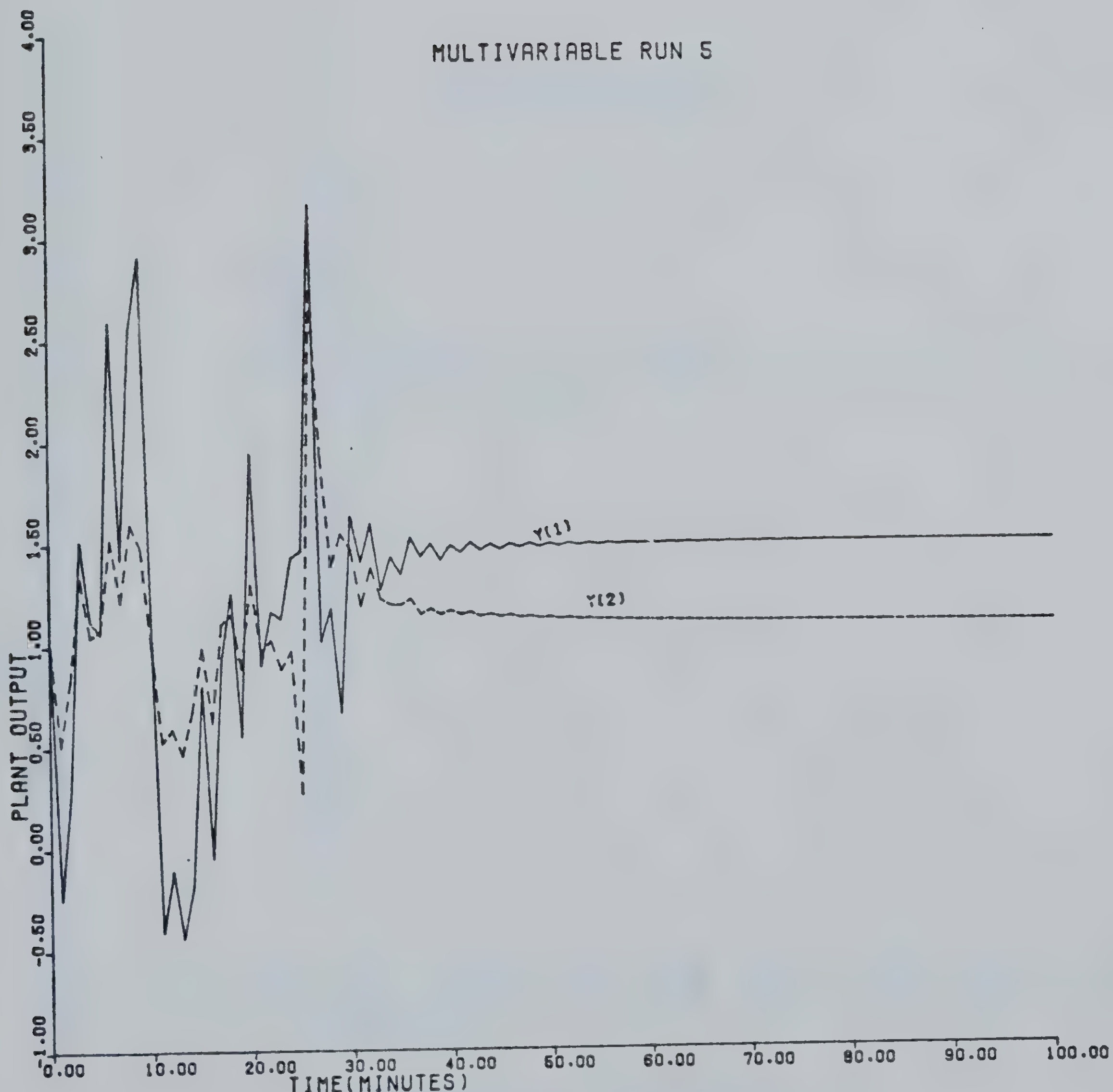


FIGURE 6.23(a): MIMO SYSTEM II
PLANT OUTPUT VS TIME

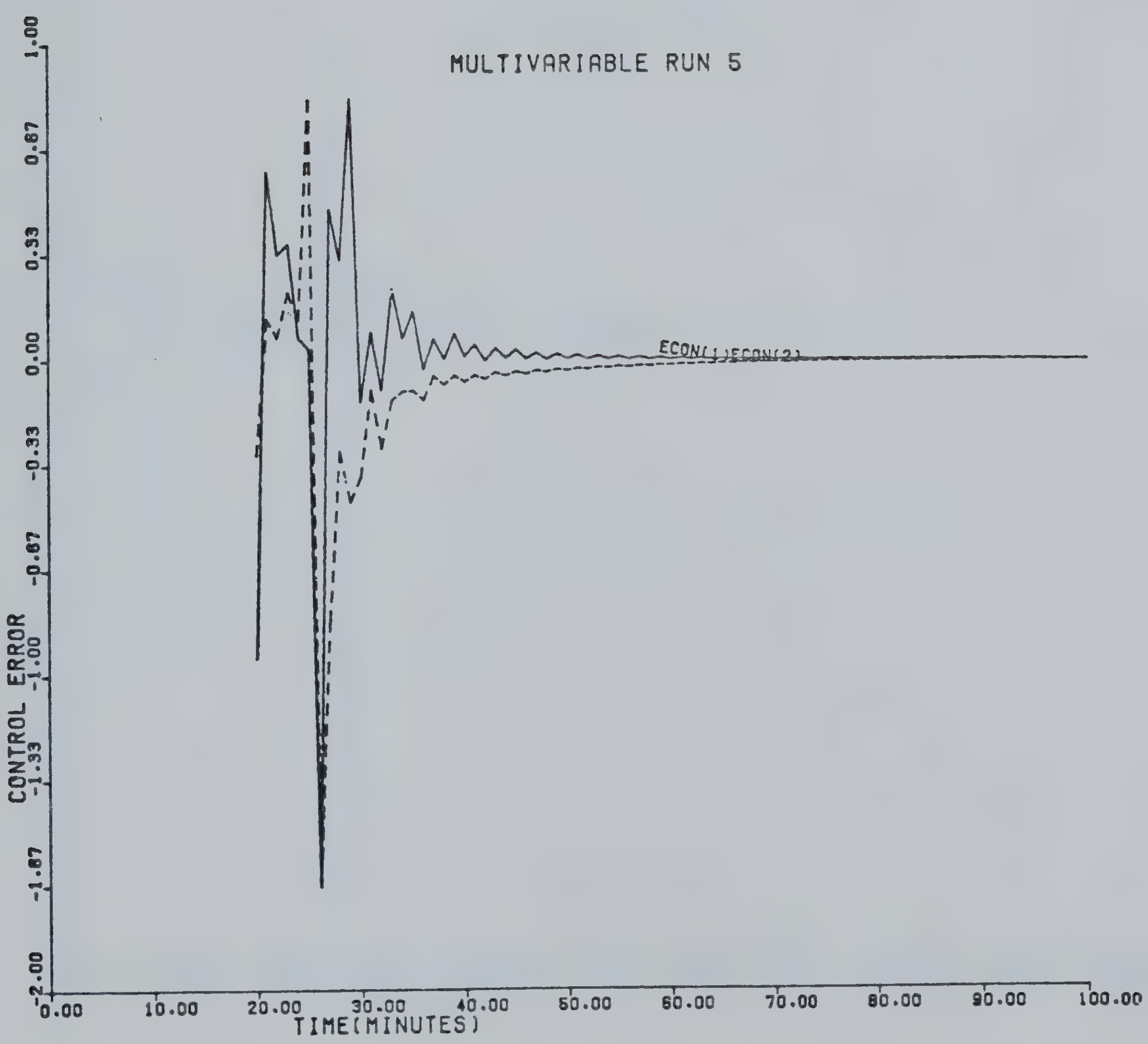


FIGURE 6.23(b): MIMO SYSTEM II
CONTROL ERROR VS TIME

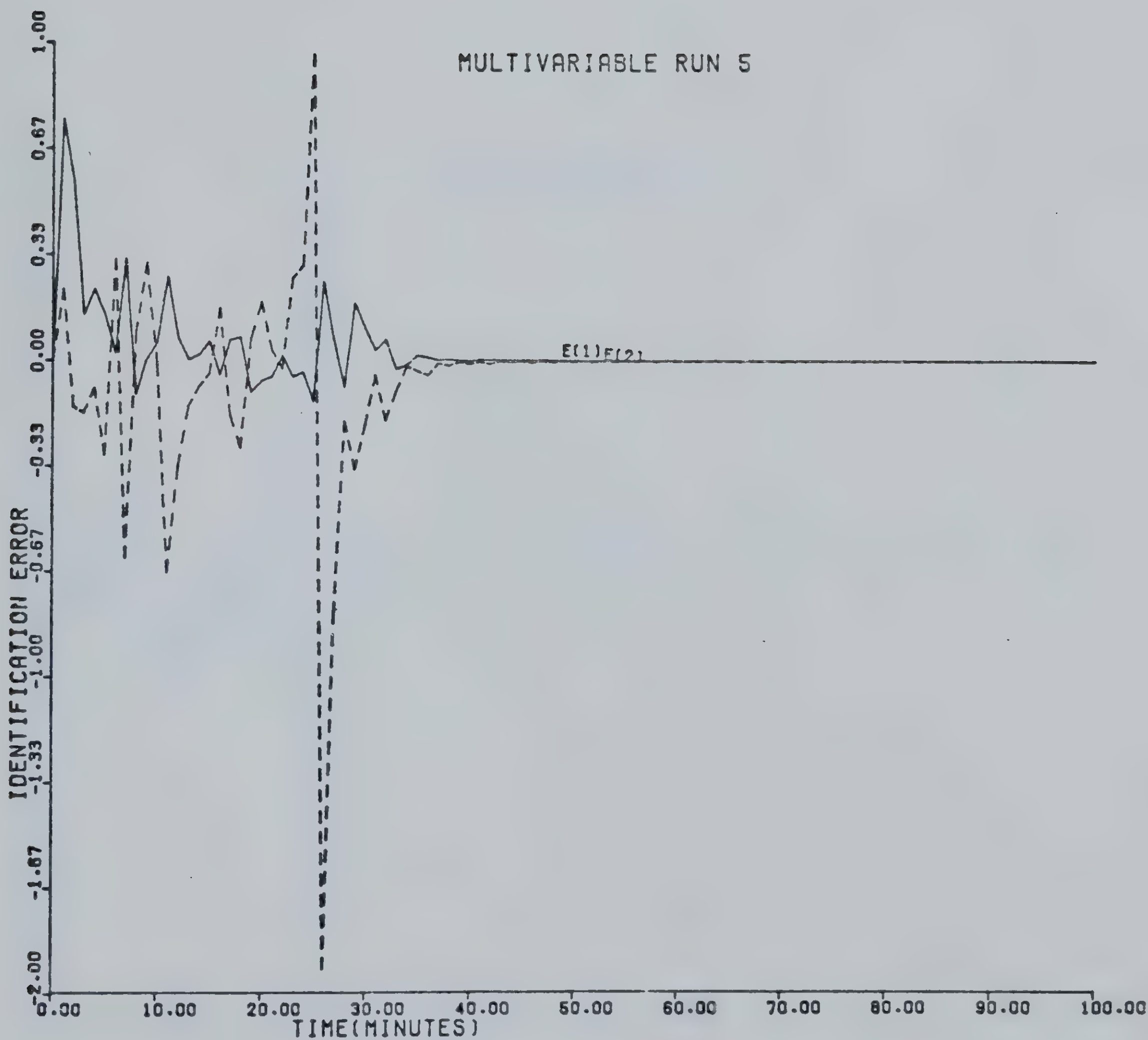


FIGURE 6.23(c): MIMO SYSTEM II
IDENTIFICATION ERROR VS TIME

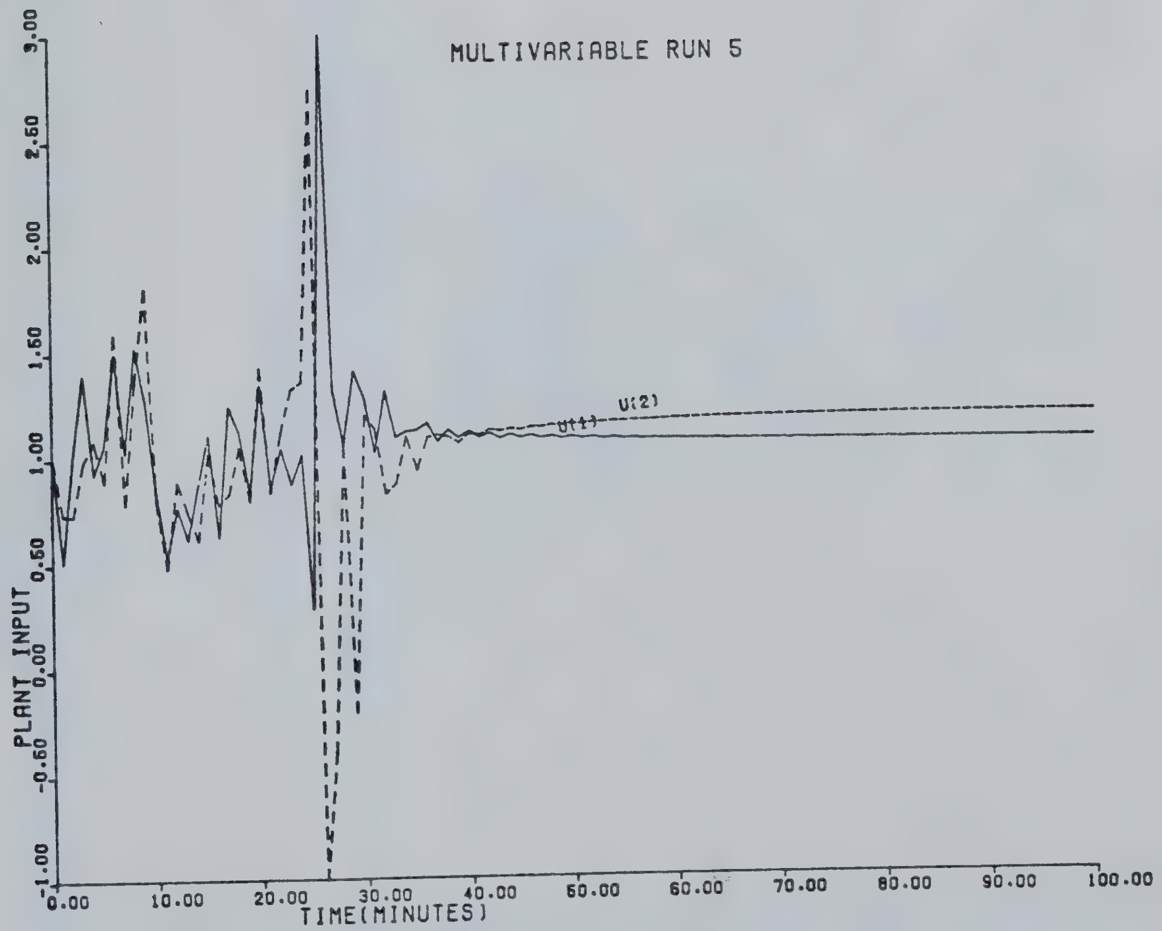


FIGURE 6.23(d): MIMO SYSTEM II
PLANT INPUT VS TIME

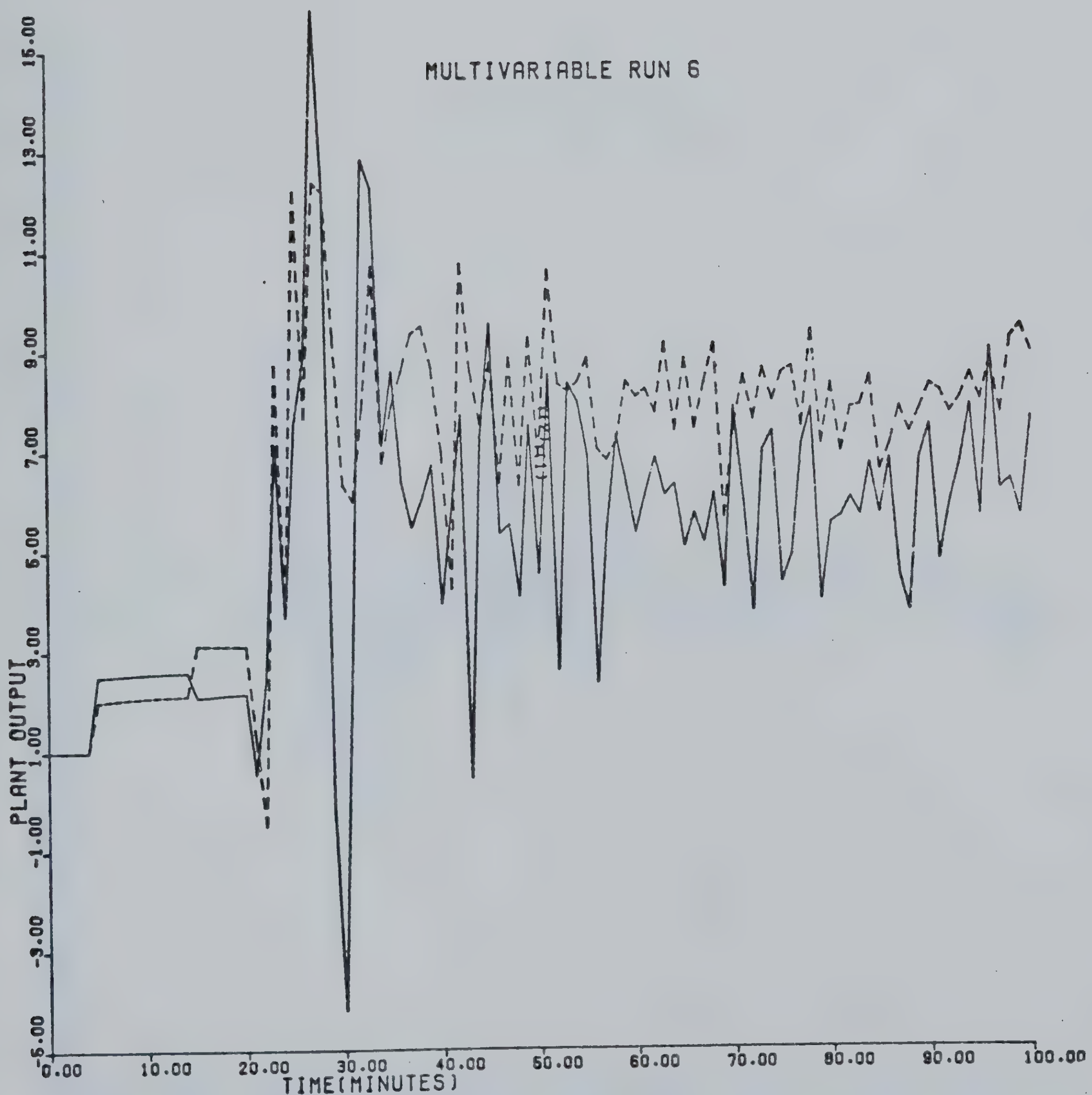


FIGURE 6.24(a): MIMO SYSTEM III
PLANT OUTPUT VS TIME

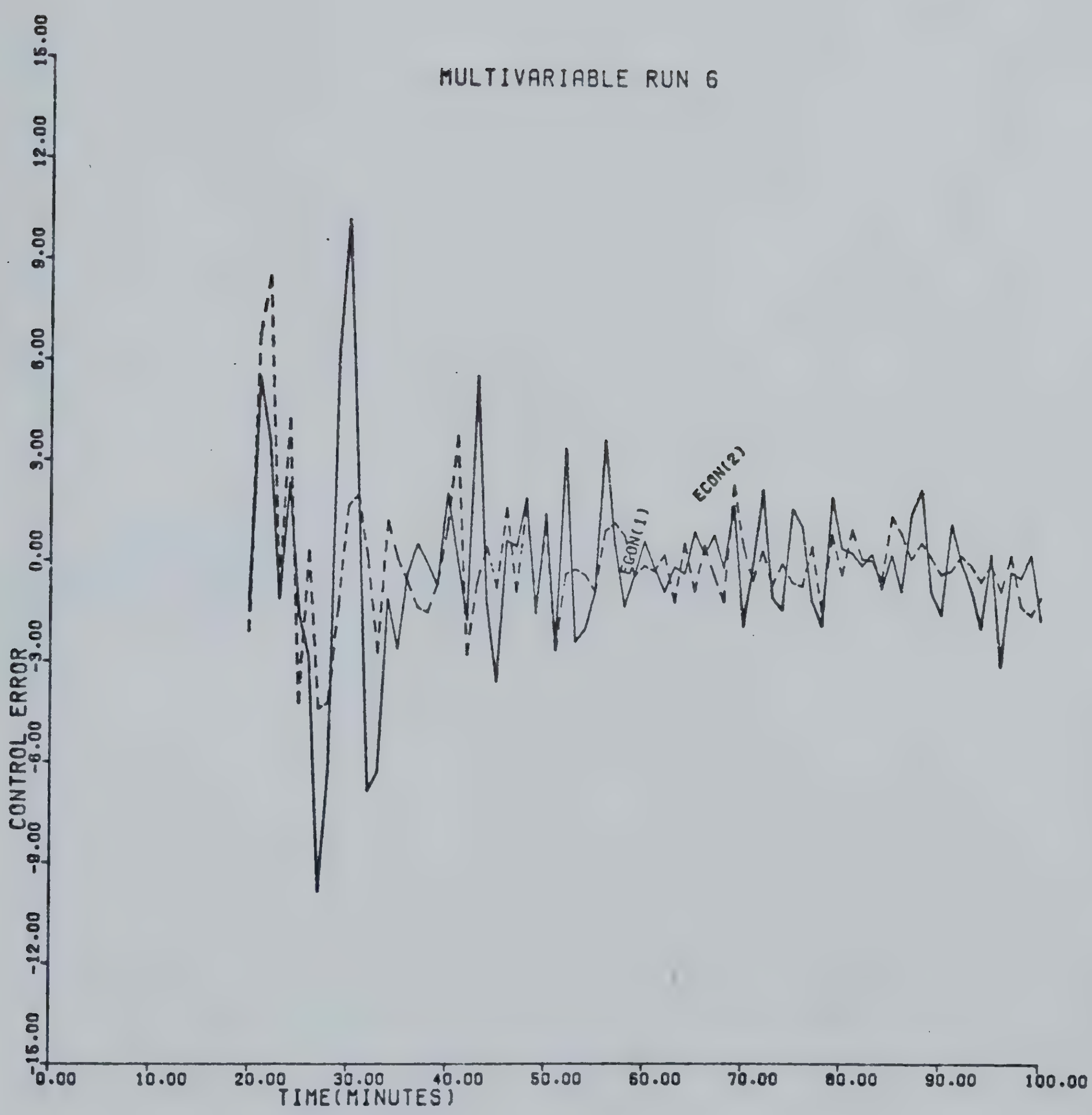


FIGURE 6.24(b): MIMO SYSTEM III
CONTROL ERROR VS TIME

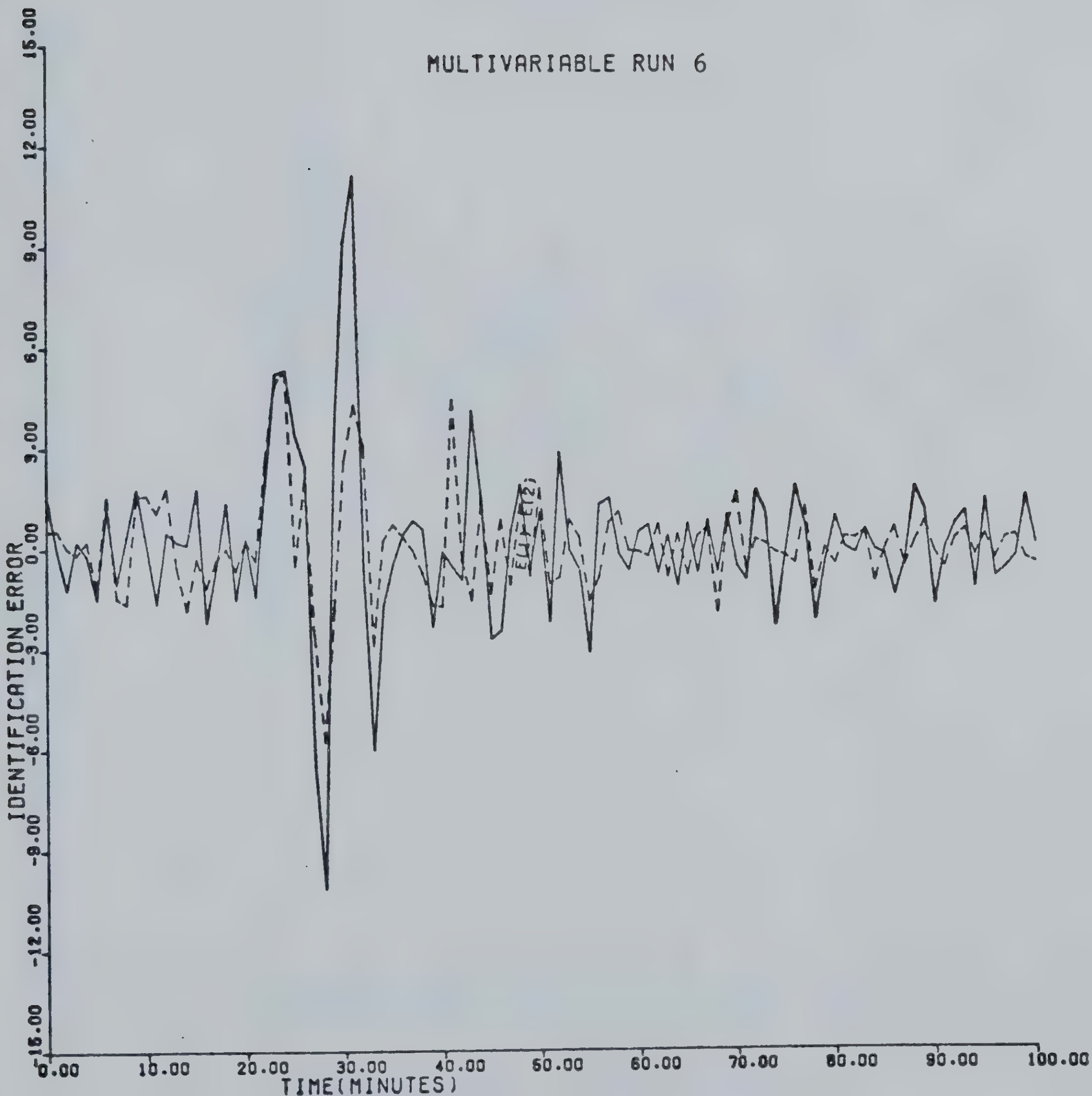


FIGURE 6.24(c): MIMO SYSTEM III
IDENTIFICATION ERROR VS TIME

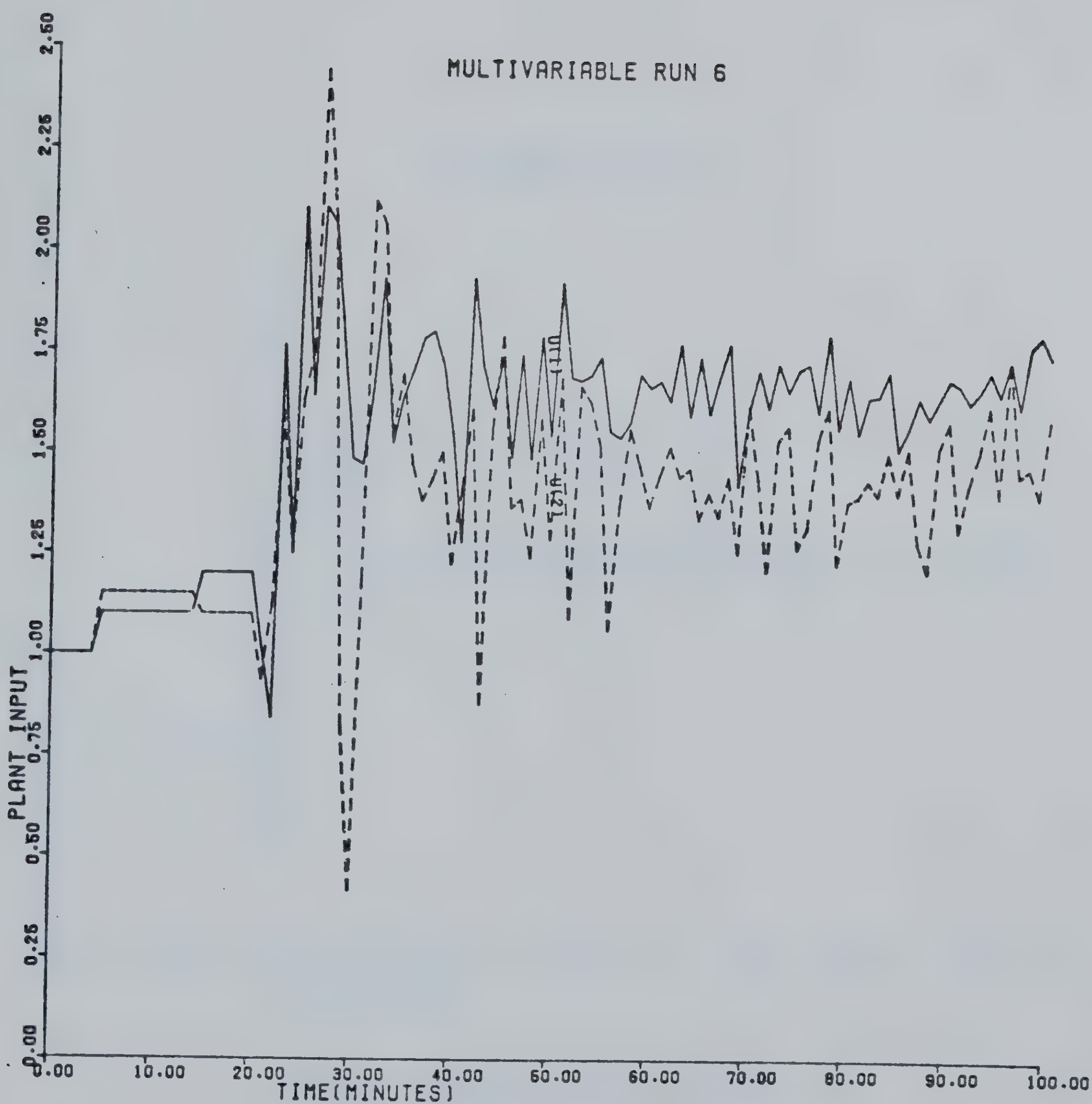


FIGURE 6.24(d): MIMO SYSTEM III
PLANT INPUT VS TIME

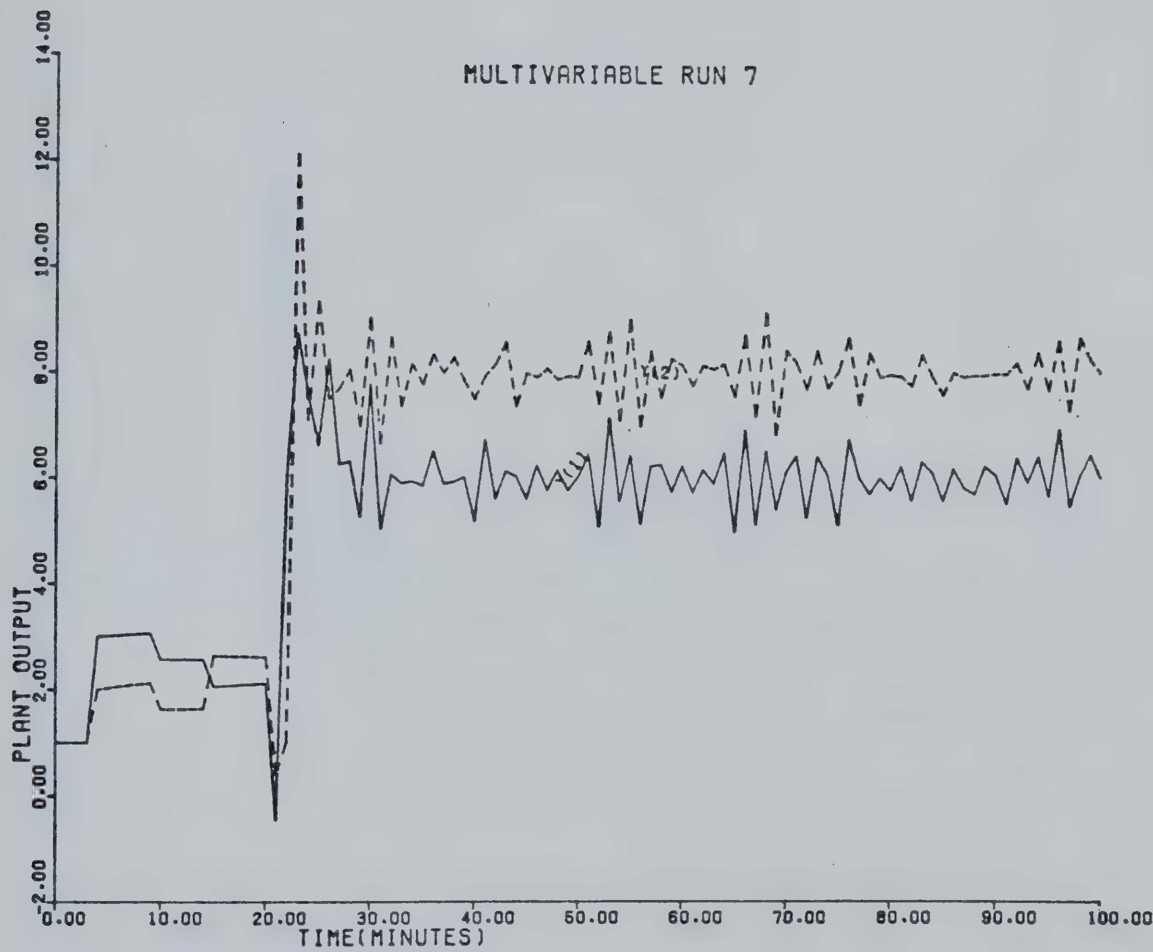


FIGURE 6.25(a): MIMO SYSTEM III
PLANT OUTPUT VS TIME

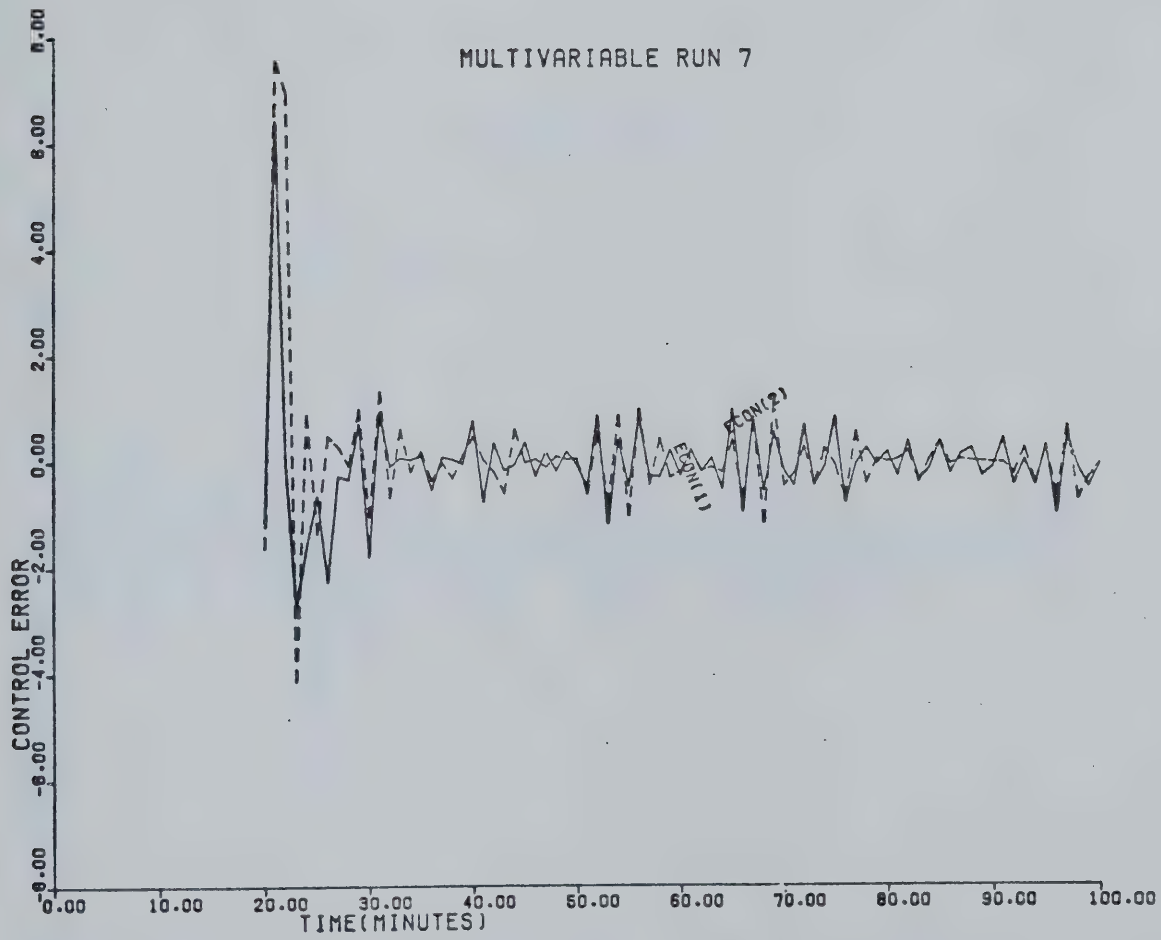


FIGURE 6.25(b): MIMO SYSTEM III
CONTROL ERROR VS TIME

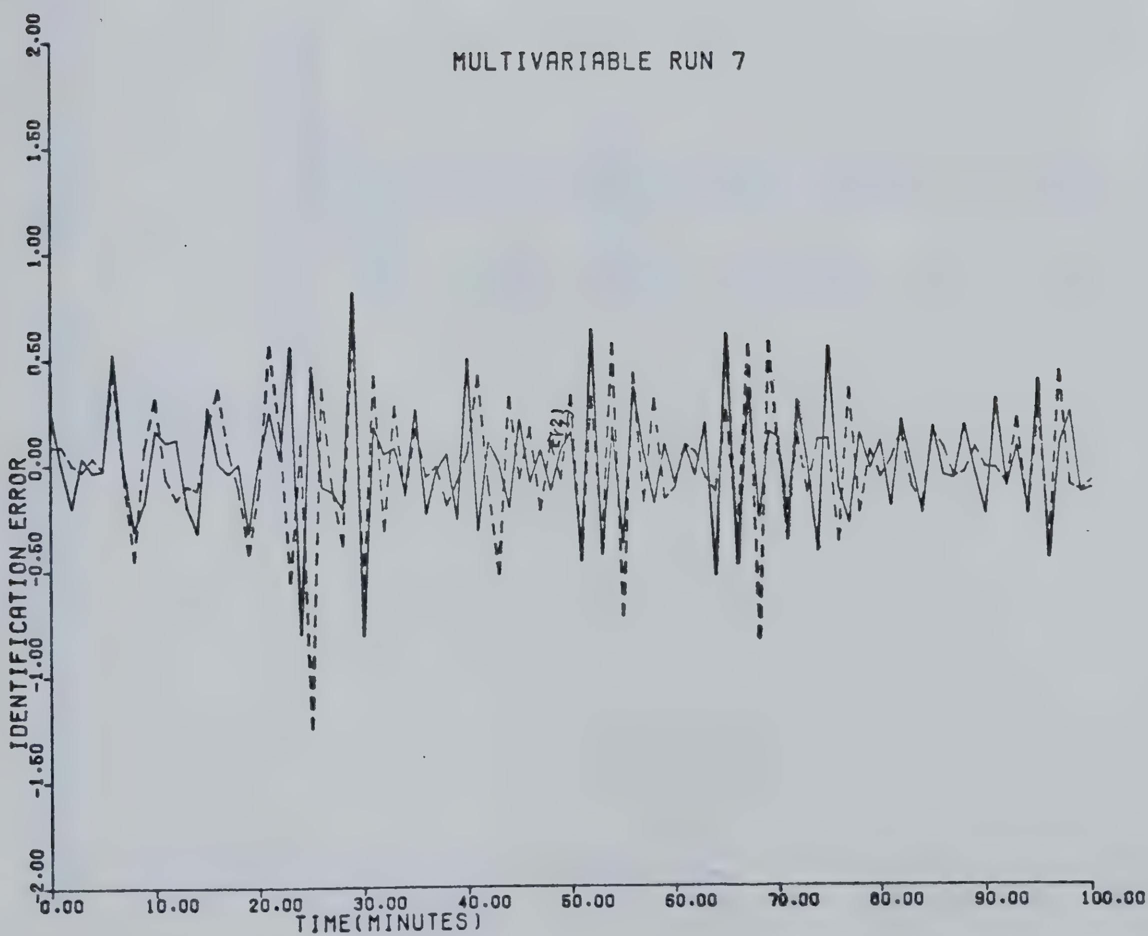


FIGURE 6.25(c): MIMO SYSTEM III
IDENTIFICATION ERROR VS TIME

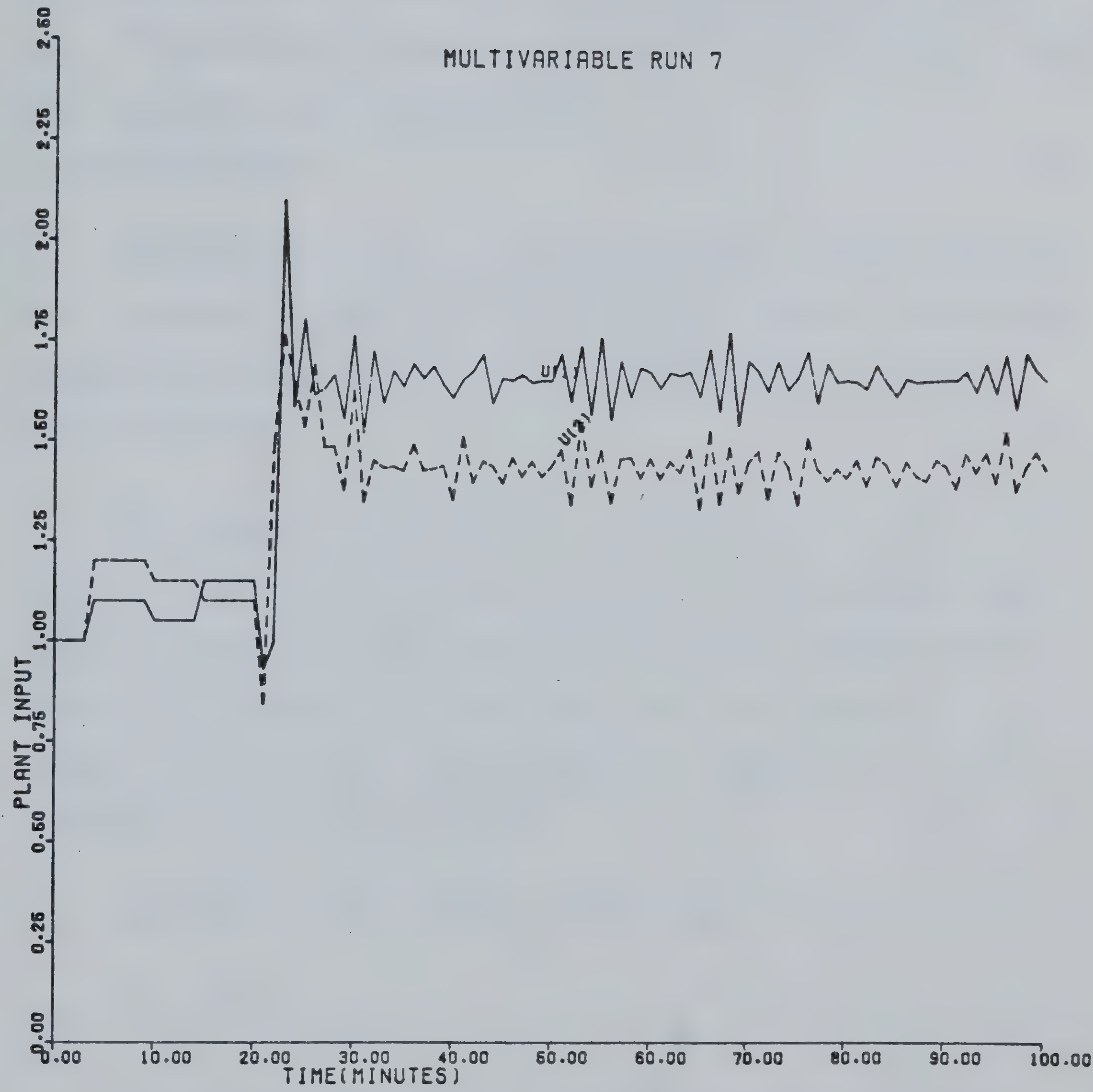


FIGURE 6.25(d): MIMO SYSTEM III
PLANT INPUT VS TIME

for multivariable systems to protect against this eventuality. This would essentially limit the identification model parameters to a fixed-space such that any combination of parameters results in a minimum phase set. Theoretical conditions for output realizability[1] under such circumstances are required.

6.8 Conclusions

Altogether a total of five systems have been examined in a simulation study to investigate the effect of various design parameters on the proposed hyperstable, adaptive control scheme.

The system has been shown to respond very well in the face of noise corrupted measurements for both single-input single-output and multi-input multi-output plants and, in general, it would be recommmended that the amount of filtering action (as determined by the value of λ) be tailored to the noise present.

Some of the multi-input multi-output runs have shown a

1

This would refer to the ability of one system to track another's outputs when their parameters are not identical.

tendency to become unstable in the presence of very noisy measurements. The explanation has been advanced that this is due to the uncertainty of the manner in which the identification model parameters adapt. It is felt that some type of projection mechanism may provide a solution to this problem. This comment also applies to the scheme put forward by Martin-Sanchez [4 - 5], as this difficulty exists there too [6].

Undeniably, there is an undesirably large amount of oscillation during the initial stages of adaptation. This relates to the identification process, in that the Gaussian excitation input signals, used to drive the initial identification sequence, is rich in frequency content but is unrelated entirely to the control actions generated in the control mode. Because the identification procedure need not provide good estimates of the actual plant parameters (the only requirement being output convergence), the parameters are apt to readapt when excited by a different class of input signal.

Finally, it is noted that the output driving function has been shown to be valid under some very stringent environmental conditions. Certainly these schemes are far more attractive than the essentially regulatory systems

investigated by Oliver [7].

Chapter Six

CHAPTER SEVEN

Dynamic Precompensation Using Model Reference Techniques

7.1 Introduction

The development of multivariable frequency domain design techniques, in the last decade, has reversed a trend in research institutions that had been prevalent for the previous fifteen years. Rijnsdorp and Seborg [1] have shown fairly clearly a general reluctance, on the industrial world's part, to have anything to do with the so-called "modern control" methodology, principally it appears, because of the reliance on linear parametric models [2].

The frequency domain methods, on the other hand, do not strictly require a parametric model as frequency data will suffice. Further, these schemes retain the desirable design features of the classical single-loop controller designs [3,4], ie.:

i) the controllers may be tuned "on line". This has obvious ramifications from an industrial viewpoint;

ii) no state estimation is necessary since outputs only are used;

iii) since the gain estimates are conservative,

there is a certain safety margin with respect to inaccuracy in the data or to drifting parameter values. These schemes are, thus, generally less sensitive to parameter variations than other methods, and

iv) a very good indication of a number of facets of the control scheme, including:

(a) stability,

(b) integrity,

(c) interactions and,

(d) performance are obtained.

Moreover, classical measures of performance can be applicable, giving the designer an enormous amount of subjective choice in configuring the system.

Some very detailed work was done at the University of Alberta in the experimental comparison of several promising methods, in this area, using a double-effect evaporator [3]. Further, a computer-aided design package was developed for use on the University's Amdahl 470 V/6 computer [5 - 7].

The main objection to these approaches, is the one which is held in common with all such apriori design

techniques -- the method depends on the relative accuracy of the data on which the design is based; data which can require extensive effort and cost to obtain.[1]

A better philosophical approach would be to require the scheme to "learn" on-line the requisite information and subsequently adjust itself in some predetermined "optimal" fashion.

The following sections are devoted to describing some alternative approaches for implementing the desired methodology using model reference techniques.

7.2 Model Reference Adaptive Decoupling

The model reference approaches of Chapter Five appear to offer an interesting and operable procedure, in conjunction with the frequency domain techniques, for the implementation of the philosophy described immediately above.

There are essentially two schemes that could be

---

[1] An obvious case in point, here, is equipment with ill-defined control objectives, such as the indurator operated by Hamersley Iron Pty. Ltd. in Dampier (Australia).

envisaged, at this time, and they are depicted in Figure 7.1.

The first of the two proposed methods is similar in concept to the example presented in Chapter Five, in which a hyperstable identification scheme is used to provide estimates of the open-loop plant parameters at each instant. These can then be used in any desired manner to update the parameters of a controller/compensator. This method, although only strictly applicable to stable plants, is the more general of the two, as there is included an explicit identification that, as such, represents a rich information stream which may be tapped in a number of ways. In particular, a scheme may be suggested using the results of Chapter Five.

Suppose that:

$$\hat{\underline{\underline{G}}}_{OL}\ \underline{\underline{K}}_{pre} = \underline{\underline{G}}_{OL_d} \quad \ldots\ldots(1)$$

where:

$\hat{\underline{\underline{G}}}_{OL}(z)$ is the identified plant transfer matrix at any instant.

$\underline{\underline{K}}_{pre}(z)$ is the decoupling precompensator transfer matrix.

and,

$\underline{\underline{G}}_{OL_d}(z)$ is the desired open-loop transfer matrix at any time.

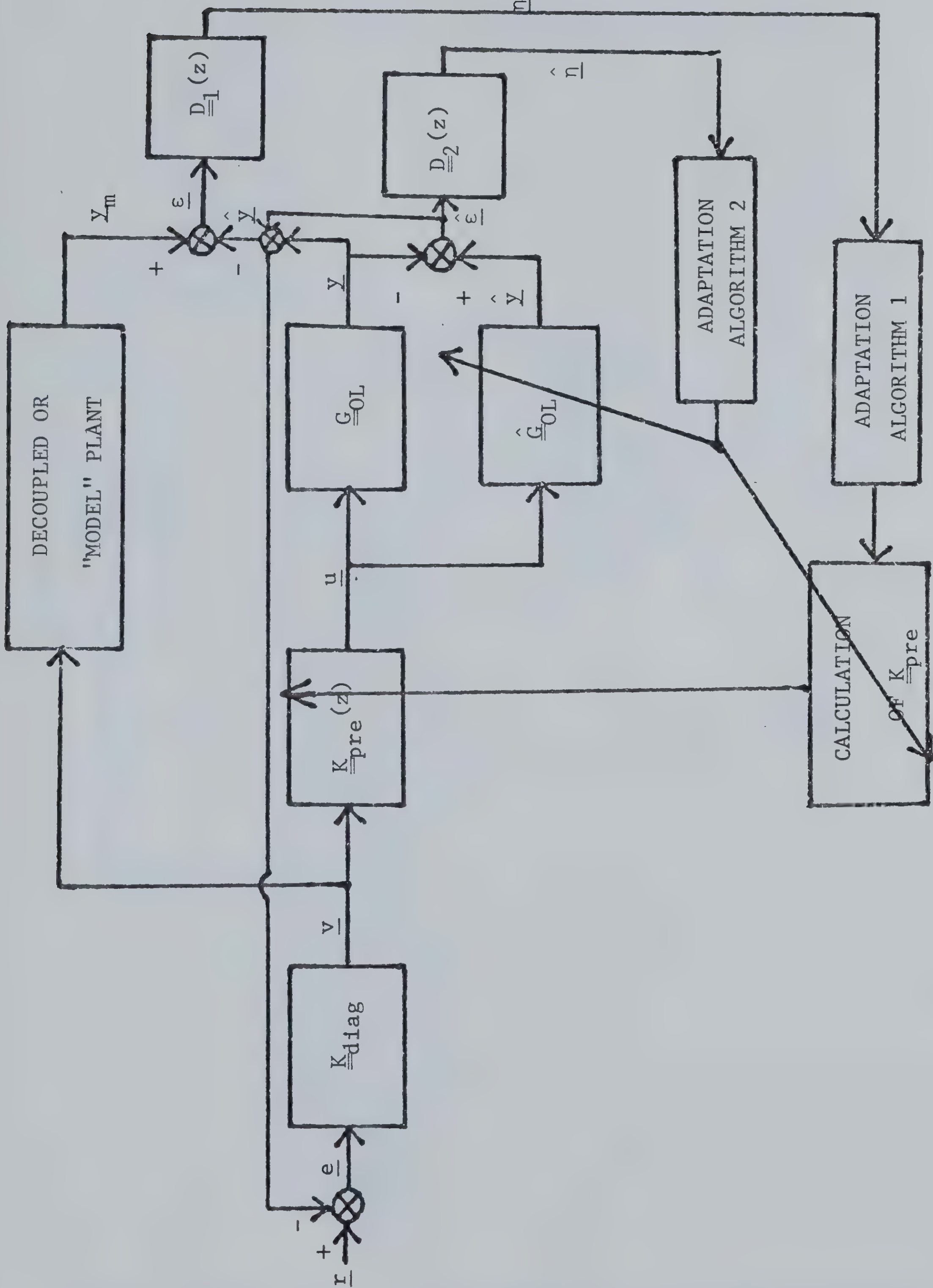


FIGURE 7.1(a) DYNAMIC ADJUSTMENT OF A PRECOMPENSATOR VIA THE AUGMENTED OUTPUT MODEL REFERENCE ADAPTIVE CONTROL METHOD

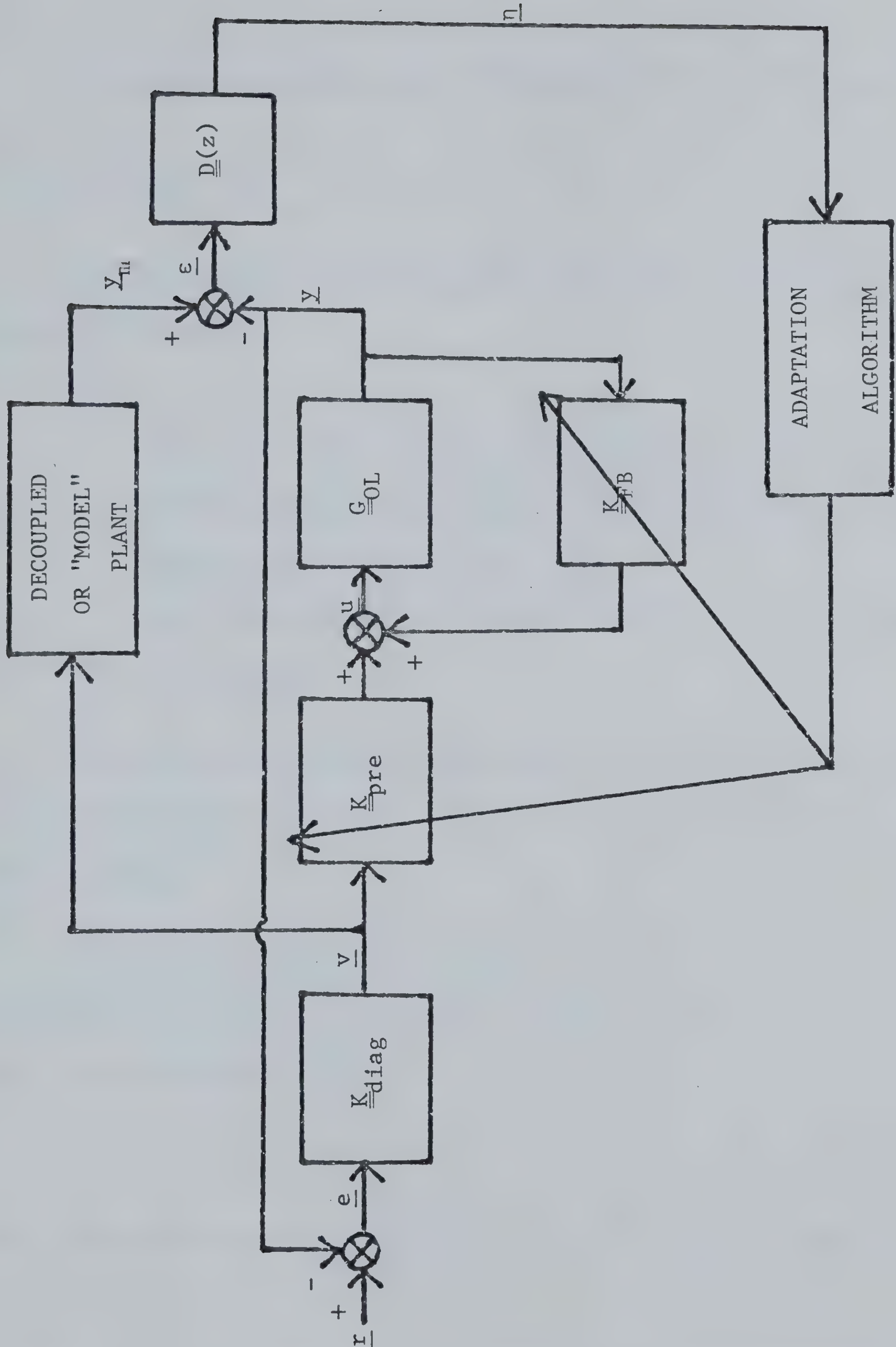


FIGURE 7.1(b) DYNAMIC ADJUSTMENT OF A PRECOMPENSATOR VIA THE DIRECT PARAMETER ADJUSTMENT MODEL REFERENCE ADAPTIVE TECHNIQUE

By simple rearrangement of equation (1), one has:

$$\underline{K}_{pre} = \underline{\underline{\hat{G}}}^{-1}_{OL} \; \underline{\underline{G}}_{OL_d} \quad \ldots\ldots (2)$$

The requirement that $\underline{\underline{\hat{G}}}_{OL}$ be invertible implies two further conditions:

(i) that there be no adaptive right-hand plane zeroes present and thus, the identification model should be minimum phase and,

(ii) the number of outputs must equal the number of inputs.

A related strategy to the above, involves the use of a signal-synthesis approach similar to that outlined in Chapter Four.

Suppose that $\underline{\underline{D}}_1(z)=0$ in Figure 7.1(a). Also let $\underline{y}_d$ be the desired output at the next instant and, $\underline{y}_m=\underline{y}_d$.
Then, from Figure 7.1(a):

$$\hat{\underline{y}} = \underline{\underline{\hat{G}}}_{OL} \; \underline{u} = \underline{y}_d \quad \ldots\ldots (3)$$

This equation may also be written in the time-domain as:

$$\underline{y}_d(k+1) = \sum_{i=1}^{h} \hat{\underline{A}}_i(k)\, \hat{\underline{y}}(k-i+1) + \sum_{j=1}^{f} \hat{\underline{B}}_j(k)\, \underline{u}(k-j+1) \quad \ldots\ldots (4)$$

where $\hat{\underline{y}}(k-i)$ and $\underline{u}(k-j)$ are n dimensional identification model output and control or input vectors, respecively. $\hat{\underline{\underline{A}}}_i$ and $\hat{\underline{\underline{B}}}_j$ are identification model parameter matrices of appropriate order.

Rearranging equation (4), one arrives at:

$$\underline{u}_1(k) = \hat{\underline{B}}_1^{-1}(k)\Big[\, \underline{y}_d(k+1) - \sum_{i=1}^{h} \hat{\underline{A}}_i(k)\, \hat{\underline{y}}(k-i+1) - \sum_{j=2}^{f} \hat{\underline{B}}_j(k)\, \underline{u}(k-j+1) \,\Big] \quad \ldots\ldots (5)$$

Since the control signal, to be applied to the plant via the primary fixed control loop, can be written as:

$$\underline{u}_2 = \underline{K}_{pre}(z)\, \underline{v} \quad \ldots\ldots (6)$$

then, utilizing equations (5) and (6), the supplementary control signal may be computed from:

$$\underline{u}^*(k) = \underline{u}_1(k) - \underline{u}_2(k) \quad \ldots\ldots (7)$$

The second technique (that depicted in Figure 7.1(b)) has been considered by Bethoux and Courtiol [8] for the general control situation.

It is supposed that the plant can be represented by an expression of the form:

$$\underline{x}(k) = \underline{\underline{A}}_p \, \underline{x}(k - 1) + \underline{\underline{B}}_p \, \underline{u}(k - 1)$$

and:

$$\underline{y}(k) = \underline{\underline{I}} \, \underline{x}(k) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

where $\underline{x}(k)$ and $\underline{y}(k)$ are n dimensional vectors

$\underline{u}(k-1)$ is an m dimensional input or control vector and,

$\underline{\underline{A}}_p$ and $\underline{\underline{B}}_p$ are process parameter matrices of appropriate order.

A control law can be defined as:

$$\underline{u}(k) = -\underline{\underline{K}}_{FB}(k) \, \underline{y}(k) + \underline{\underline{K}}_{pre}(k) \, \underline{v}(k) \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

$\underline{\underline{K}}_{FB}$ (k) and $\underline{\underline{K}}_{FF}$ (k) are time-varying feedback and precompensator matrices, respectively.

Substituting equation (9) into equation (8), one has:

$$\underline{y}(k) = [\, \underline{\underline{A}}_p - \underline{\underline{B}}_p \, \underline{\underline{K}}_{FB}(k) \,] \, \underline{y}(k - 1) + \underline{\underline{B}}_p \, \underline{\underline{K}}_{pre}(k) \, \underline{v}(k - 1)$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

A decoupled model reference plant is now specified as:

$$\underline{y}_m(k) = \underline{\underline{A}}_m \, \underline{y}_m(k-1) + \underline{\underline{B}}_m \, \underline{v}(k-1) \qquad (11)$$

where $\underline{y}_m(k)$ is an nx1 model output vector.

$\underline{v}(k-1)$ is an nx1 model input vector and,

$\underline{\underline{A}}_m$ and $\underline{\underline{B}}_m$ are diagonal model parameter matrices of appropriate order.

Defining an error vector, $\underline{\varepsilon}(k)$, as:

$$\underline{\varepsilon}(k) = \underline{y}_m(k) - \underline{y}(k) \qquad (12)$$

and:

$$\underline{n}(k) = \sum_{i=o}^{p_1} \underline{\underline{D}}_i \, \underline{\varepsilon}(k-i)$$

$$\underline{\underline{D}}_o = \underline{\underline{I}} \qquad (13)$$

where equation (13) defines the input-output relationship of a linear compensator, $\underline{\underline{D}}(z)$, one can derive, using equations (8) - (12):

$$\underline{\varepsilon}(k) = \underline{\underline{A}}_m \, \underline{\varepsilon}(k-1) + (\underline{\underline{A}}_m - \underline{\underline{A}}_p + \underline{\underline{B}}_p \, \underline{\underline{K}}_{FB}(k)) \, \underline{y}(k-1) +$$

$$(\underline{\underline{B}}_m - \underline{\underline{B}}_p \, \underline{\underline{K}}_{pre}(k)) \, \underline{v}(k-1) \qquad (14)$$

Letting:

$$\underline{A}_m - \underline{A}_p = \underline{B}_p \underline{A}o$$

and:

$$\underline{B}_m = \underline{B}_p \underline{B}o \quad \text{.......................................}(15)$$

equation (14) becomes:

$$\underline{\varepsilon}(k) = \underline{A}_m \underline{\varepsilon}(k - 1) + \underline{B}_p[(\underline{A}o + \underline{K}_{FB}(k)) \underline{y}(k - 1) +$$

$$(\underline{B}o - \underline{K}_{pre}(k)) \underline{v}(k - 1)] \quad \text{..........................}(16)$$

Now let:

$$\underline{w}(k) = (\underline{A}o + \underline{K}_{FB}(k)) \underline{y}(k - 1) + (\underline{B}o - \underline{K}_{pre}(k)) \underline{v}(k - 1)$$

$$\text{..}(17)$$

and:

$$\underline{w}_1(k) = -\underline{w}(k) \quad \text{..}(18)$$

Thus:

$$\underline{\varepsilon}(k) = \underline{A}_m \underline{\varepsilon}(k - 1) + \underline{B}_p \underline{w}(k) \quad \text{.........................}(19)$$

Now, we further restrict the system description space to those trajectories upon which the vector pairs ($\underline{\eta}$, $\underline{w}_1$) satisfy the following inequality condition:

$$\mu(k_0,k_1) = \sum_{k=k_0}^{k_1} \underline{n}^T(k)\, \underline{w}_1(k) \geq -\lambda_0^2$$

$$\forall k_1 \geq k_0 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots (20)$$

where λ_o is a finite constant, only dependent on the initial state of the system.

Equations (16) - (19) and inequality (20) define a system which can be depicted as in Figure 7.2.

It is obvious, therefore, from the previous development, that the discrete hyperstability theorem [9] may be invoked once more to obtain adaptation laws for the $\underline{\underline{K}}_{FB}$ and $\underline{\underline{K}}_{pre}$ matrices. This leads to the following theorem.

<u>Theorem 7.1</u>

Sufficient conditions for the adaptive decoupling system described by equations (8) - (9), (11) - (13), (17) - (18) and inequality (20) to be asymptotic hyperstable are:

(i) The transfer matrix, $\underline{\underline{G}}(z)$, defined by:

$$\underline{\underline{G}}(z) = \underline{\underline{D}}(z)\,(\underline{\underline{I}} - \underline{\underline{A}}_m\, z^{-1})^{-1}\, \underline{\underline{B}}_p$$

must be positive real discrete;

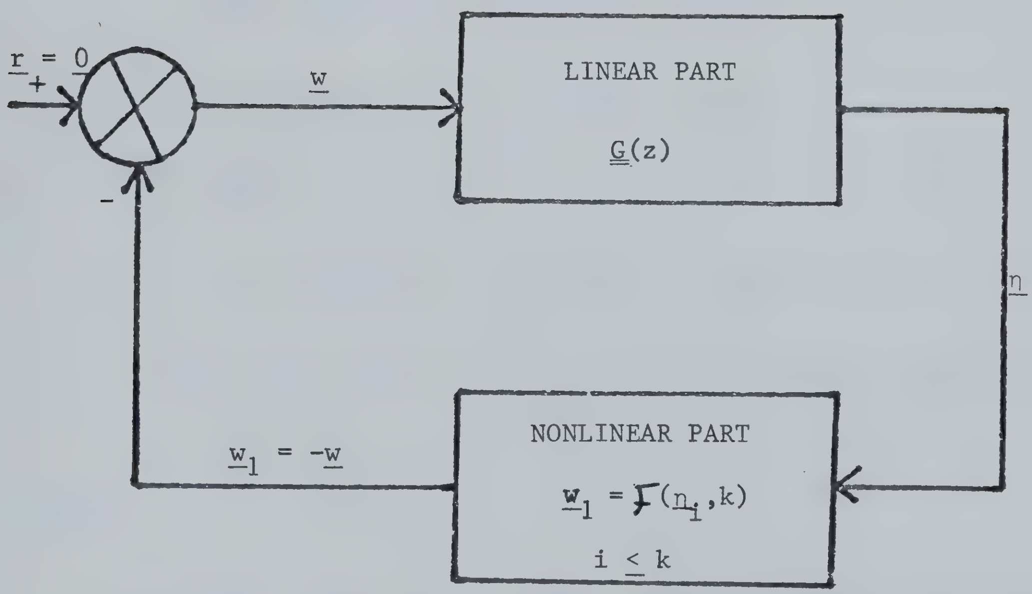


FIGURE 7.2 EQUIVALENT AUTONOMOUS NONLINEAR FEEDBACK SYSTEM

(ii)

$$(\underline{A}_0 + \underline{K}_{FB}(k))\, \underline{y}(k-1)$$

$$(\underline{B}_0 - \underline{K}_{pre}(k))\, \underline{v}(k-1)$$

and $\underline{n}(k)$ must all be of the same dimension and,

(iii) the adaptation laws for $\underline{\underline{K}}_{FB}(k)$ and $\underline{\underline{K}}_{pre}(k)$ must admit the following nonlinear functions[1]

$$\underline{\Phi}(k) = - \sum_{l=k_0}^{k} \underline{n}(l) \left[\underline{R}^I \underline{y}(l-1) \right]^T$$

and:

$$\underline{\Delta}(k) = \sum_{l=k_0}^{k} \underline{n}(l) \left[\underline{Q}^I \underline{v}(l-1) \right]^T$$

$\underline{\underline{R}}^I$ and $\underline{\underline{Q}}^I$ are positive definite or semi-definite matrices.

<u>Proof</u>

The first condition of the Theorem can be shown to be sufficient by using Theorem 5.1 and the equivalent system of Figure 7.2.

1

Only "integral adaptation" [8] has been considered here, as the proportional term may be considered as a simple extension.

From the discrete hyperstability theorem it is sufficient, for the system of Figure 7.2 to be asymptotic hyperstable, that:

(i) $\underline{\underline{G}}(z)$ must be positive real discrete and,

(ii) an inequality of the form of (20) must be satisfied.

From equation (19), one obtains:

$$\varepsilon(z) = (\underline{I} - \underline{A}_m z^{-1})\ \underline{B}_p\ \underline{w}(z) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(21)$$

or using equation (13):

$$\underline{n}(z) = \underline{D}(z)\ (\underline{I} - \underline{A}_m z^{-1})^{-1}\ \underline{B}_p\ \underline{w}(z) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(22)$$

So that:

$$\underline{G}(z) = \underline{D}(z)\ (\underline{I} - \underline{A}_m z^{-1})^{-1}\ \underline{B}_p$$

and condition (i) of the theorem is thus proved.

The second condition merely ensures that $\underline{w}(k)$ and hence, $\underline{w}_1(k)$, are defined and the product $\underline{n}^T(k)\underline{w}_1(k)$ is sensible.

It thus remains to show that the adaptation laws, characterized by condition (iii) of the theorem, ensures an

inequality of the type (20) is verified.

Utilizing equations (17) and (18), (20) becomes:

$$\sum_{k=k_0}^{k_1} \underline{n}^T(k) \left[-(\underline{A}_0 + \underline{K}_{FB}(k))\, \underline{v}(k-1) - (\underline{B}_0 - \underline{K}_{pre}(k))\, \underline{v}(k-1) \right] \geq -\lambda\hat{\delta}$$

$$\forall k_1 \geq k_0 \quad \ldots\ldots(23)$$

Introducing the adaptation laws of the theorem:

$$\sum_{k=k_0}^{k_1} \underline{n}^T(k) \left[\left(\sum_{l=k_0}^{k} \underline{n}(l) \left[\underline{R}^I \underline{v}(l-1) \right]^T - \underline{A}_0 \right) \underline{v}(k-1) + \left(\sum_{l=k_0}^{k} \underline{n}(l) \left[\underline{Q}^I \underline{v}(l-1) \right]^T - \underline{B}_0 \right) \underline{v}(k-1) \right] \geq -\lambda\hat{\delta}$$

$$\forall k_1 \geq k_0 \quad \ldots\ldots(24)$$

Sufficient conditions for inequality (24) to be satisfied are:

$$\sum_{k=k_0}^{k_1} \underline{n}^T(k) \left[\left(\sum_{l=k_0}^{k} \underline{n}(l) \left[\underline{R}^I \underline{v}(l-1) \right]^T - \underline{A}_0 \right] \underline{v}(k-1) \geq -\lambda\hat{\gamma}$$

$$\sum_{k=k_0}^{k_1} \underline{n}^T(k) \left[\left(\sum_{l=k_0}^{k} \underline{n}(l) \left[\underline{\Omega}^I \underline{v}(l-1) \right]^T - \underline{B}_0 \right] \underline{v}(k-1)$$

$$\geq -\lambda \xi^2$$

$$\forall k_1 \geq k_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

These conditions can be verified by noting the property [8,9]:

$$\sum_{k=k_0}^{k_1} \underline{E}_k^T \left[\sum_{i=k_0}^{k} \underline{E}_i + \underline{E}_0 \right] \geq -1/2\ \underline{E}_0^T \underline{E}_0 \quad \ldots\ldots\ldots\ldots\ldots (26)$$

This proves Theorem 7.1.

Remarks

At each instant, one needs to calculate $\underline{\underline{K}}_{FB}(k+1)$ and $\underline{\underline{K}}_{pre}(k+1)$. Unfortunately, these are functions of the, as yet, unknown $\underline{n}(k+1)$. From equations (12) and (13):

$$\underline{n}(k+1) = \underline{y}_m(k+1) - \underline{y}(k+1) + \sum_{i=1}^{p_1} \underline{D}_i\ \underline{\varepsilon}(k-i+1) \quad (27)$$

which can be written as:

$$\underline{n}(k+1) = [\ \underline{A}_m\ \underline{y}_m(k) + \underline{B}_m\ \underline{v}(k)\ ] - [\ \underline{A}_p\ \underline{y}(k) -$$

$$\underline{B}_p\ \underline{K}_{FB}(k+1)\ \underline{y}(k)\ ] - \underline{B}_p\ \underline{K}_{pre}(k+1)\ \underline{v}(k) + \sum_{i=1}^{p_1} \underline{D}_i$$

$$\underline{\varepsilon}(k-i+1) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

using equations (10) and (11).

Introducing the adaptation laws in equation (28), one has:

$$\underline{n}(k+1) = [\,\underline{A}_m\,\underline{y}_m(k) + \underline{B}_m\,\underline{v}(k)\,] - [\,\underline{A}_p - \underline{B}_p(\,-\sum_{l=k_0}^{k}\underline{n}(l)\,[\,\underline{R}^I\,\underline{y}(l-1)\,]^T)\,]\,\underline{y}(k) - \underline{B}_p(\,\sum_{l=k_0}^{k}\underline{n}(l)\,[\,\underline{Q}^I\,\underline{v}(l-1)\,]^T)\,\underline{v}(k) + \sum_{i=1}^{p_1}\underline{D}_i\,\underline{\varepsilon}(k-i+1) \quad \text{.........................(29)}$$

or:

$$\underline{n}(k+1) = [\,\underline{A}_m\,\underline{y}_m(k) + \underline{B}_m\,\underline{v}(k)\,] - [\,\underline{A}_p - \underline{B}_p(-\underline{n}(k+1)\,[\,\underline{R}^I\,\underline{y}(k)\,]^T + \underline{K}_{FB}(k))\,]\,\underline{y}(k) - \underline{B}_p\,[\,\underline{n}(k+1)\,[\,\underline{Q}^I\,\underline{v}(k)\,]^T + \underline{K}_{pre}(k)\,]\,\underline{v}(k) + \sum_{i=1}^{p_1}\underline{D}_i\,\underline{\varepsilon}(k-i+1) \quad \text{.................(30)}$$

or, finally:

$$\underline{n}(k+1) = [\,\underline{A}_m\,\underline{y}_m(k) + \underline{B}_m(k)\,\underline{v}(k)\,] - [\,\underline{A}_p - \underline{B}_p\,\underline{K}_{FB}(k)\,]\,\underline{y}(k) - \underline{B}_p\,\underline{K}_{pre}(k)\,\underline{v}(k) - \underline{B}_p(\underline{n}(k+1)\,[\,\underline{R}^I\,\underline{y}(k)\,]^T\,\underline{y}(k) + \underline{n}(k+1)\,[\,\underline{Q}^I\,\underline{v}(k)\,]^T\,\underline{v}(k)\,) + \sum_{i=1}^{p_1}\underline{D}_i\,\underline{\varepsilon}(k-i+1) \quad \text{...(31)}$$

Rearranging equation (31), we obtain:

$$\underline{n}(k+1) = (\underline{I} + \underline{B}_p([\underline{R}^I \underline{v}(k)]^T \underline{v}(k) + [\underline{Q}^I \underline{v}(k)]^T$$

$$\underline{v}(k))^{-1} (\underline{A}_m \underline{y}_m(k) + \underline{B}_m \underline{v}(k) - [\underline{A}_p - \underline{B}_p \underline{K}_{FB}(k)] \underline{y}(k) +$$

$$\underline{B}_p \underline{K}_{pre}(k) \underline{v}(k)) \text{.....................................} (32)$$

A major difficulty arises for the practical implementation of equation (32), ie. a complete knowledge of the process parameter matrices is strictly required.

In order to minimize these difficulties, two assumptions are made [8]:

(i) the estimated performance error vector:

$$\underline{\varepsilon}(k+1) = \underline{y}_m(k+1) - \underline{y}(k+1) \text{......................} (33)$$

can be approximated by $\underline{\varepsilon}^o(k)$, where:

$$\underline{y}(k+1) = [\underline{A}_p - \underline{B}_p \underline{K}_{FB}(k)] \underline{y}(k) + \underline{B}_p \underline{K}_{pre}(k) \underline{v}(k) \text{.} (34)$$

(ii) the setpoint changes, $\underline{r}(k)$, must be known one sampling instant ahead.

The second assumption merely ensures that $\underline{v}(k)$ can be always made available. Thus, for practical applications:

$$\underline{\eta}(k+1) = (\underline{\underline{I}} + \underline{\underline{B}}_q([\underline{\underline{R}}^I \underline{v}(k)]^T \underline{v}(k) + [\underline{\underline{Q}}^I \underline{v}(k)]^T \underline{v}(k)]^{-1}$$

$$(\underline{\varepsilon}(k) + \sum_{i=1}^{p_1} \underline{\underline{D}}_i \, \underline{\varepsilon}(k-i+1)) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35)$$

provided $(\underline{\underline{I}} + \underline{\underline{B}}_q(\underline{\underline{R}}^I \underline{v}(k))^T \underline{v}(k) + (\underline{\underline{Q}}^I \underline{v}(k))^T \underline{v}(k)))$ is always invertible where $\underline{\underline{B}}_q$ is chosen such that it falls within the range of the variation of $\underline{\underline{B}}_p$.

7.3 Conclusions

Much of the original model reference adaptive control work has been directed to forcing a system to follow any desired trajectory. Whilst this certainly is a worthwhile objective, in many cases it simply is not feasible to specify an arbitrary reference model. Because of the enormous investment in apriori design techniques and the number of methods available, it would appear that a more sensible approach would be to use the adaptive mechanism to correct the designed controllers under changing process conditions, etc.

This procedure supplies two desirable aspects:

(i) the configuration yields a two-level control

technique which is essentially able to "learn" how to cope with its environment "on-line", but with the added integrity of a primary control loop and, (ii) all the process knowledge which goes into the design of the primary control system becomes available for the adaptive loop design.

This chapter has introduced methods for dynamic precompensation of a plant when sufficient knowledge is available to design controllers based on multivariable frequency domain techniques. The methods have been based on hyperstable adaptive systems, developed previously, purely for control purposes.

It is manifest that this concept is infinitely extendable, in theory, since the reference model is arbitrarily specified and could include some sort of directed logic supervisory mode [10 - 11].

CHAPTER EIGHT

Conclusions

8.1 Introduction

The prime motivation for this work was the desire to classify the field of model reference adaptive control. Although there are several excellent surveys available [1 - 7], these have been concerned primarily with indicating the various aspects of the design work, rather than the similarities inherent in the schemes presented.

Further, there was felt to be a need to conceptually re-examine the results of the last decade to gain some insight into which direction it would be most profitable to venture in the future.

Of the two model reference design methods based on stability techniques -- the Liapunov technique [8] and schemes based on Popov's hyperstability theory [9 - 10], the latter appears to be the more promising [1,10 - 11]. The schemes investigated from Chapter Three on, have restricted themselves to this basis.

In the last few years there has been a general re-evaluation of the state-space control theory, particularly in response to several practical objections

that have been raised [12 - 15]. This has resulted in a more careful examination of input-output type relations and their usefulness in design. Not only have transfer function techniques been extended to incorporate multivariable systems [16 - 19], but the state-space theory, itself, is being developed to consider more techniques based on output (or incomplete state) feedback [20 - 25].

Conceptually the requirement at the present time appears to be the following; based on input-output measurements of an unknown plant, design an adaptive system which will guarantee the control objectives of the entire controlled configuration. Since the fixed parameter techniques are essentially special cases of the adaptive control superset, it is not surprising that work in this direction is being considered by adaptive system designers.

Martin-Sanchez [26 - 27] appears to have been the first to consider a specific design technique to satisfy the above requirement, although the claimed general applicability is questionable [28].

This work has considered a very general theoretical presentation of the adaptive problem, with specific reference to the design of a technique similar to the one

proposed by Martin-Sanchez, although starting from the theory presented by Landau [10 - 11]. The validity of the method has further been tested by a simulation study and several suggestions as to practical calculational procedures have been included. Moreover, since this scheme is essentially dependent on the estimation system convergence, the results of Ljung [29] have been mentioned.

It will be noted that regulation per se is not of principle concern here, as it is a subset of the output driving problem. There are some cases, however, in which regulation can be a prime goal. In such cases the adaptation mechanism can be more profitably employed in updating conventional controllers. This may take place intermittently, avoiding the problem of large computational overheads. Of particular interest is the so-called dynamic precompensation approach, in which an adaptation scheme is responsible for updating a precompensator, which in turn maintains a particular system feature (eg. diagonal dominance). The primary control task is meanwhile achieved by a fixed parameter controller.

The following sections are concerned with summarizing the cognitive categorizations of the model reference adaptive approaches mentioned in the previous chapters. A

brief note towards the end of the chapter will examine possible modifications of existing state-space methods to include incomplete state feedback without the use of observers.

8.2 Model Reference Adaptive Control System Definition

Conceptually all model reference adaptive control techniques belong to one of two classes. The first of these includes a mechanism whereby the parameters of the controller, itself, are adjusted directly. The controller is contained within a conventional control configuration and control action is exercised through this primary loop. Whilst this approach might seem very attractive, there appears to be several limitations for general implementation purposes. For example, these schemes are restricted to formulations of relative degree one with some known apriori information about the variation of the plant parameters assumed [30,41-43].

The second cognitive approach relies on a recursive estimation scheme to supply an updated estimate of the open-loop plant parameters. The parameter set may then be used to compute the control action required, in any convenient manner. This method may be termed an indirect technique as achievement of the control objective is

dependent on the output convergence of the estimation scheme.

These two classes of adaptive control system are depicted in Figures 8.1 and 8.2, respectively. It should be noted that in these diagrams, the reference model per se has given way to what has been termed a driver block [26]. This approach has been taken so that it is realized that the reference model need not be an accurate representation of the plant. The main criterion for a good driver block design is that it properly designates the desired output of the system.

8.3 Practical Adaptive Control System Definitions

This thesis has considered the theoretical development of practical schemes based on both the classes of control system depicted in Figures 8.1 and 8.2. Figures 8.3 through 8.6 schematically represent four general configurations of adaptive system.

This particular demarkation summarizes the schemes that have been suggested to date. None are perfectly general, although all may be considered as a subset of one of the designs presented in Figures 8.3 to 8.6.

There appears to have been a strict division in the

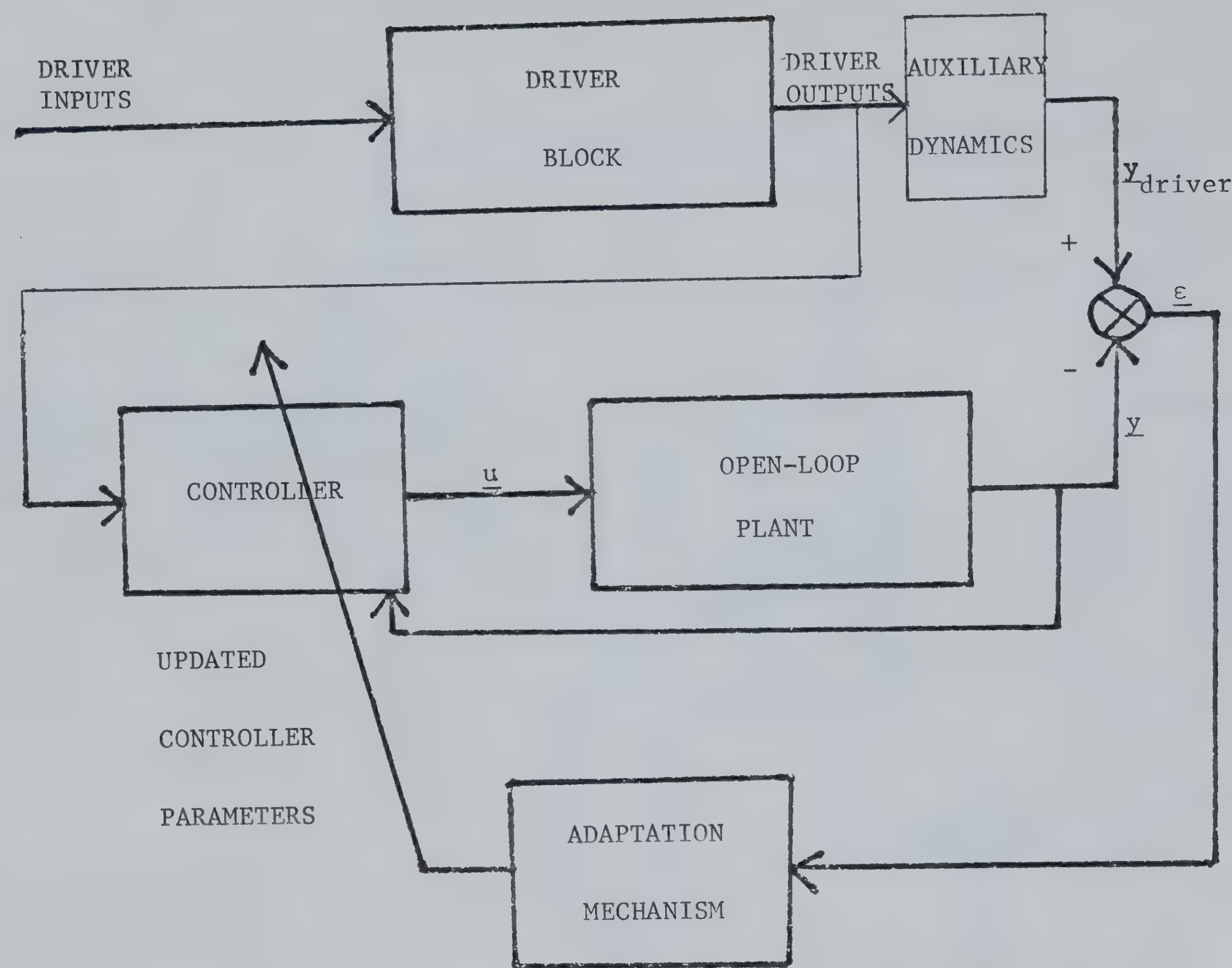


FIGURE 8.1 "DIRECT" MODEL REFERENCE ADAPTIVE CONTROL SYSTEM CONFIGURATION

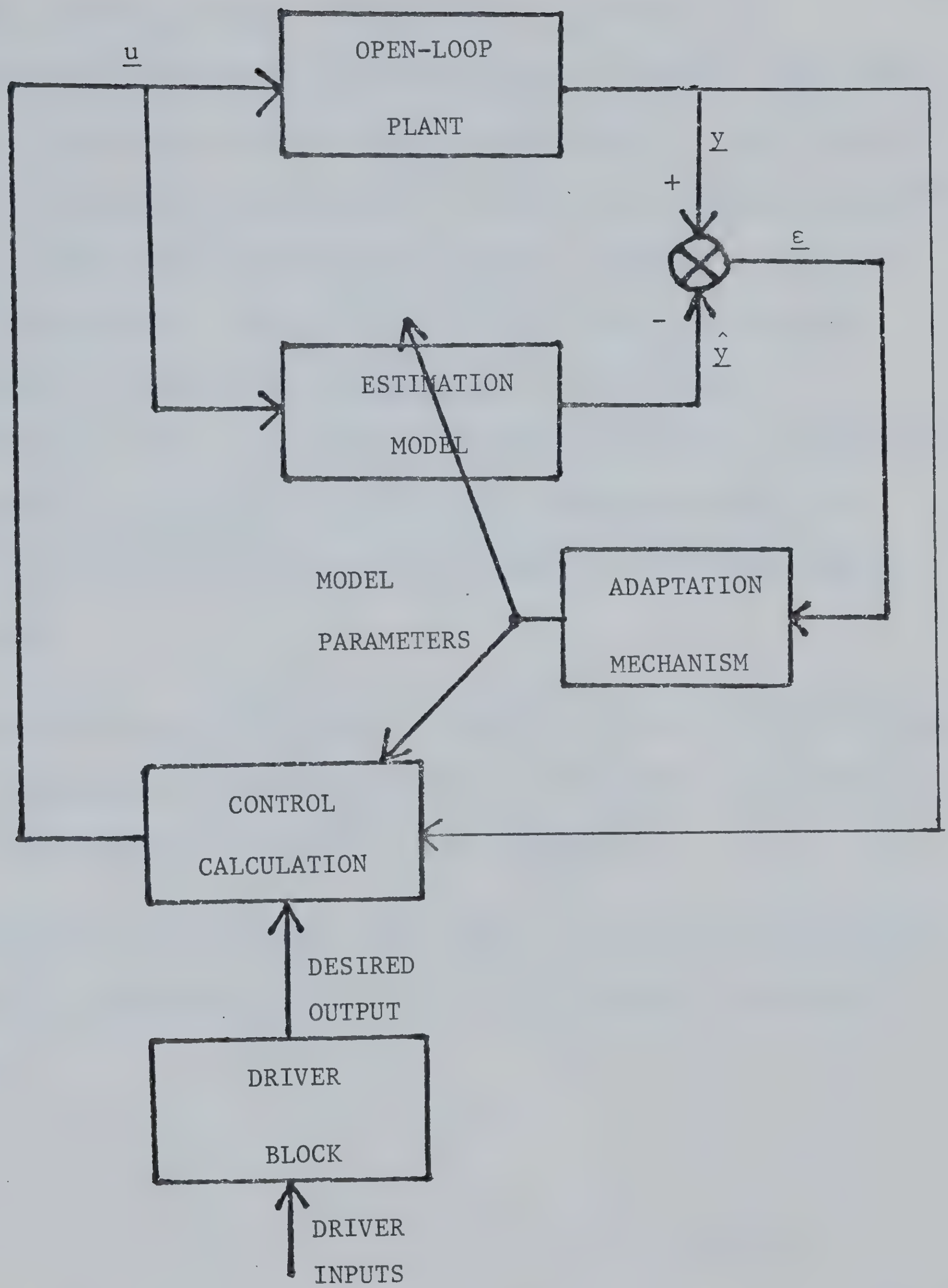


FIGURE 8.2 "INDIRECT" MODEL REFERENCE ADAPTIVE CONTROL SYSTEM CONFIGURATION

literature between those schemes which utilize an input-output formulation as their starting point, and those which are based on a state-space methodology. For instance, the systems depicted in Figures 8.3 and 8.4, usually require an input-output description to be used, whereas those of Figures 8.5 and 8.6 are strictly state-space techniques. The decision is based on mathematical tractability considerations, for at present, the only general method for calculating the control signal using the approaches of Figures 8.3 and 8.4, relies on an adaptive inverse [26,31] computation.

The state-space methods are not so restricted, although if true incomplete state feedback is considered an on-line minimal realization routine may be required[1].

The next section will summarize the design freedom and theoretical restrictions of each of the general control schemes.

1

See Section 8.5.

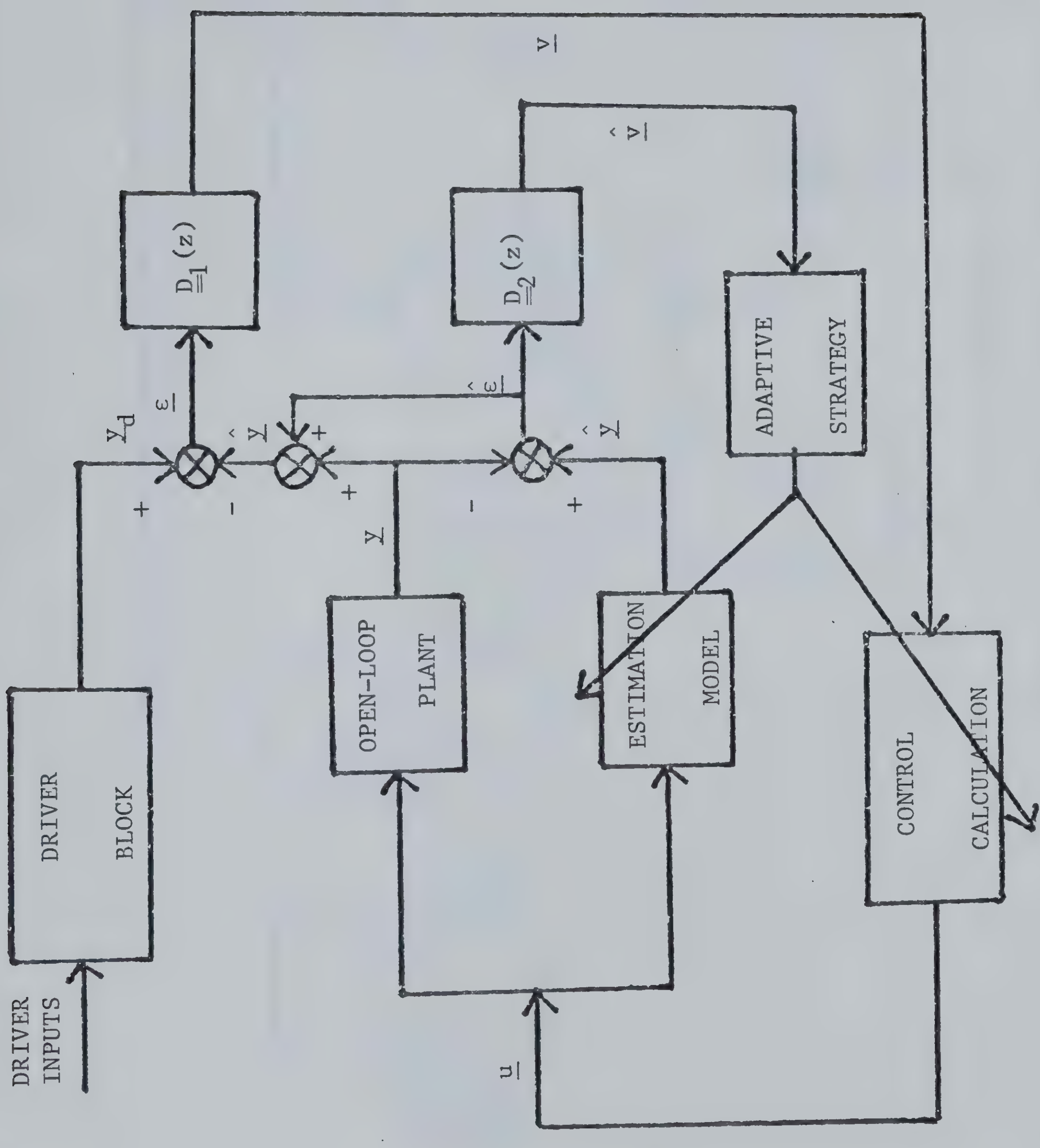


FIGURE 8.3 A GENERAL MODEL REFERENCE ADAPTIVE CONTROL SYSTEM BASED ON PRIOR IDENTIFICATION

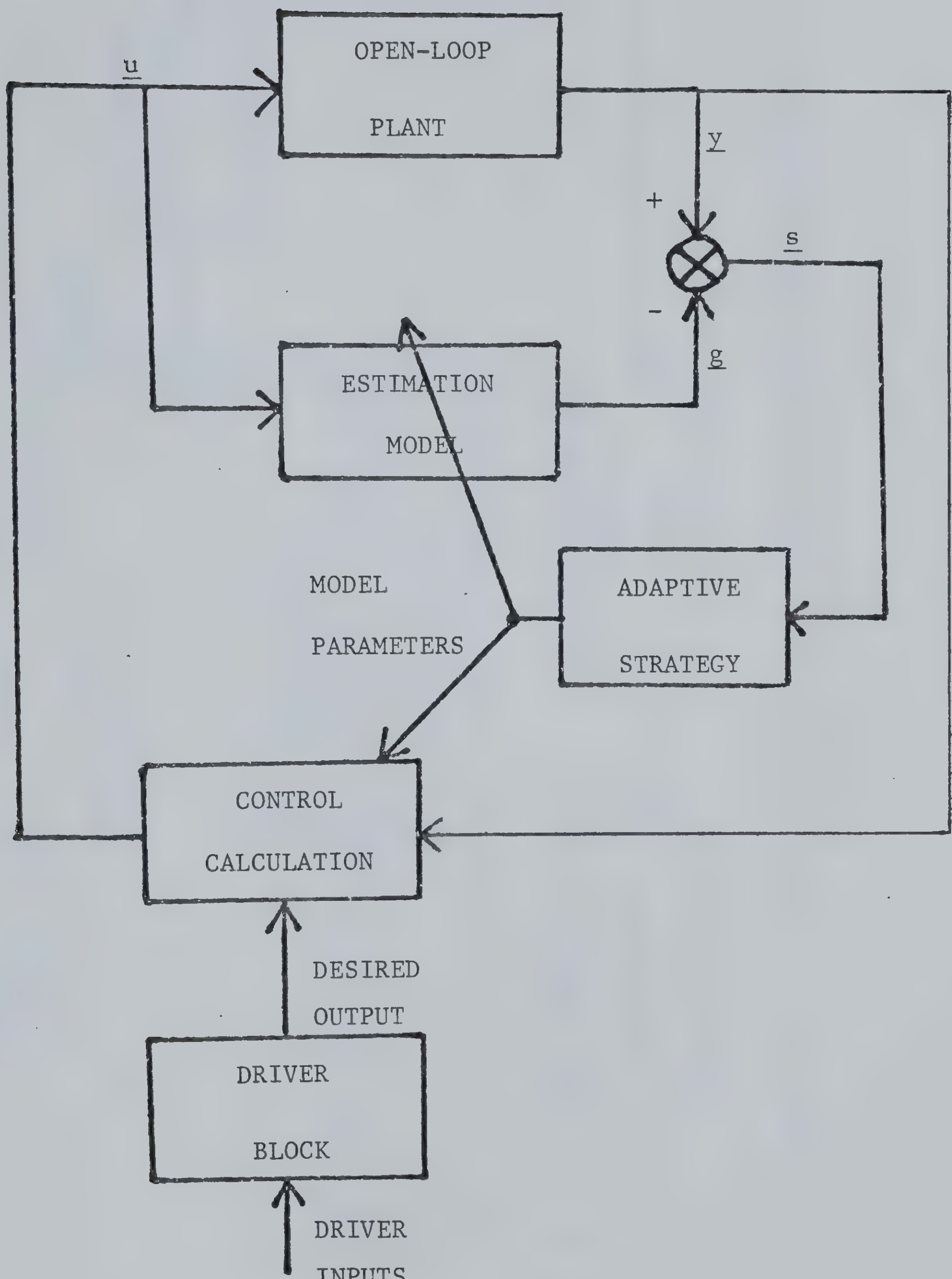


FIGURE 8.4 MARTIN-SANCHEZ'S ADAPTIVE-PREDICTIVE CONTROL SYSTEM [26]

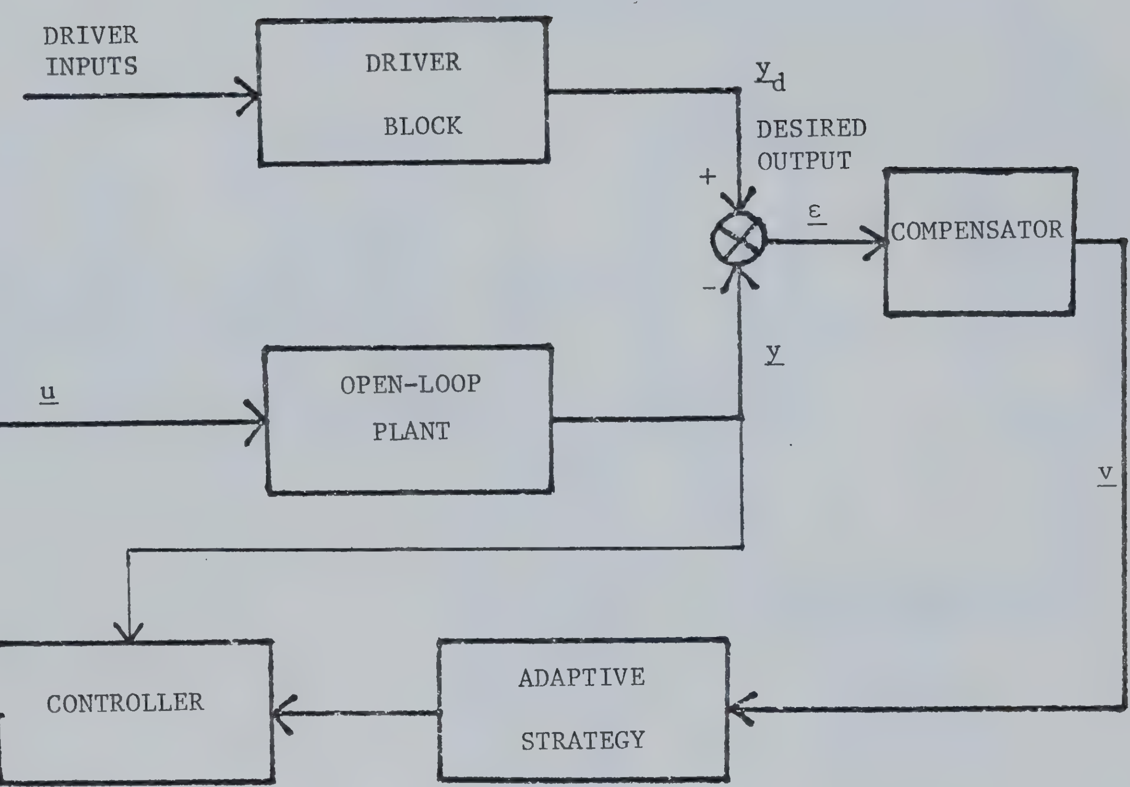


FIGURE 8.5 STATE-SPACE DIRECT CONTROLLER PARAMETER ADAPTIVE CONTROL TECHNIQUE [30]

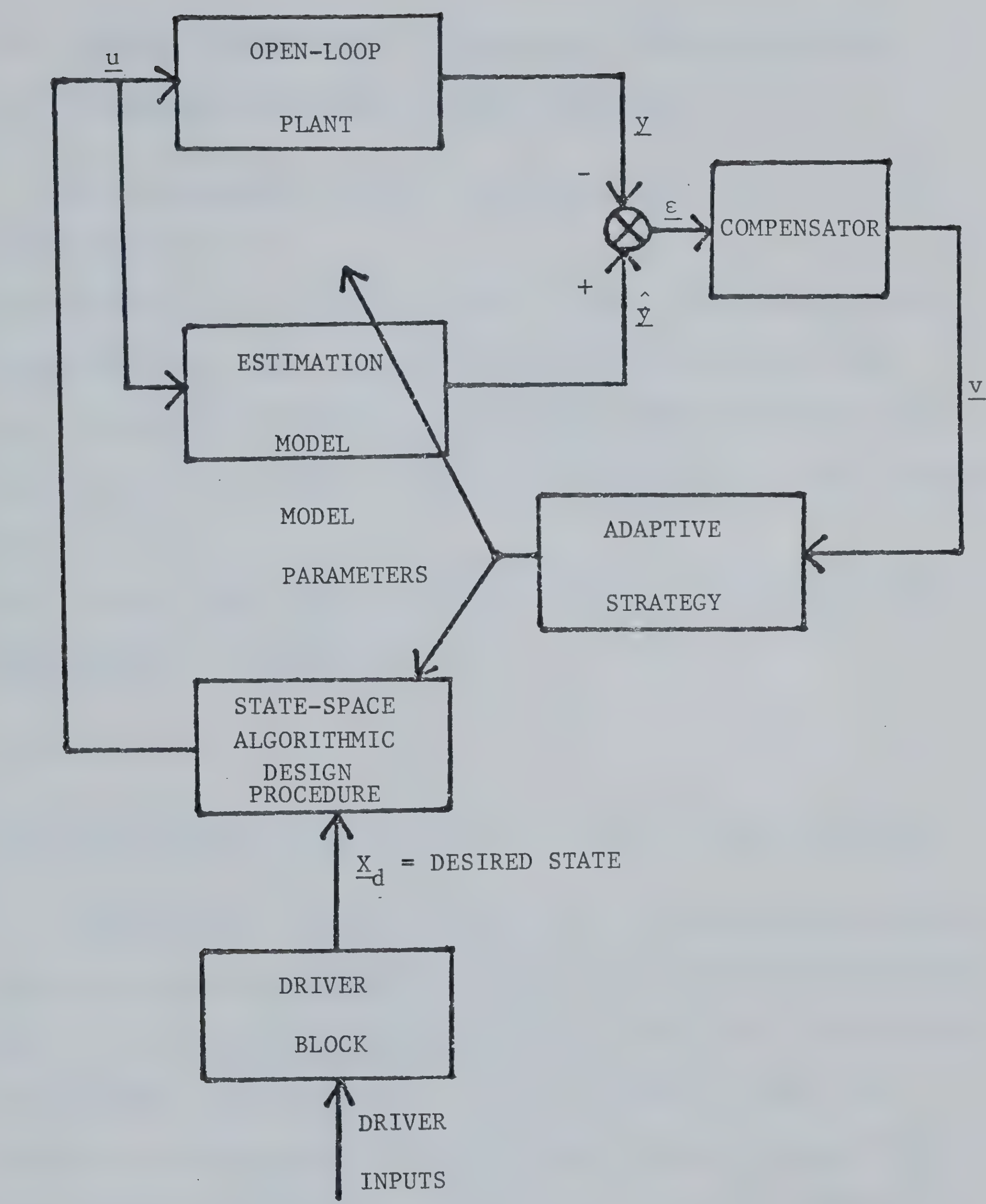


FIGURE 8.6 GENERAL STATE-SPACE ADAPTIVE OUTPUT FEEDBACK SYSTEM WITH PRIOR IDENTIFICATION

8.4 Design Freedom and Theoretical Restrictions of Practical Adaptive Control Systems

The discussion may be organized intc three distinct subdivisions:

(i) Input-Output Methods Based on Prior Identification -- represented by the techniques of Figures 8.3 and 8.4,

(ii) State-Space Methods with Direct Adaptation of the Controller Parameters. Figure 8.5 represents these methods, and,

(iii) State-Space Methods Based on Prior Identification, as represented by Figure 8.6.

(i) Input-Output Methods Based on Prior Identification

Figures 8.3 and 8.4 conceptually describe the most promising adaptive control schemes that can be envisioned, since they are by definition based solely on input-output measurements of the plant, asymptotically stable and absolutely convergent to the desired system state. Due to mathematical tractability conditions, however, there are several practical restrictions which must be accepted.

a) Compensator Design

In the particular example of the design presented in Chapter Five, a compensator was introduced which ensured the positive realness of a linear transfer matrix[1] [11,33].

It is unclear in what way the compensator order affects the performance of the control system, although obviously if unstable plant poles are known exactly, these may be cancelled by corresponding zeroes of the identification model compensator.

b) Identification Scheme

In both the methods shown in Figures 8.3 and 8.4, the convergence of the control system is based on the convergence of an estimation scheme. Thus, theoretically, any asymptotically convergent identification technique which can be operated recursively can be used. For instance, any of the following are obvious candidates for such a choice:

1. Hyperstable Recursive Identification [34],
2. Extended Least Squares [35],
3. Instrumental Variables [36],
4. Recursive Maximum Likelihood [29] and,
5. Extended Kalman Filter [37].

1

See Chapter Five -- Section 5.3.

In all of these there is a requirement that in the absence of any exact knowledge of the plant parameters, the plant must be open-loop stable [29].

Because of the manner in which the control action is computed, there is a certain freedom of adaptation in that, strictly, parameter convergence is not necessary; the only requirement being that the output error between the plant and the identification model go to zero. This alleviates the problem which faces all identification techniques when embedded in a closed-loop control configuration ie. the requisite "richness" of the input signals is seldom sufficient for accurate parameter identification in a closed-loop envircnment.

c) Control Computation

At present, there is only one method available for the calculation of the input signal based on desired output information and the identification model parameters -- the adaptive inverse approach. In both its forms:[1]

1. Controller Transfer Matrix Computation and,

1

See Chapter Five.

2. Direct Control Signal Calculation,

there is a distinct difficulty with identification models which contain right-half plane zeroes[1] [28]. This problem arises during the calculation of the control signal because the algorithm uses the inverse of the identification model. Hence the right-half plane zeroes become right-half plane poles and the consequent control action becomes unstable. The difficulty is more readily recognized when it is remembered that the identification scheme is only required to facilitate output convergence and the parameters may be adaptively placed anywhere in the parameter space. Martin-Sanchez has suggested that this problem does not present any problems since the driver block (reference model) can be chosen such that the situation never arises [38]. This is difficult to rationalize since the schemes do not place any restrictions on the parameter placement. A more general solution would entail the specification of a specific parameter space which is known to be minimum-phase for all parameter sets. The prime consideration then would be the question of output

---

[1] Here the w-plane is considered, as this representation is more familiar to practising engineers.

realizability ie. the ability of one system to follow another eventhough their parameters are not identical.

d) Driver Block or Reference Model Specification

Whilst this choice is to all intentional purposes completely arbitrary, it would normally be expected that the specification would realize a stable and physically meaningful desired output signal.

(ii) State-Space Methods with Direct Adaptation of the Controller Parameters (Figure 8.5)

These schemes are restricted to completely observable and controllable systems where all the states are accessible.

a) Compensator Design

The specification of a compensator ensures the positive realness of a certain linear transfer matrix [30]. In this case some of the plant parameters must be known, at least approximately, so an additional requirement is that the transfer matrix be positive real over the entire range of the plant parameter variation space. Karmarkar [39 - 40] has presented a scheme for designing compensators to ensure

this property.

There appear to be no restrictions on the open-loop plant configuration in these techniques, although the reference model would normally be assumed to be stable.

b) Control Calculation

The control action is calculated using an updated controller. At present the control law is restricted to the form:

$$\underline{u} = -\underline{\underline{K}}_{FB}\,\underline{x} + \underline{\underline{K}}_{SP}\,\underline{r} + \underline{\underline{K}}_{FF}\,\underline{\xi} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where $\underline{u}$ is an mx1 control input vector,

$\underline{x}$ and $\underline{r}$ are nx1 state and setpoint vectors, respectively,

$\underline{\xi}$ is a px1 measurable disturbance vector,

$\underline{\underline{K}}_{FB}$ is an mxn feedback control matrix,

$\underline{\underline{K}}_{SP}$ is an mxn setpoint control matrix, and

$\underline{\underline{K}}_{FF}$ is an mxp feedforward control matrix.

Note:

The original theory for this type of system was developed by Bethoux and Courtiol [30] for a purely state-space formulation. Recently it has been shown that extensions to include input-output formulations

are feasible [41 - 43].

Whilst for implementation purposes these techniques have a number of advantages, they are at present restricted to:

(i) plants with uni-signed $\underline{\underline{B}}_{ip}$ matrices (i.e. all the elements of a particular $\underline{\underline{B}}_{ip}$ matrix are of the same sign) [41 - 43] and,

(ii) input-output formulations of relative degree one [41 - 43].

The extension due to Johnstone [41] does not require that all the elements of the $\underline{\underline{B}}_{ip}$'s be of the same sign, although the compensator design would require some knowledge of the variation of these parameters.

(iii) State-Space Methods Based on Prior Identification (Figure 8.6)

This approach may almost be termed the "classical" approach to control law parameter adaptation as there are a large number of procedures which are intuitively similar [44 - 46].

Two methods may be envisaged using the hyperstable system approach:

1. hyperstable identification with all states accessible followed by controller calculation based on the identified model, or

2. hyperstable identification using only input-output data followed by a minimal realization routine [32] to obtain an equivalent state-space representation. The controller calculation would then be based on this information as with 1., above.

The result of both is the same, state-space representations of the identified plant. The control may then be calculated using any of the available state-space design techniques.[1]

a) Compensator Design

Here, as with the pure input-output methods, it is a general requirement that, in the absence of any knowledge of the plant parameters, the open-loop plant must

---

[1] A particular example is described in the next section.

be stable. The compensator is specified such that a particular linear transfer matrix is positive real.

b) Identification Scheme

The estimation scheme can be any of a number of methods (see above) provided it is asymptotically stable and absolutely output convergent. Once again there is no requirement here for exact parameter matching.

c) Control Calculation

The choice of the control calculation technique is influenced by the form of the reference model or driver block specification. There are two ways of defining a desired output:

1. as the output of a reference model (or driver block), or
2. through the description of the reference model itself (ie. eigenvalue placement, etc.).

Whilst the first category requires the least computational overheads, the second is inherently the more flexible as it includes the whole gambit of state-space design techniques from optimal methods through to such techniques as eigenvalue placement [47 - 50]. More recently, these have been extended to include output

feedback and output derivative feedback [20 - 25].

d) Driver Block or Reference Model Specification

Here, as has been mentioned above, it is possible to specify the driver block in a number of ways. The control system convergence is then dependent on the estimation scheme convergence plus the controller design.

8.5 A Particular Example of a State-Space Method With Prior Identification

This example is illustrative of class (iii) of the systems described in the previous section. The primary control loop is based on output feedback and the identification scheme used is the the input-output version of Landau's hyperstable recursive technique [34] described in Chapter Five. The reference model specification consists of an eigenvalue placement decision.

A pictorial representation of the system appears in Figure 8.7.

In a recent paper, Paraskevopoulos [51] has presented two techniques for solving the problem of pole assignment via proportional plus derivative output feedback.

The controller is specified as:

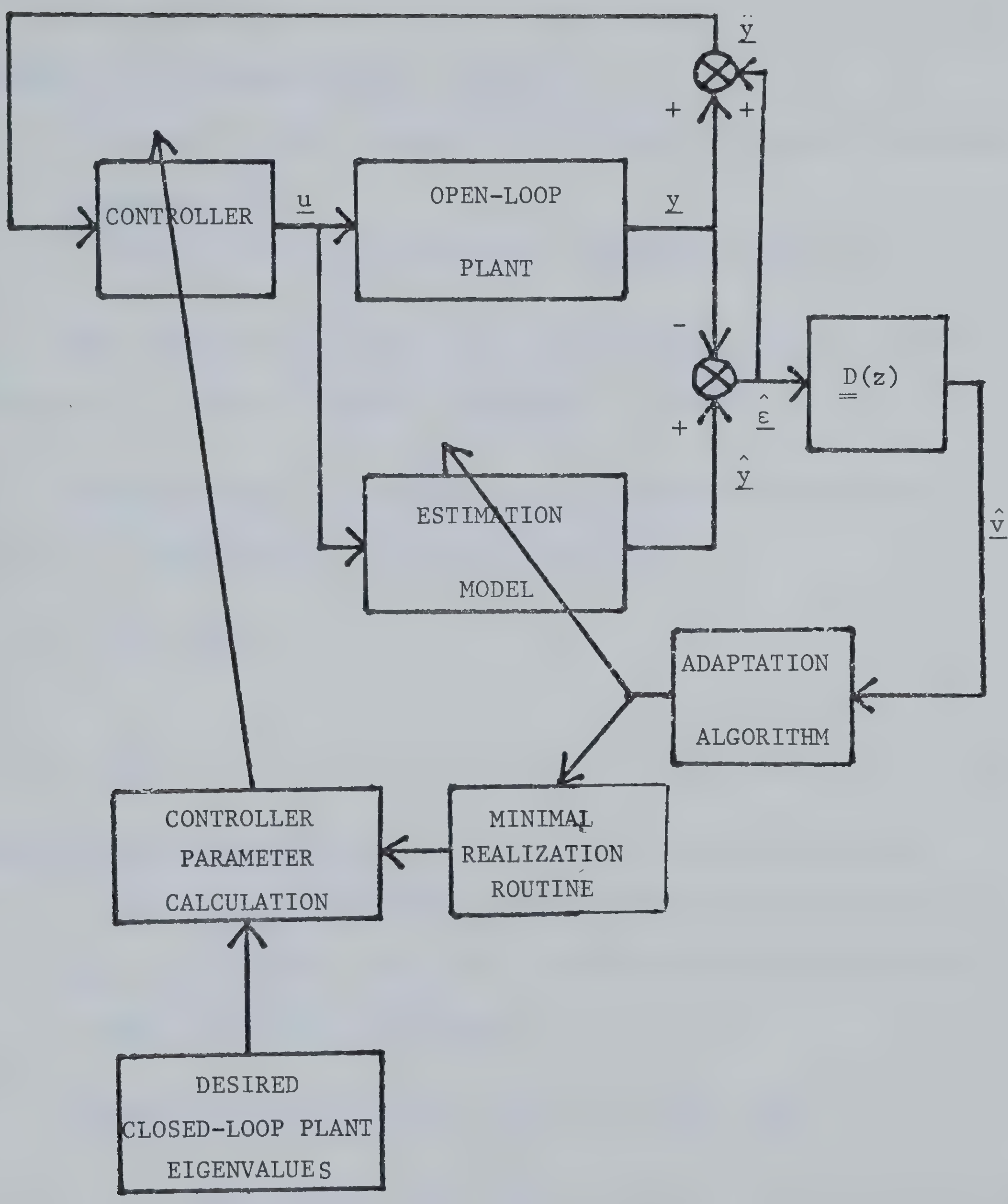


FIGURE 8.7 EXAMPLE OF A STATE-SPACE METHOD OF ADAPTIVE CONTROL BASED ON PRIOR IDENTIFICATION

$$\underline{u} = \underline{\underline{P}}\,\underline{y} + \underline{\underline{D}}\,\dot{\underline{y}} \quad \ldots\ldots(2)$$

where $\underline{u}$ is the mx1 control vector,

$\underline{y}$ and $\dot{\underline{y}}$ are nx1 output and derivative output vectors, respectively,

$\underline{\underline{P}}$ and $\underline{\underline{D}}$ are mxn controller parameter matrices.

The control algorithm for the adaptive configuration of Figure 8.7 can be briefly described as follows.

Suppose that the output of the minimal realization block of the Figure can be described as:

$$\dot{\hat{\underline{x}}} = \hat{\underline{\underline{A}}}\,\hat{\underline{x}} + \hat{\underline{\underline{B}}}\,\underline{u}$$

$$\hat{\underline{y}} = \hat{\underline{\underline{C}}}\,\hat{\underline{x}} \quad \ldots\ldots(3)$$

where $\hat{\underline{x}}$ denotes nx1 equivalent model "state" vectors,

$\hat{\underline{y}}$ is the rx1 identification model output vector and,

$\hat{\underline{\underline{A}}}$, $\hat{\underline{\underline{B}}}$ and $\hat{\underline{\underline{C}}}$ are equivalent state-space identification model parameter matrices.

Using equations (2) and (3), one may write:

$$\dot{\hat{\underline{x}}} = (\underline{\underline{I}} - \hat{\underline{\underline{B}}}\,\underline{\underline{D}}\,\hat{\underline{\underline{C}}})^{-1}\,(\hat{\underline{\underline{A}}} + \hat{\underline{\underline{B}}}\,\underline{\underline{P}}\,\hat{\underline{\underline{C}}})\,\hat{\underline{x}} = \underline{\underline{A}}_c\,\hat{\underline{x}} \quad \ldots\ldots(4)$$

provided $(\underline{\underline{I}} - \underline{\underline{\hat{B}D\hat{C}}})$ is invertible.

Let $\underline{\underline{H}}$ be an nxn matrix with exactly known eigenvalues ($\underline{\underline{H}}$ can be upper triangular or in phase canonical form). A sufficient condition for pole assignment then is to have:

$$\underline{\underline{A}}_c = \underline{\underline{H}} \quad \text{...} (5)$$

or $\det(s\underline{\underline{I}} - \underline{\underline{A}}_c) = \det(s\underline{\underline{I}} - \underline{\underline{H}})$.

From equations (4) and (5):

$$\hat{\underline{\underline{B}}}\ \underline{\underline{P}}\ \hat{\underline{\underline{C}}} + \hat{\underline{\underline{B}}}\ \underline{\underline{D}}\ \hat{\underline{\underline{C}}}\ \underline{\underline{H}} = \underline{\underline{H}} - \hat{\underline{\underline{A}}} \quad \text{..............................} (6)$$

If the following definitions are allowed:

$$\hat{\underline{\underline{B}}}\ \underline{\underline{K}}\ \underline{\underline{M}} = \underline{\underline{V}} \qquad \underline{\underline{K}} = [\ \underline{\underline{P}} \quad \underline{\underline{D}}\ ]$$

$$\underline{\underline{M}} = \begin{bmatrix} \hat{\underline{\underline{C}}} \\ \\ \hat{\underline{\underline{C}}}\ \underline{\underline{H}} \end{bmatrix} \qquad \underline{\underline{V}} = \underline{\underline{H}} - \hat{\underline{\underline{A}}}$$

then:

$$\underline{\underline{R}}\ \underline{\underline{K}} = \underline{\underline{V}} \quad \text{...} (7)$$

where $\underline{\underline{R}} = \underline{\underline{M}}^T * \underline{\underline{B}}$ and * represents the Kronecker product

$$\underline{\underline{M}}^T * \underline{\underline{B}} = \begin{bmatrix} b_{11}\,\underline{\underline{M}}^T & \cdots\cdots\cdots & b_{1m}\,\underline{\underline{M}}^T \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ b_{n1}\,\underline{\underline{M}}^T & \cdots\cdots\cdots & b_{nm}\,\underline{\underline{M}}^T \end{bmatrix}$$

$$\underline{\underline{K}} = \begin{bmatrix} \underline{k}_1^T \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \underline{k}_m^T \end{bmatrix} \qquad \underline{\underline{V}} = \begin{bmatrix} \underline{v}_1^T \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \underline{v}_n^T \end{bmatrix}$$

$\underline{K}_i$ and $\underline{v}_j$ are i^{th} and j^{th} rows of $\underline{\underline{K}}$ and $\underline{\underline{V}}$, respectively.

Equation (7) may then be solved for $\underline{\underline{K}}$, and $\underline{\underline{P}}$ and $\underline{\underline{D}}$ subsequently determined.

The computation overheads for this scheme are quite significant and so it would be recommended that such a scheme is useful mainly for adaptation of slowly varying systems in a regulatory environment.

8.6 Suggestions for Future Research

Since this work has been mainly exploratory in nature,

there is a requirement for a future commitment to several areas of research.

Specifically, there is an urgent demand for controller design techniques which are based on input-cutput descriptions of the open-loop plant. This would allow the full flexibility of the hyperstable input-output identification schemes to be employed. At present these methods are restricted to adaptive inverse approaches which:

a) can require large and sudden control inputs to be stipulated and,

b) result in unstable controllers when right-half plane zeroes are present in the identification model.

The methods described herein all require a fairly large computational overhead to be borne. Whilst the software does exist tc carry out these calculational tasks, it is often wasteful in time and storage. Algorithms to reduce both these factors are available [52] and further work in this area would be helpful.

As far as the author can ascertain, there has been no attempt to investigate the use of adaptation techniques outside the ill-defined regions of so-called regulatory

control or output driving. There are a large number of practical situations in which the conceptual notions of adaptation could prove beneficial. This thesis has mentioned one.

In relation to the hyperstable schemes examined, there is a need to investigate the use of the compensator, $\underline{\underline{D}}(z)$. For the present it remains as a necessity to ensure the positive realness of a particular transfer matrix. Its effect on the performance of the system is unknown. There is a further problem with the adaptively placed identification model parameters ie. right-half plane identification model zeroes can result in unstable control inputs. A solution based on limiting the identification model parameter space to a compact subset of the open left-half complex plane has been suggested. The mechanism of this procedure and the effect on the output realizability of the control system remain to be defined.

Of a more prosaic nature are requirements for experimental evaluation of the various schemes. These would be envisaged as directed towards the design choices which must be made apriori to the application eg. adaptation gains, initial model parameters, etc.

8.7 Conclusions

This chapter has attempted to unify the conceptual notions that have appeared in the remainder of the thesis. It has been stated that all model reference adaptation methods are subclasses of two general approaches:

1. Direct adaptive control schemes based on direct adaptation of the parameters of a conventional controller, and,
2. Indirect adaptive control schemes based on an identified model of the open-loop plant.

Of specific interest has been the second class and various schemes have been mentioned, both within an input-output descriptive plane and based on familiar state-space methodology.

The design freedom and restrictions demanded for mathematical tractability have been outlined and suggestions for future research to complete the study, mentioned.

Throughout much of this thesis there has been a need to remain very specific to certain schemes. The bias has been firmly entrenched in the input-output formulation, although state-space methodology may allow a more acceptable solution in the short-term.

REFERENCES FOR CHAPTER ONE

1. Rijnsdorp, J.E. and Seborg, D.E., "A Survey of Experimental Applications of Multivariable Control to Process Control Problems", Part C of a report presented at the Engineering Foundation Conf. on Chemical Process Control, Asilomar Conf. Site (1976).

2. Foss, A.S., "Critique of Chemical Process Control Theory", AIChEJ, 19, 209 - 214 (1973).

3. Lee, W. and Weekman, V.W., "Advanced Control Practice in the Chemical Process Industry: A View from Industry", AIChEJ, 22, 27 - 38 (1976).

4. Weekman, V.W., "Industrial Process Models -- State of the Art", Third Int. React. Eng. Symp. (1974).

5. Kestenbaum, A., Shinnar, R. and Thau, F.E., "Design Concepts for Process Control", Ind. Eng. Chem., Process Design Develop., 15, 2 - 13 (1976).

6. Seborg, D.E. and Fisher, D.G., "Experience With Experimental Applications of Multivariable Computer Control", presented at the Interkama Conf., Dusseldorf (1977).

7. Fisher, D.G. and Seborg, D.E., "Multivariable Computer Control -- A Case Study", American Elsevier, New York (1976).

8. Kuon, J.F., "Multivariable Frequency-Domain Design Techniques", Ph.D. Thesis, Univ. of Alberta (1975).

9. Shah, S.L., "The Role of Eigenvectors and Eigenvalues in Multivariable Control Systems Design.", Ph.D. Thesis, Univ. of Alberta (1977).

10. Chintapalli, P., Seborg, D.E. and Fisher, D.G., "Model Reference Identification of State Space Models for a Double Effect Evaporator", Can. J. Chem. Eng., 55, 213 - 220 (1977).

11. Martin-Sanchez, J.M., "A New Solution to Adaptive Control", Proc. IEEE, 64, 1209 - 1218 (1976).

12. Siljak, D.D., "Nonlinear Systems", John Wiley and Sons Inc., New York (1969).

REFERENCES FOR CHAPTER TWO

1. Tsypkin, Ya. Z., "Adaptation and Learning in Automatic Systems", Academic Press, New York (1970).

2. Davies, W.D.T., "System Identification for Self-Adaptive Control", John-Wiley and Sons, Ltd. (1970).

3. Newell, R.B., "Multivariable Computer Control of an Evaporator", Ph.D. Thesis, Univ. of Alberta (1971).

4. Landau, I.D., "A Survey of Model-Reference Adaptive Techniques - Theory and Applications", Automatica, 10, 353 - 379 (1974).

5. ----------, "Model-Reference Adaptive Systems A Survey (MRAS - What is Possible and Why?)", Trans. ASME J. Dyn. Systems, Meas. Control, 94, 119 - 132 (1972).

6. Asher, R.B., Andrisani, D. and Dorato, P., "Bibliography on Adaptive Control Systems", Proc. IEEE, 64, 1226 - 1230 (1976).

7. Hang, C.C. and Parks, P.C., "Comparative Studies of Model-Reference Adaptive Control Systems", IEEE Trans. on Auto. Control, AC-18, 419 - 428 (1973).

8. Unbehauen, H. and Schmid, C., "Status and Industrial Application of Adaptive Control Systems", Automatic Control Theory and Applic., 3, 1 - 12 (1975).

9. Narendra, K.S. and Valavani, L.S., "Stable Adaptive Observers and Controllers", Proc. IEEE, 64, 1198 - 1208 (1976).

10. Batterham, R.J., Smythe, R.L., Turner, R.E., and Thornton, G.J., "A New Control Technique for the Induration of Iron Ore", Paper presented at XIIth Int. Min. Process Congress (1977).

11. Hendry, J.E., Hendy, R.J., and White, E.T., "Application of Directed Logic Methods for the Optimising Control of Chemical Plant", from "Chemeca 77" 5th Aust. Conf. on Chem. Eng., 149 - 153 (1977).

12. Parks, P.C., "Liapunov Redesign of Model Reference Adaptive Control Systems", IEEE Trans. on Auto. Control, AC-11, 362 - 367 (1966).

13. Newton, G.C., Gould, L.A. and Kaiser, J.E., "Analytical Design of Linear Feedback Controls", John Wiley and Sons, New York (1957).

14. Nims, P.T., "Some Design Criteria for Automatic Controls", Trans. AIEE, 70, 606 (1951).

15. Graham, D. and Lathrop, R.C., "The Synthesis of Optimum Transient Response; Criteria and Standard Forms", Trans. AIEE, 72, 278 (1953).

16. Wescott, J.H., "The Minimum-Moment-of-Error-Squared Criterion; A New Performance Index for Servos", Proc. IEE, 101, 471 - 480 (1954).

17. Schultz, W.C. and Rideout, V.C., "The Selection and Use of Servo Performance Criteria", Trans. AIEE, 76, 383 (1957).

18. James, H.M., Nichols, N.B. and Phillips, R.S., "Theory of Servomechanisms", McGraw-Hill, New York (1947).

19. Truxal, J.G., "Automatic Feedback Control System Synthesis", McGraw-Hill, New York (1955).

20. Murphy, G.J. and Bold, N.T., "Optimization Based on a Square-Error Criterion with an Arbitrary Weighting Function", Trans. IEEE on Auto. Control, AC-5, (1960).

21. Eveleigh, V.W., "Adaptive Control Systems", Electro-Technology, 80 (1963).

22. Reskasuis, Z.V., "A General Performance Index for Analytical Design of Control Systems", IRE Trans. on Auto. Control, AC-6, 217 - 222 (1961).

23. Nightingale, J.M., "Practical Optimizing Systems-Part I. Continuous Parameter Adjustment", Control Engineering, 11, 76 - 81 (1964).

24. Jacobs, O.L.R., "Introduction to Control Theory", Clarendon Press, Oxford (1974).

25. Eveleigh, V.W., "Adaptive Control and Optimization Techniques", McGraw-Hill, New York (1967).

26. Stromer, P.R., "A Selected Bibliography on Sampled Data Systems", IRE Trans. on Auto. Control, AC-4, 112 - 114 (1959).

27. Wilde, D.J., "Optimum Seeking Methods", Prentice-Hall Inc., New Jersey (1964).

28. Gibson, J.E. and McVey, E.S., "Multidimensional Adaptive Control", Proc. Nat. Electron. Conf., 15, 17 - 26 (1959).

29. Eveleigh, V.W., "A Comparison of Two Approaches to Extrema Searching Adaptive Systems", Doctoral Dissertation, Purdue Univ. (1961).

30. ----------, "General Stability Analysis of Sinusoidal Perturbation Extrema Searching Adaptive Systems", Proc. 2nd Int. Congress of IFAC, (1963).

31. Smyth, R.K. and Nahi, N.E., "On the Stability and Design of Dither Adaptive Systems", ibid.

32. Beall, W.J. and Sollecito, W.E., "System Design of High Performance Pneumatic Servomechanisms with Large Variable Friction Loads", TIS no. 60GL109, General Electric Co. (1960).

33. Kochenburger, R.J., "Self-Adaptive Method for Accommodating Large Variations of Plant Gain in Control Systems", Proc. 2nd Int. Congress of IFAC (1963).

34. Margolis, M. and Leondes, C.T., "A Parameter Tracking Servo for Adaptive Control Systems", IRE WESCON Convention Record, part 4, 104 - 116 (1959).

35. Draper, C.S. and Li, Y.T., "Principles of Optimalizing Control Systems and an Application to the Internal Combustion Engine", ASME, New York (1951).

36. Tsien, H.S., "Engineering Cybernetics", McGraw-Hill, New York (1954).

37. Gibson, J.E., "Non-Linear Automatic Control", McGraw-Hill (1963).

38. Morosonov, I.S., "Methods of Extremum Control", Automation and Remote Control, 18, no. 11 (1957)

39. Eveleigh, V.W., "Limit Cycle Conditions in Optimalizing Controllers", Proc. 2nd IFAC Symp. on the Theory of Self-Adaptive Systems, 281 - 289 (1965).

40. Merrian, C.W. III, "Use of a Mathematical Error Criterion in the Design of Adaptive Control Systems", AIEE Trans. Paper 59-1158 (1959).

41. Marx, M.F., "Recent Adaptive Control Work at the General Electric Co.", Proc. Self Adaptive Flight Controls Symp., Wright Air Development Center (1959).

42. Whitaker, H.P., "The MIT Adaptive Autopilot", ibid.

43. Bongiorno, J.J. (Jr.), "Stability and Convergence Properties of Adaptive Control Systems", IRE Trans. on Auto. Control, AC-7, 30 - 41 (1962).

44. Clark, D.C., "The Model-Reference Self Adaptive Control System as Applied to the Flight Control of a Supersonic Transport", Proc. JACC Conf. (1964).

45. Schuk, O.H., "Honeywell's History and Philosophy in the Adaptive Control Field", Proc. Self Adaptive Flight Controls Symp., Wright Air Development Center (1959).

46. Anderson, G.W., Buland, R.W. and Cooper, G.R., "The Aeronutronic Self-Optimizing Automatic Control System", ibid.

47. Gibson, J.E., Leedham, C.D., McVey, E.S. and Rekasius, Z.V., "Specifications and Data Presentation in Linear Control Systems", Purdue Univ., Control and Information Systems Laboratory (1959).

48. Centner, R.M. and Idelsohn, J.M., "Adaptive Controller for a Metal-Cutting Process", Proc. JACC Conf. (1963).

49. Beadle, R.G., "On-Line Computer Control of Hot-Strip Finishing Mill for Steel", Proc. 2nd Int. Congress of IFAC (1963).

50. Fujii, S. and Kanda, N., "An Optimalizing Control of Boiler Efficiency", ibid.

51. Powell, N.R., "Controlled Parameter Phase Feedback FM Demodulation", Proc. Symp. Space Electron. Telemetry (1964).

52. Hu, M.J.C., "Application of the Adaline System to Weather Forecasting", Tech. Rept. 6775-1, Systems Theory Laboratory, Stanford Univ. (1964).

53. Gibson, J.E., "Making Sense Out of the Adaptive Principle", Control Engineering, 7, 113 - 119 (1960).

54. Butchart, R.L. and Shackcloth, B., "Synthesis of Model Reference Adaptive Control Systems by Lyapunov's Second Method", Proc. IFAC Symp. Adaptive Control, 145 - 142 (1965).

55. Winsor, C.A. and Roy, R.J., "Design of Model Reference Adaptive Control Systems by Liapunov's Second Method", IEEE Trans. on Auto. Control, AC-13, 204 (1968).

56. Popov, V.M., "The Solution of a New Stability Problem for Controlled Systems", Automation and Remote Control, 24, 1 - 23 (1963).

57. Landau, I.D., "A Hyperstability Criterion for Model Reference Adaptive Control Systems", IEEE Trans. on Auto. Control, AC-14, 552 - 555 (1969).

58. Tsypkin, Ya. Z., "A Frequency Criterion for Absolute Stability of Non-Linear Sampled-Data Systems", Avtom. Telemekh, No.3, 261 - 267 (1964).

59. ----------, and Popkov, Yu. S., "Theory of Non-Linear Sampled-data Systems", (in Russian), Nauka (1973).

60. Jakubovic, V.A., "Frequency Conditions for the Absolute Stability and Dissipativity of Control Systems with a Single Differentiable Non-Linearity", Soviet Math., 6, 98 - 101 (1965).

61. Lefshetz, S., "Stability of Non-Linear Systems", Academic Press, New York (1965).

62. Hsu, J.C. and Meyer, A.U., "Modern Control Principles and Applications", McGraw-Hill, New York, (1968).

63. Zames, G., "On the Input-Output Stability of Time-Varying Non-Linear Feedback Systems - Part II - Condition Involving Circles in the Frequency Plane and Sector Non-Linearities", IEEE Trans. on Auto. Control, AC-11, 465 - 476 (1966).

64. Desoer, C.A., "A Generalization of the Popov Criterion", IEEE Trans. on Auto. Control, AC-10, 182 - 185 (1965).

65. Popov, V.M. and Halanay, A., "On the Stability of Non-Linear Automatic Control Systems with Lagging Argument", Automation and Remote Control, 23, 783 - 786 (1963).

66. Martin-Sanchez, J.M., "A New Solution to Adaptive Control", Proc. IEEE, 64, 1209 - 1218 (1976).

67. Ljung, L., "On Positive Real Transfer Functions and the Convergence of Some Recursive Schemes", IEEE Trans. on Auto. Control, AC-22, 539 - 550 (1977).

68. ----------, "Analysis of Recursive Stochastic Algorithms", ibid.

69. "Learning Systems", A Symp. of the AACC Theory Committee (1973).

70. Lerner, A. Ya., "A Survey of Soviet Contributions to Control Theory", Control and Dynamic Systems, Volume 11, Academic Press, New York (1974).

References for Chapter Two

REFERENCES FOR CHAPTER THREE

1. Landau, I.D., "A Survey of Model-Reference Adaptive Techniques -- Theory and Applications", Automatica, 10, 353 - 379 (1974).

2. ----------, "Model-Reference Adaptive Systems A Survey (MRAS - What is Possible and Why?)", Trans. ASME J. Dyn. Systems, Meas. Control, 94, 119 - 132 (1972).

3. Asher, R.B., Andrisani, D. and Dorato, P., "Bibliography on Adaptive Control Systems", Proc. IEEE, 64, 1226 - 1240 (1976).

4. Hang, C.C. and Parks, P.C., "Comparative Studies of Model-Reference Adaptive Control Systems", IEEE Trans. on Auto. Control, AC-18, 419 - 428 (1973).

5. Unbehauen, H. and Schmid, C., "Status and Industrial Application of Adaptive Control Systems", Automatic Control Theory and Applic., 3, 1 - 12 (1975).

6. Narendra, K.S. and Valavani, L.S., "Stable Adaptive Observers and Controllers", Proc. IEEE, 64, 1198 - 1208 (1976).

7. Lindorff, D.P. and Carroll, R.L., "Survey of Adaptive Control Using Liapunov Design", Int. J. of Control, 18, 897 - 914 (1973).

8. Mondey, D., (ed.), "The International Encylopedia of Aviation", Octopus Books Ltd., Hong Kong (1977).

9. Eveleigh, V.W., "Adaptive Control and Optimization Techniques", McGraw-Hill Book Company, Toronto (1967).

10. Osburn, P.V., Whitaker, H.P. and Keezer, A., "New Developments in the Design of Adaptive Control Systems", Inst. Aeronautical Sciences, Paper 61-39 (1961).

11. Whitaker, H.P., "The MIT Adaptive Autopilot", Proc. Self Adaptive Flight Controls Symp., Wright Air Development Center (1959).

12. Donalson, D.D. and Leondes, C.T., "A Model Referenced Parameter Tracking Technique for Adaptive Control Systems", IEEE Trans. Appl. Ind., 241 - 262 (1963).

13. Dressler, R.M., "An Approach to Model-Reference Adaptive Control Systems", IEEE Trans. on Auto. Control, AC-12, 75 - 80 (1967).

14. Price, C.F., "An Accelerated Gradient Method for Adaptive Control", Proc. 9th IEEE Symp. Adaptive Processes Decision and Control, IV 4.1 - 4.9 (1970).

15. Winsor, C.A., "Model-Reference Adaptive Design", NASA-CR-98453 (1968).

16. Monopoli, R.V., Gilbart, J.W. and Thayer, W.D., "Model Reference Adaptive Control Based on Lyapunov-like Techniques", Proc. 2nd IFAC Symp. System Sensitivity and Adaptivity, F.24 - F.36 (1968).

17. Parks, P.C., "Liapunov Redesign of Model Reference Adaptive Control Systems", IEEE Trans. on Auto. Control, AC-11, 362 - 367 (1966).

18. Butchart, R.L. and Shackcloth, B., "Synthesis of Model Reference Adaptive Control Systems by Lyapunov's Second Method", Proc. IFAC Symp. Adaptive Control, 145 - 152 (1965).

19. Phillipson, P.H., "Design Methods for Model Reference Adaptive Systems", Proc. Inst. Mechanical Engineers, 183, 695 - 706 (1968).

20. Gilbart, J.W., Monopoli, R.V. and Price, C.F., "Improved Convergence and Increased Flexibility in the Design of Model Reference Adaptive Control Systems", Proc. 9th IEEE Symp. Adaptive Processes Decision and Control, IV 3.1 - 3.10 (1970).

21. Winsor, C.A. and Roy, R.J., "Design of Model Reference Adaptive Control Systems by Liapunov's Second Method", IEEE Trans. on Auto. Control, AC-13, 204 (1968).

22. Porter, B. and Tatnall, M.L., "Performance Characteristics of Multivariable Model Reference Adaptive Systems Synthesized by Liapunov's Direct Method", Int. J. of Control, 10, 241 - 257 (1969).

23. ----------, "Stability Analysis of a Class of Multivariable Model-Reference Adaptive Systems Having Time-Varying Process Parameters", Int. J. of Control, 11, 741 - 757 (1970).

24. Lion, P.M., "Rapid Identification of Linear and Non-Linear Systems", AIAA J., 5, 1835 - 1842 (1967).

25. Mendel, J.M. (ed.), "Gradient Identification for Linear Systems in Adaptive, Learning and Pattern Recognition Systems", Academic Press, New York (1970).

26. Pazdera, J.S. and Pottinger, H.J., "Linear System Identification Via. Liapunov Design Techniques", Proc. JACC Conf., 795 - 801 (1969).

27. Rang, E.G., "Adaptive Controllers Derived by Stability Considerations", Minneapolis-Honeywell Regulator Co., Memorandum MR 7905 (1962).

28. Astrom, K.J. and Eykhoff, P., "System Identification -- A Survey", Automatica, 7, 123 - 162 (1971).

29. Chintapalli, P., Seborg, D.E. and Fisher, D.G., "Model Reference Identification of State Space Models for a Double Effect Evapcrator", Can. J. of Chem. Eng., 55, 213 - 220 (1977).

30. Kudva, P. and Narendra, K.S., "An Identification Procedure for Discrete Multivariable Systems", IEEE Trans. on Auto. Control, AC-19, 549 - 552 (1974).

31. Udink Ten Cate, A.J., "Gradient Identification of Multivariable Discrete Systems", Electronics Letters, 11, 98 - 99 (1975).

32. Oliver, W.K., "Model Reference Adaptive Control: Hybrid Computer Simulation and Experimental Verification", MSc. Thesis, Univ. of Alberta (1972).

33. Fisher, D.G. and Seborg, D.E., "Multivariable Computer Control -- A Case Study", American Elsevier, New York (1976).

34. Newell, R.B., "Multivariable Computer Control of an Evaporator", PhD Thesis, Univ. of Alberta (1971).

35. Newell, R.B. and Fisher, D.G., "Implementation of Optimal, Multivariable Setpoint Changes on a Pilot Plant Evaporator", 2nd IFAC Symp. of Multivariable Control Systems (1971).

36. Currie, M.G. and Stear, E.B., "State Space Structure of Model Reference Adaptive Control for a Noisy System", Proc. 2nd Asilomar Conf. Circuits and Systems, 401 - 405 (1968).

37. Carroll, R.L. and Lindorff, D.P., "An Adaptive Observer for Single-Input Single-Output Linear Systems", IEEE Trans. on Auto. Control, AC-18, 428 - 435 (1973).

38. Luders, G. and Narendra, K.S., "An Adaptive Observer and Identifier for a Linear System", IEEE Trans. on Auto. Control, AC-18, 496 - 499 (1973).

39. ----------, "A New Canonical Form for an Adaptive Observer", IEEE Trans. on Auto. Control, AC-19, 117 - 119 (1974).

40. Kudva, P. and Narendra, K.S., "Synthesis of an Adaptive Observer Using Liapunov's Direct Method", Int. J. of Control, 18, 1201 - 1210 (1973).

41. Sutherlin, D.W. and Boland, J.S. III, "Model-Reference Adaptive Control System Design Technique", Trans. ASME J. Dyn. Systems Meas. Control, 94, 373 - 379 (1973).

42. Iwai, Z., Seborg, D.E. and Fisher, D.G., "Model Reference Adaptive Control Using Output Feedback and Observers", Proc. JACC Conf. (1977).

43. Hang, C.C., "On the Design of Multivariable Model-Reference Adaptive Control Systems", Int. J. of Control, 19, 365 - 372 (1974).

44. Popov, V.M., "The Solution of a New Stability Problem for Controlled Systems", Automation and Remote Control, 24, 1 - 23 (1963).

45. ----------, "Hyperstability of Automatic Control Systems with Several Non-Linear Elements", Rev. Roum. Sci. Tech., 9, 35 - 45 (1964).

46. ----------, "Hyperstability of Control Systems", Springer-Verlag (1973).

47. Landau, I.D., "A Hyperstability Criterion for Model Reference Adaptive Control Systems", IEEE Trans. on Auto. Control, AC-14, 552 - 555 (1969).

48. Anderson, B.D.O., "A Simplified Viewpoint of Hyperstability", IEEE Trans. on Auto. Control, AC-13, 292 - 294 (1968).

49. Landau, I.D., "Synthesis of Hyperstable Discrete Model Reference Adaptive Systems", 5th Asilomar Conf. Circuits and Systems (1971).

50. ----------, "Synthesis of Discrete Model Reference Adaptive Systems", IEEE Trans. on Auto. Control, AC-16, 507 - 508 (1971).

51. ----------, "Design of Discrete, Model Reference Adaptive Systems Using the Positivity Concept", Proc. 3rd IFAC Symp. Sensitivity, Adaptivity and Optimality, 307 - 314 (1973).

52. Hitz, L. and Anderson, B.D.O., "Discrete Positive Real Functions and Their Applications to System Stability", Proc. IEE, 116, 153 - 155 (1969).

53. Landau, I.D., "A Generalization of the Hyperstability Conditions for Model Reference Adaptive Systems", IEEE Trans. on Auto. Control, AC-17, 246 - 247 (1972).

54. ----------, "Design of Multivariable Adaptive Model Following Control Systems", Proc. 3rd IFAC Symp. Sensitivity, Adaptivity and Optimality, 315 - 322 (1973).

55. ----------, "Design of Multivariable Adaptive Model Following Control Systems", Automatica, 10, 483 - 494 (1974).

56. ---------- and Courtiol, B., "Adaptive Model Following Control Systems for Flight Control and Simulation", Proc. AIAA 10th Aerospace Sciences Meeting, paper 72 - 95 (1972).

57. ----------, "Adaptive Model Following Systems for Flight Control and Simulation", J. Aircraft, 9, 668 - 674 (1972).

58. ----------, Sinner, E. and Courtiol, B., "Model Reference Adaptive Systems, Some Examples", Proc. ASME Winter Meeting, (1972).

59. Bethoux, G. and Courtiol, B., "A Hyperstable Discrete Model Reference Adaptive Control System", Proc. 3rd IFAC Symp. Sensitivity, Adaptivity and Optimality, 282 - 289 (1973).

60. Landau, I.D., "Hyperstability Concepts and Their Application to Discrete Control Systems", Proc. 13th JACC Conf., 373 - 381, Stanford University (1972).

61. Hang, C.C., "A New Form of Stable Adaptive Observer", IEEE Trans. on Auto. Control, AC-21, 544 - 547 (1976).

62. ----------, Ph.D. dissertation, Dept. Eng. Warwick Univ. (1973).

63. Narendra, K.S. and Kudva, P., "Stable Adaptive Schemes for System Identification and Control", IEEE Trans. Syst. Man. Cybern., SMC-4, 552 - 560 (1974).

64. Monopoli, R.V., "Model-Reference Adaptive Control With an Augmented Error Signal", IEEE Trans. on Auto. Control, AC-19, 474 - 484 (1974).

65. Thathachar, M.A.L. and Gajendran, F., "Convergence Problems in a Class of Model Reference Adaptive Control Systems", Proc. IEEE Conf. on Decision and Control, 1036 - 1041 (1977).

66. Suzuki, T. and Dohimoto, Y., "A Modified Scheme for the Model Reference Adaptive Control With Augmented Error Signal", Int. J. of Control, 27, 199 - 211 (1978).

67. Landau, I.D., "Hyperstability and Identification", Proc. IEEE Symp. Adaptive Processes (1970).

68. ----------, "Hyperstability and Identification", Proc. 9th IEEE Conf. Decision and Control (1970).

69. ----------, "An Hyperstable Algorithm for Identification Through the Equation Error Method", Proc. 5th Hawaii Int. Conf. System Sciences, 300 - 302 (1972).

70. ----------, "An Asymptotic Unbiased Recursive Identifier for Linear Systems", Proc. IEEE Conf. Decision and Control, 288 - 294 (1974).

71. ----------, "Unbiased Recursive Identification Using Model Reference Adaptive Techniques", IEEE Trans. on Auto. Control, AC-21, 194 - 202 (1976).

72. Courtiol, B., "On a Multidimensional System Identification Method", IEEE Trans. on Auto. Control, AC-17, 390 - 394 (1972).

73. Hang, C.C., "An Experimental Study of Model Reference System Identification Method", Int. J. of Control, 23, 393 - 401 (1976).

74. ----------, "The Design of Model Reference Parameter Estimation Systems Using Hyperstability Theories", Proc. 3rd IFAC Symp. System Identification, 741 - 744 (1973).

75. Martin-Sanchez, J.M., "A New Solution to Adaptive Control", Proc. IEEE, 64, 1209 - 1218 (1976).

76. ----------, "Implementation of an Adaptive Autopilot Scheme for the F-8 Aircraft", Report - Charles Stark Draper Labs. Inc. (1975).

77. ----------, "A General Solution to the Adaptive Control of Linear Time-Variant Processes With Time-Delays", Parts I and II, Univ. of Alberta (1976).
(submitted in revised form for publication).

78. Erzberger, H., "On the Use of Algebraic Methods in the Analysis and Design of Model Following Control Systems", Tech. note D-4663 NASA.

79. Anderson, B.D.O., "A System Theory Criterion for Positive Real Transfer Matrices", SIAM J. Control, 5, 171 - 182 (1967).

80. Leondes, C.T., (ed.), "Theory and Application of Kalman Filtering" -- NATO-AGARD (1970).

REFERENCES FOR CHAPTER FOUR

1. Landau, I.D., "An Asymptotic Unbiased Recursive Identifer for Linear Systems", Proc. IEEE Conf. on Decision and Control, 288 - 294 (1974).

2. ----------, "Unbiased Recursive Identification Using Model Reference Adaptive Techniques", IEEE Trans. on Auto. Control, AC-21, 194 - 202 (1976).

3. Courtiol, B., "On a Multidimensional System Identification Method", IEEE Trans. on Auto. Control, AC-17, 390 - 394 (1972).

4. Iwai, Z., Seborg, D.E. and Fisher, D.G., "Model Reference Adaptive Control Using Output Feedback and Observers", Proc. JACC Conf. (1977).

5. Rucker, R.A., "Real Time System Identification in the Presence of Noise", Proc. Western Electron. Conv., paper 23 (1963).

6. Hang, C.C., "On State-Variable Filters for Adaptive System Design", IEEE Trans. on Auto. Control, AC-21, 874 - 876 (1976).

7. Carroll, R.L. and Lindorff, D.P., "An Adaptive Observer for Single-Input Single-Output Linear Systems", IEEE Trans. on Auto. Control, AC-18, 428 - 435 (1973).

8. Luders, G. and Narendra, K.S., "An Adaptive Observer and Identifier for a Linear System", IEEE Trans. on Auto. Control, AC-18, 496 - 499 (1973).

9. Narendra, K.S. and Kudva, P., "Stable Adaptive Schemes for System Identification and Control", Parts I and II, IEEE Trans. Sys., Man. and Cybern., SMC-4, 542 - 560 (1974).

10. Morse, A.S., "Representation and Parameter Identification of Multi-Output Linear Systems", IEEE Conf. on Decision and Control, 301 - 306 (1974).

11. Morgan, A.P. and Narendra, K.S., "On the Stability of Nonautonomous Differential Equations $\dot{x}=(A + B(t))x$, With Skew Symmetric Matrix $B(t)$", SIAM J. of Contr. and Optim., 15, 163 - 176 (1977).

12. Kudva, P. and Narendra, K.S., "An Identification Procedure for Discrete Multivariable Systems", IEEE Trans. on Auto. Control, AC-19, 549 - 552 (1974).

13. Narendra, K.S. and Valavani, L.S., "Stable Adaptive Observers and Controllers", Proc. IEEE, 64, 1198 - 1208 (1976).

14. ----------, "Stable Adaptive Controller Design -- Part I -- Direct Control", Proc. IEEE Conf. on Decision and Control, 881 - 886 (1977).

15. Nikiforuk, P.N., Ohta, H. and Gupta, M.M., "A Two-Level Adaptive Controller for Application to Flight Control Systems", AIAA Guidance and Control Conf., 401 - 407 (1977).

16. ----------, "Adaptive Observer and Identifier Design for Multi-Input Multi-Output Systems", Proc. 4th IFAC International Symp. on Multivariable Technological Systems, 189 - 196 (1977).

17. Monopoli, R.V., "Model Reference Adaptive Control With an Augmented Error Signal", IEEE Trans. on Auto. Control, AC-19, 474 - 484 (1974).

18. Suzuki, T. and Dohimoto, Y., "A Modified Scheme for the Model-Reference Adaptive Control With Augmented Error Signal", Int. J. Control, 27, 199 - 211 (1978).

19. Martin-Sanchez, J.M., "A New Solution to Adaptive Control", Proc. IEEE, 64, 1209 - 1218 (1976).

20. ----------, "Implementation of an Adaptive Autopilot Scheme for the F-8 Aircraft", Report - Charles Stark Draper Labs Inc. (1975)

21. ----------, "A General Solution to the Adaptive Control of Linear Time-Variant Processes With Time-Delays", Parts I and II, Univ. of Alberta (1976).
(submitted in revised form for publication).

22. Landau, I.D., "Synthesis of Discrete Model Reference Adaptive Systems", IEEE Trans. on Auto. Control, AC-16, 507 - 508 (1971).

23. ----------, "Hyperstability Concepts and Their Application to Discrete Control Systems", Proc. 13th JACC Conf., 373 - 381, Stanford University (1972).

24. Landau, I.D., "An Asymptotic Unbiased Recursive Identifier for Linear Systems", Proc. IEEE Conf. on Decision and Control, 288 - 294 (1974).

25. ----------, "Unbiased Recursive Identification Using Model Reference Techniques", IEEE Trans. on Auto. Control, AC-21, 194 - 202 (1976).

26. Godbole, S.S. and Smith, C.F., "A New Control Approach Using the Inverse System", IEEE Trans. on Auto. Control, AC-17, 698 - 702 (1972).

27. Sain, M.K. and Massey, J.L., "Invertibility of Linear Time-Invariant Dynamical Systems", IEEE Trans. on Auto. Control, AC-14, 141 - 149 (1969).

28. Silverman, L.M., "Inversion of Multivariable Linear Systems", IEEE Trans. on Auto. Control, AC-14, 270 - 276 (1969).

29. Singh, S.P. and Liu, R., "A Survey of Inverse Systems", Proc. Nat. Electron. Conf., 380 - 384 (1970).

30. Forney, G.D., "Acceptable Inverses of Linear Systems", Proc. Hawaii Int. Conf. Syst. Theory, 191 - 194 (1970).

31. Silverman, L.M., "Properties and Application of Inverse Systems", IEEE Trans. on Auto. Control, AC-13, 436 - 437 (1968).

32. Johnson, C.R. (Jr.) and Larimore, M.G., "Comments on 'A New Solution to Adaptive Control'", Proc. IEEE, 65, 587 (1977).

33. Martin-Sanchez, J.M., "Reply to Johnson and Larimore", ibid, 587 - 588 (1977).

34. Ljung, L., "On Positive Real Transfer Functions and the Convergence of Some Recursive Schemes", IEEE Trans. on Auto. Control, AC-22, 539 - 550 (1977).

REFERENCES FOR CHAPTER FIVE

1. Landau, I.D., "A Survey of Model-Reference Adaptive Techniques -- Theory and Applications", Automatica, 10, 353 - 379 (1974).

2. Asher, R.B., Andrisani, D. and Dorato, P., "Bibliography on Adaptive Control Systems", Proc. IEEE, 64, 1226 - 1240 (1976).

3. Hang, C.C. and Parks, P.C., "Comparative Studies of Model-Reference Adaptive Control Systems", IEEE Trans. on Auto. Control, AC-18, 419 - 428 (1973).

4. Unbehauen, H. and Schmid, C., "Status and Industrial Application of Adaptive Control Systems", Automatic Control Theory and Applic., 3, 1 - 12 (1975).

5. Narendra, K.S. and Valavani, L.S., "Stable Adaptive Observers and Controllers", Proc. IEEE, 64, 1198 - 1208 (1976).

6. Popov, V.M., "Hyperstability of Control Systems", Springer-Verlag (1973).

7. Landau, I.D., "A Hyperstability Criterion for Model Reference Adaptive Control Systems", IEEE Trans. on Auto. Control, AC-14, 552 - 555 (1969).

8. Foss, A.S., "Critique of Chemical Process Control Theory", AIChEJ, 19, 209 - 214 (1973).

9. Lee, W. and Weekman, V.W., "Advanced Control Practice in the Chemical Process Industry: A View From Industry", AIChEJ, 22, 27 - 38 (1976).

10. Rosenbrock, H.H., "Distinctive Problems of Process Control", Chem. Eng. Prog., 58, 43 - 50 (1962).

11. ---------- and McMorran, P.D., "Good, Bad or Optimal?", IEEE Trans. on Auto. Control, AC-16, 552 - 554 (1971)

12. Kestenbaum, A., Shinnar, R. and Thau, F.E., "Design Concepts for Process Control", Ind. Eng. Chem., Process Design Develop., 15, 2 - 13 (1976).

13. Weekman, V.W., "Industrial Process Models -- State of the Art", Third Int. React. Eng. Symp. (1974).

14. Landau, I.D., "Unbiased Recursive Identification Using Model-Reference Techniques", IEEE Trans. on Auto. Control, AC-21, 194 - 202 (1976).

15. Bethoux, G. and Courtiol, B., "A Hyperstable Discrete Model Reference Adaptive Control System", Proc. 3rd IFAC Symp., Sensitivity, Adaptivity and Optimality, 282 - 289 (1973).

16. Landau, I.D. and Courtiol, B., "Adaptive Model Following Systems for Flight Control and Simulation", J. Aircraft, 9, 668 - 674 (1972).

17. Johnstone, R. McG., "Report no. 1", Chem. Eng. Departmental Report, Univ. of Alberta, Dec. (1977).

18. Nikiforuk, P.N., Ohta, H. and Gupta, M.M., "A Two-Level Adaptive Controller for Application to Flight Control Systems", AIAA Guidance and Control Conf., 401 - 407 (1977).

19. ----------, "Adaptive Observer and Identifier Design for Multi-Input, Multi-Output Systems", Proc. 4th IFAC Int. Symp. on Multivariable Technological Systems, 189 - 196 (1977).

20. Godbole, S.S. and Smith, C.F., "A New Control Approach Using the Inverse System", IEEE Trans. on Auto. Control, AC-17, 698 - 702 (1972).

21. Martin-Sanchez, J.M., "A New Solution to Adaptive Control", Proc. IEEE, 64, 1209 - 1218 (1976).

22. Landau, I.D., "Hyperstability Concepts and Their Application to Discrete Control Systems", Proc. 13th JACC Conf., 373 - 381, Stanford University (1972).

23. ----------, "Synthesis of Discrete Model Reference Adaptive Systems", IEEE Trans. on Auto. Control, AC-16, 507 - 508 (1971).

24. ----------, "Design of Discrete Model Reference Adaptive Systems Using the Positivity Concept", Proc. 3rd IFAC Symp. on Sensitivity, Adaptivity and Optimality, 307 - 317 (1973).

25. Hitz, L. and Anderson, B.D.O., "Discrete Positive Real Functions and Their Applications to System Stability", Proc. IEE, 116, 153 - 155 (1969).

26. Ljung, L., "On Positive Real Transfer Functions and the Convergence of Some Recursive Schemes", IEEE Trans. on Auto. Control, AC-22, 539 - 550 (1977).

27. Astrom, K.J. and Eykhoff, P., "System Identification -- A Survey", Automatica, 7, 123 - 162 (1971).

28. Landau, I.D., "An Addenum to 'Unbiased Recursive Identification Using Model Reference Adaptive Techniques'", IEEE Trans. on Auto. Control, AC-23, 97 - 99 (1978).

29. Kosut, R.L., "The Determination of the System Transfer Function Matrix for Flight Control Systems", IEEE Trans. on Auto. Control, AC-13, 214 (1968).

30. Munro, N. and Zakian, V., "Inversion of Rational Polynomial Matrices", Electronics Letters, 6, 629 - 639 (1970).

31. Karmarkar, J., "Application of Positive Real Functions in Hyperstable Discrete Model Reference Adaptive System Design", Proc. 5th Int. Hawaii Conf. on System Sciences, 382 - 384 (1972).

32. ---------- and Siljak, D.D., "A Computer-Aided Regulator Design", 9th Annual Allerton Conf. on Circuits and Systems, 585 - 593 (1971).

33. Anderson, B.D.O., "A Simplified Viewpoint of Hyperstability", IEEE Trans. on Auto. Control, AC-13, 292 - 294 (1968).

34. Jacobs, O.L.R., "Introduction to Control Theory", Clarendon Press, Oxford (1974).

35. Hsu, J.C. and Meyer, A.U., "Modern Control Principles and Applications", McGraw-Hill, New York (1968).

36. Park, H., "Control System Design by Eigenvalue Assignment", MSc. Thesis, Univ. of Alberta (1974).

37. Topaloglu, T., "Modal Control and Eigenvalue Assignment", MSc. Thesis, Univ. of Alberta (1973).

38. Davies, W.D.T., "System Identification for Self-Adaptive Control", John-Wiley and Sons, Ltd. (1970).

39. Seborg, D.E. and Fisher, D.G., "Experience With Experimental Applications of Multivariable Computer Control", presented at the Interkama Conf., Dusseldorf (1977).

40. Oliver, W.K., "Model Reference Adaptive Control: Hybrid Computer Simulation and Experimental Verification", MSc. Thesis, Univ. of Alberta (1972).

41. Thathachar, M.A.L. and Gajendran, F., "Convergence Problems in a Class of Model Reference Adaptive Control Systems", Proc. IEEE Conf. on Decision and Control, 1036 - 1041 (1977).

42. Popov, V.M., "The Solution of a New Stability Problem for Controlled Systems", Automation and Remote Control, 24, 1 - 23 (1963).

43. Naumov, B.N. and Tsypkin, Ya. Z., "A Frequency Criterion for Absolute Process Stability in Non-Linear Automatic Control Systems", (in Russian), Automation and Remote Control, 25, 852 - 867 (1964).

44. Yakubovich, V.A., "The Method of Matrix Inequalities in the Stability Theory of Non-Linear Control Systems, Part II, Absolute Stability in a Class of Non-Linearities With a Condition on the Derivative", Automation and Remote Control, 26, 577 - 592 (1965).

45. Desoer, C.A., "A Generalization of the Popov Criterion", IEEE Trans. on Auto. Control, AC-10, 182 - 185 (1965).

46. Jury, E.I. and Lee, B.W., "The Absolute Stability of Systems With Many Non-Linearities", (in Russian), Automation and Remote Control, 26, 945 - 965 (1965).

47. Tokumaru, H. and Saito, N., "On the Stability of Automatic Control Systems With Many Non-Linear Characteristics", Mem. Fac. Eng., Kyoto Univ., 27, 347 - 379 (1965).

48. Brockett, R.W. and Willems, J.W., "Frequency Domain Stability Criteria", Parts I and II, IEEE Trans. on Auto. Control, AC-10, 255 - 261, 407 - 413 (1965).

49. ----------, "The Status of Stability Theory for Deterministic Systems", ibid, AC-11, 596 - 606 (1966).

50. Dewey, A.G. and Jury, E.I., "A Stability Inequality for a Class of Non-Linear Feedback Systems", ibid, AC-11, 54 - 62 (1966).

51. Anderson, B.D.O., "Stability of Control Systems With Multiple Non-Linearities", J. Franklin Institute, 3, 183 - 187 (1967).

52. Kuon, J.F., "Multivariable Frequency-Domain Design Techniques", Ph.D. Thesis, Univ. of Alberta (1975).

53. Kuon, J.F. and Fisher, D.G., "Multivariable Frequency Domain Techniques User's Manual", Research Report No. 750410, Dept. of Chem. Eng., Univ. of Alberta (1975).

54. Holtzman, J.M., "A Local Bounded-Input Bounded-Output Condition for Non-Linear Feedback Systems", IEEE Trans. on Auto. Control, AC-13, 585 - 586 (1968).

55. ----------, "A General Solution to the Adaptive Control of Linear Time-Variant Processes With Time-Delays", Parts I and II, Univ. of Alberta (1976).
(submitted in revised form for publication).

REFERENCES FOR CHAPTER SIX

1. Chen, C.T., "Introduction to Linear System Theory", Holt, Rinehart and Winston Inc., New York (1970).

2. Johnstone, R. McG., "Adaptive Control Simulation Programs", Dept. of Chem. Eng., Univ. of Alberta (to appear).

3. Ljung, L., "On Positive Real Transfer Functions and the Convergence of Some Recursive Schemes", IEEE Trans. on Auto. Control, AC-22, 539 - 550 (1977).

4. Martin-Sanchez, J.M., "A New Solution to Adaptive Control", Proc. IEEE, 64, 1209 - 1218 (1976).

5. ----------, "A General Solution to the Adaptive Control of Linear Time-Variant Processes With Time-Delays", Parts I and II, Univ. of Alberta (1976). (submitted in revised form for publication).

6. Johnson, C.R. (Jr.) and Larimore, M.G., "Comments on 'A New Solution to Adaptive Control'", Proc. IEEE, 65, 587 (1977).

7. Oliver, W.K., "Model Reference Adaptive Control: Hybrid Computer Simulation and Experimental Verification", MSc. Thesis, Univ. of Alberta (1972).

8. Elliot, R.F., "Direct Digital Control of a Simulated Evaporator", M.Eng. Report, Univ. of Alberta (1975).

REFERENCES FOR CHAPTER SEVEN

1. Rijnsdorp, J.E. and Seborg, D.E., "A Survey of Experimental Applications of Multivariable Control to Process Control Problems", AIChE Symposium Series, no. 159, 72, 112 - 123 (1976).

2. Wood, R.K., "Improved Control by Application of Advanced Control Techniques", ISA Trans., 16, 32 - 39 (1977).

3. Kuon, J.F., "Multivariable Frequency-Domain Design Techniques", Ph.D. Thesis, Univ. of Alberta (1975).

4. Bode, H.W., "Network Analysis and Feedback Amplifier Design", Van Nostrand (1945)

5. Kuon, J.F. and Fisher, D.G., "Multivariable Frequency Domain Techniques User's Manual", Research Report No. 750410, Dept. of Chem. Eng., Univ. of Alberta (1975).

6. ----------, "Design Example of a Controller in the Multivariable Frequency Domain Using GEMSCOPE", Research Report No. 750411, Dept. of Chem. Eng., Univ. of Alberta (1975).

7. ----------, "Documentation of Modifications to GEMSCOPE:Multivariable Frequency Domain Techniques", Research Report No. 741020, Dept. of Chem. Eng., Univ. of Alberta (1974).

8. Bethoux, G. and Courtiol, B., "A Hyperstable Discrete Model Reference Adaptive Control System", Proc. 3rd IFAC Symposium on Sensitivity, Adaptivity and Optimality, 282 - 289 (1972).

9. Landau, I.D., "Synthesis of Discrete Model Reference Adaptive Systems", IEEE Trans. on Auto. Control, AC-16, 507 - 508 (1971).

10. Hendry, J.E., Hendy, R.J., and White, E.T., "Application of Directed Logic Methods for the Optimising Control of Chemical Plant", from "Chemeca 77" 5th Aust. Conf. on Chem. Eng., 149 - 153 (1977).

11. Batterham, R.J., Smythe, R.L., Turner, R.E., and Thornton, G.J., "A New Control Technique for the Induration of Iron Ore", Paper presented at XIIth Int. Min. Process Congress (1977).

REFERENCES FOR CHAPTER EIGHT

1. Landau, I.D., "A Survey of Model Reference Adaptive Techniques -- Theory and Applications", Automatica, 10, 353 - 379 (1974).

2. ----------, "Model-Reference Adaptive Systems A Survey (MRAS -- What is Possible and Why?)", Trans. ASME J. Dyn. Systems Meas. Control, 94, 119 - 132 (1972).

3. Asher, R.B., Andrisani, D. and Dorato, P., "Bibliography on Adaptive Control Systems", Proc. IEEE, 64, 1226 - 1240 (1976).

4. Hang, C.C. and Parks, P.C., "Comparative Studies of Model-Reference Adaptive Control Systems", IEEE Trans. on Auto. Control, AC-18, 419 - 428 (1973).

5. Unbehauen, H. and Schmid, C., "Status and Industrial Application of Adaptive Control Systems", Automatic Control Theory and Applic., 3, 1 - 12 (1975).

6. Narendra, K.S. and Valavani, L.S., "Stable Adaptive Observers and Controllers", Proc. IEEE, 64, 1198 - 1208 (1976).

7. Lindorff, D.P. and Carroll, R.L., "Survey of Adaptive Control Using Liapunov Design", Int. J. of Control, 18, 897 - 914 (1973).

8. Parks, P.C., "Liapunov Redesign of Model Reference Adaptive Control Systems", IEEE Trans. on Auto. Control, AC-11, 362 - 367 (1966).

9. Popov, V.M., "The Solution of a New Stability Problem for Controlled Systems", Automation and Remote Control, 24 1 - 23 (1963).

10. Landau, I.D., "A Hyperstability Criterion for Model Reference Adaptive Control Systems", IEEE Trans. on Auto. Control, AC-14, 552 - 555 (1969).

11. ----------, "Synthesis of Discrete Model Reference Adaptive Systems", IEEE Trans. on Auto. Control, AC-16, 507 - 508 (1971).

12. Foss, A.S., "Critique of Chemical Process Control Theory", AIChEJ, 19, 209 - 214 (1973).

13. Lee, W. and Weekman, V.W., "Advanced Control Practice in the Chemical Process Industry: A View from Industry", AIChEJ, 22, 27 - 38 (1976).

14. Rosenbrock, H.H. and McMorran, P.D., "Good, Bad or Optimal?", IEEE Trans. on Auto. Control, AC-16, 552 - 554 (1971).

15. Kestenbaum, A., Shinnar, R. and Thau, F.E., "Design Concepts for Process Control", Ind. Eng. Chem. Process Design Develop., 15, 2 - 13 (1976).

16. Chen, C.T., "Introduction to Linear System Theory", Holt, Rinehart and Winston Inc., New York (1970).

17. Kuon, J.F., "Multivariable Frequency-Domain Design Techniques", Ph.D. Thesis, Univ. of Alberta (1975).

18. Rosenbrock, H.H., "Design of Multivariable Control Systems Using the Inverse Nyquist Array", Proc. IEE, 116, 1929 - 1936 (1969).

19. MacFarlane, A.G.J., "Multivariable-Control-System Design Techniques:A Guided Tour", Proc. IEE, 117, 1039 - 1047 (1970).

20. Paraskevopoulos, P.N., "Modal Output by Output Feedback", Int. J. of Control, 24, 209 - 216 (1976).

21. ----------, "A General Solution to the Output Feedback Eigenvalue -- Assignment Problem", ibid., 509 - 528 (1976).

22. ----------, and King, R.E., "A Kronecker Product Approch to Pole Assignment by Output Feedback", ibid., 325 - 334 (1976).

23. Seraji, H. and Tarokh, M., "Design of Proportional Plus Derivative Output Feedback for Pole Assignment", Proc. IEEE, 128, 729 - 732 (1977).

24. ----------, "On Pole-Shifting Using Output Feedback", Int. J. of Control, 20, 721 - 726 (1976).

25. Munro, N. and Vardoulakis, A., "Pole-Shifting Using Output Feedback", ibid., 18, 1267 - 1273 (1973).

26. Martin-Sanchez, J.M., "A New Solution to Adaptive Control", Proc. IEEE, 64, 1209 - 1218 (1976).

27. Martin-Sanchez, J.M., "A General Solution to the Adaptive Control of Linear Time-Variant Processes With Time-Delays", Parts I and II, Univ. of Alberta (1976). (submitted in revised form for publication).

28. Johnson, C.R. (Jr.) and Larimore, M.G., "Comments on 'A New Solution to Adaptive Control'", Proc. IEEE, 65, 587 (1977).

29. Ljung, L., "On Positive Real Transfer Functions and the Convergence of Some Recursive Schemes", IEEE Trans. on Auto. Control, AC-22, 539 - 550 (1977).

30. Bethoux, G. and Courtiol, B., "A Hyperstable Discrete Model Reference Adaptive Control System", Proc. 3rd IFAC Symp. Sensitivity, Adaptivity and Optimality, 282 - 289 (1973).

31. Godbole, S.S. and Smith, C.E., "A New Control Approach Using the Inverse System", IEEE Trans. on Auto. Control, AC-17, 698 - 720 (1972).

32. Sinha, N.K., "Minimal Realization of Transfer Function Matrices A Comparative Study of Different Methods", Int. J. Control, 22, 627 - 639 (1975).

33. Anderson, B.D.O., "A Simplified Viewpoint of Hyperstability", IEEE Trans. on Auto. Control, AC-13, 292 - 294 (1968).

34. Landau, I.D., "Unbiased Recursive Identification Using Model-Reference Adaptive Techniques", IEEE Trans. on Auto. Control, AC-21, 194 - 202 (1976).

35. Panuska, V., "An Adaptive Recursive Least Squares Identification Algorithm", Proc. 8th IEEE Symp. Adaptive Processes (1969).

36. Young, P.C., "An Instrumental Variable Method for Real Time Identification of a Noisy Process", Automatica, 6, 271 - 288 (1970).

37. Ljung, L., "The Extended Kalman Filter as a Parameter Estimator for Linear Systems" (to be published).

38. Martin-Sanchez, J.M., "Reply to Johnson and Larimore", Proc. IEEE, 65, 587 - 588 (1977).

39. Karmarkar, J., "Application of Positive Real Functions in Hyperstable Discrete Model Reference Adaptive System Design", Proc. 5th Int. Hawaii Conf. on System Sciences, 382 - 384 (1972).

40. ----------- and Siljak, D.D., "A Computer-Aided Regulator Design", 9th Annual Allerton Conf. on Circuits and Systems, 585 - 593 (1971).

41. Johnstone, R. McG., "Technical note 78.08.14", Dept. of Chem. Eng., Univ. of Alberta (1978).

42. Karim, N., "Multivariable MRAC Algorithm Input/Output Formulation", Dept. of Chem. Eng., Univ. of Alberta (1978).

43. Shah, S.L., "Technical note 13.08.78", Dept. of Chem. Eng., Univ. of Alberta (1978).

44. Landau, I.D. and Courtiol, B., "Adaptive Model Following Systems for Flight Control and Simulation", J. of Aircraft, 9, 668 - 674 (1972).

45. Preusche, G., "A Two-Level Model Following Control System and its Application to the Power Control of a Steam-Cooled Fast Reactor", Automatica, 8, 145 - 151 (1972).

46. Nikiforuk, P.N., Ohta, H. and Gupta, M.M., "A Two-Level Adaptive Controller for Application to Flight Control Systems", Proc. of the AIAA Guidance and Control Conf., 401 - 407 (1977).

47. Hsu, J.S. and Meyer, A.U., "Modern Control Principles and Applications", McGraw-Hill (1968).

48. Ogata, K., "State-Space Analysis of Control Systems", Prentice-Hall Inc., N.J. (1967).

49. Jacobs, O.L.R., "Introduction to Control Theory", Clarendon Press, Oxford (1974).

50. Newell, R.B., "Multivariable Computer Control of an Evaporator", PhD. Thesis, Univ. of Alberta (1971).

51. Paraskevopoulos, P.N., "On Pole Assignment by Proportional-Plus-Derivative Output Feedback", Electronics Letters, 14, 34 - 36 (1978).

52. Munro, N., "A Flexible Fortran Storage Technique Suitable for Transfer Function Matrices", Control Systems Centre Report No. 91, Univ. of Manchester Institute of Science and Technology (1971).

References for Chapter Eight

APPENDIX 5.1

A Practical Calculational Procedure for the Implementation of the Augmented Output Technique for Model Reference Adaptive Control

For the general case of the algorithm, described in Chapter Five, it is required to compute two compensator/controllers such that:

$$\underline{K}_1(z) = \hat{\underline{G}}_{OL}^{-1}(z)\, \underline{R}_1(z)\, (\underline{I} - \underline{R}_1(z))^{-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

and:

$$\underline{K}_2(z) = \hat{\underline{G}}_{OL}^{-1}(z)\, (\underline{I} + \hat{\underline{G}}_{OL}(z)\, \underline{K}_1(z))\, \underline{R}_2(z) - \hat{\underline{G}}_L(z) \quad \ldots\ldots (2)$$

or, using equation (1):

$$\underline{K}_2(z) = \hat{\underline{G}}_{OL}^{-1}(z)\, (\underline{I} + \underline{R}_1(z)\, (\underline{I} - \underline{R}_1(z))^{-1})\, \underline{R}_2(z) - \hat{\underline{G}}_L(z) \quad (3)$$

Here, the possibilities of measurable disturbances (or alternatively disturbance prediction) has been allowed for (see Appendix 5.2).

$\hat{\underline{G}}_{OL}$, the estimate of the open-loop transfer matrix is given by:

$$\hat{\underline{G}}_{OL} = (\underline{I} - \sum_{i=1}^{h} \hat{\underline{A}}_i(k)\, z^{-i})^{-1} \sum_{j=1}^{f} \hat{\underline{B}}_j(k)\, z^{-j} \quad \ldots\ldots\ldots\ldots (4)$$

$\hat{\underline{\underline{G}}}_L$, the estimate of the load transfer matrix is given by:

$$\hat{\underline{\underline{G}}}_L = (\underline{\underline{I}} - \sum_{i=1}^{h} \hat{\underline{\underline{A}}}_i(k)\, z^{-i})^{-1} \sum_{l=1}^{g} \hat{\underline{\underline{D}}}_l(k)\, z^{-l} \quad \ldots\ldots\ldots\ldots\ldots(5)$$

The closed-loop transfer matrices, $\underline{\underline{R}}_1(z)$ and $\underline{\underline{R}}_2(z)$ can be represented, at any time, by:

$$\underline{\underline{R}}_1(z) = (\underline{\underline{I}} - \sum_{i=1}^{H} \underline{\underline{A}}_{ip}\, z^{-i})^{-1} \sum_{j=1}^{F} \underline{\underline{B}}_{jp}\, z^{-j} \quad \ldots\ldots\ldots\ldots\ldots(6)$$

and:

$$\underline{\underline{R}}_2(z) = (\underline{\underline{I}} - \sum_{i=1}^{H} \underline{\underline{A}}_{ip}\, z^{-i})^{-1} \sum_{l=1}^{G} \underline{\underline{D}}_{lp}\, z^{-l} \quad \ldots\ldots\ldots\ldots\ldots(7)$$

$\hat{\underline{\underline{A}}}_i$, $\hat{\underline{\underline{B}}}_j$, and $\hat{\underline{\underline{D}}}_l$ are time-varying estimates of the open-loop parameter matrices.

$\underline{\underline{A}}_{ip}$, $\underline{\underline{B}}_{jp}$, and $\underline{\underline{D}}_{lp}$ are time-varying estimates of the closed-loop parameter matrices.

The problem of computation of these quantities, though not as horrendous as it first appears, since $\hat{\underline{\underline{A}}}_i$ and $\underline{\underline{A}}_{ip}$ are all diagonal[1], is still quite formidable.

To see what approximations may be desirable, let us

---

[1]

This condition is simply a ramification of the independence cf the outputs on one another.

consider equation (1) first.

Equation (1) may be written as:

$$\underline{K}_1(z) = \underline{X}^{-1}(z)\ \underline{Y}(z)\ (\underline{I} - \underline{Y}(z))^{-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

where:

$$\underline{X}(z) = \begin{bmatrix} \dfrac{\sum_{j=1}^{f} b_{jm11}\, z^{-j}}{1 - \sum_{i=1}^{h} a_{im11}\, z^{-i}} & \cdots\cdots\cdots & \dfrac{\sum_{j=1}^{f} b_{jm1n}\, z^{-j}}{1 - \sum_{i=1}^{h} a_{im11}\, z^{-i}} \\ \vdots & & \vdots \\ \dfrac{\sum_{j=1}^{f} b_{jmn1}\, z^{-j}}{1 - \sum_{i=1}^{h} a_{imnn}\, z^{-i}} & \cdots\cdots\cdots & \dfrac{\sum_{j=1}^{f} b_{jmnn}\, z^{-j}}{1 - \sum_{i=1}^{h} a_{imnn}\, z^{-i}} \end{bmatrix}$$

$$\underline{Y}(z) = \begin{bmatrix} \dfrac{\sum_{j=1}^{F} b_{jp11}\, z^{-j}}{1 - \sum_{i=1}^{H} a_{ip11}\, z^{-i}} & \cdots\cdots\cdots & \dfrac{\sum_{j=1}^{F} b_{jp1n}\, z^{-j}}{1 - \sum_{i=1}^{H} a_{ip11}\, z^{-i}} \\ \vdots & & \vdots \\ \dfrac{\sum_{j=1}^{F} b_{jpn1}\, z^{-j}}{1 - \sum_{i=1}^{H} a_{ipnn}\, z^{-i}} & \cdots\cdots\cdots & \dfrac{\sum_{j=1}^{F} b_{jpnn}\, z^{-j}}{1 - \sum_{i=1}^{H} a_{ipnn}\, z^{-i}} \end{bmatrix}$$

Thus, in order to calculate $\underline{\underline{K}}_1(z)$, directly in this manner there would be a requirement that $\underline{\underline{X}}(z)$ and $(\underline{\underline{I}} - \underline{\underline{Y}}(z))$ be invertible.

Suppose, however, that use is made of the control configuration. So that:

$$\underline{u}_1 = \underline{\underline{K}}_1\ \underline{e} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Substituting equation (8) in (9) yields:

$$\underline{u}_1 = \underline{\underline{X}}^{-1}\ \underline{\underline{Y}}\ (\underline{\underline{I}} - \underline{\underline{Y}})^{-1}\ \underline{e} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

or:

$$\underline{\underline{X}}\ \underline{u}_1 = \underline{\underline{Y}}\ (\underline{\underline{I}} - \underline{\underline{Y}})^{-1}\ \underline{e} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

which eliminates one of the on-line matrix polynomial inversions. The on-line computation is still formidable, however, for large systems.

The next simplification is to assume that the $\underline{\underline{B}}_{jp}$'s are diagonal. This corresponds to the case in which the closed-loop plant is assumed to respond as a number of single-input single-output systems. The reference model

would thus, also, be assumed to be non-interactive[1]. $\underline{\underline{K}}_1$ takes on a special significance, in such circumstances, as it is now apparent that it is acting as a combined decoupler/controller.

With this assumption, we may again write:

$$\underline{\underline{K}}_1(z) = \underline{\underline{X}}^{-1}(z)\ \underline{\underline{Y}}(z)\ (\underline{\underline{I}} - \underline{\underline{Y}}(z))^{-1}$$

or equivalently:

$$\underline{\underline{X}}(z)\ \underline{u}_1 = \underline{\underline{Y}}(z)\ (\underline{\underline{I}} - \underline{\underline{Y}}(z))^{-1}\ \underline{e}$$

where, this time:

$$\underline{\underline{Y}} = \begin{bmatrix} \dfrac{\sum_{j=1}^{F} b_{jp11}\, z^{-j}}{1 - \sum_{i=1}^{H} a_{ip11}\, z^{-i}} & 0 & \cdots & 0 \\ 0 & & & \vdots \\ \vdots & & & 0 \\ 0 & \cdots & 0 & \dfrac{\sum_{j=1}^{F} b_{jpnn}\, z^{-j}}{1 - \sum_{i=1}^{H} a_{ipnn}\, z^{-i}} \end{bmatrix}$$

---

[1] This approach would seem to be a most attractive one, in practice, for the familiar indices of performance could be applied to the plant in the classical way.

and:

$$\underline{\underline{Y}}(\underline{\underline{I}} - \underline{\underline{Y}})^{-1} = \begin{bmatrix} \dfrac{\sum_{j=1}^{F} b_{jp11}\, z^{-j}}{1 - \sum_{i=1}^{H} a_{ip11}\, z^{-i} - \sum_{j=1}^{F} b_{jp11}\, z^{-j}} & 0 & \cdots & 0 \\ 0 & & & \vdots \\ \vdots & & & 0 \\ 0 & \cdots & & \dfrac{\sum_{j=1}^{F} b_{jpnn}\, z^{-j}}{1 - \sum_{i=1}^{H} a_{ipnn}\, z^{-i} - \sum_{j=1}^{F} b_{jpnn}\, z^{-j}} \end{bmatrix}$$

$\underline{\underline{X}}(z)$ is as before.

Considering equation (10) and the forms of $\underline{\underline{X}}(z)$ and $\underline{\underline{Y}}(z)$, we can write after some algebraic manipulation, that:

$$\sum_{i=1}^{H+f} d_{ij1}\, z^{-i}\, u_1 + \cdots\cdots\cdots + \sum_{i=1}^{H+f} d_{ijn}\, z^{-i}\, u_n =$$

$$\sum_{i=1}^{F+h} g_{ijj}\, z^{-i}\, e_j \qquad (j = 1 \rightarrow n) \quad \text{.................(12)}$$

and:

$$\sum_{i=1}^{H+f} d_{ijk}\, z^{-j} = (1 - \sum_{i=1}^{H} a_{ipjj}\, z^{-i} - \sum_{i=1}^{F} b_{ipjj}\, z^{-i})$$

$$(\sum_{i=1}^{f} b_{ikj}\, z^{-i}) \quad \text{.....................................(13)}$$

$j = 1 \rightarrow n, \; k = 1 \rightarrow n$

$$\sum_{i=1}^{h+F} g_{ijj} \; z^{-j} = (1 - \sum_{i=1}^{h} a_{imjj} \; z^{-i})(\sum_{i=1}^{f} b_{imjj} \; z^{-i}) \quad \ldots(14)$$

$j = 1 \rightarrow n$

Equation (12) can be written in the input-output formulation as:

$$\sum_{i=1}^{H+f} d_{ij1} \; u_1(k-i) + \ldots\ldots\ldots\ldots\ldots + \sum_{i=1}^{H+f} d_{ijn} \; u_n(k-i) =$$

$$\sum_{i=1}^{H+f} g_{ijj} \; e_j(k-i) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(15)$$

$j = 1 \rightarrow n$

Now, at each sampling instant, it is required to compute $\underline{u}_1(k)$, ..., $\underline{u}_n(k)$.

From equation (15) it can be shown that, therefore:

$$d_{1j1} \; u_1(k) + \ldots\ldots\ldots\ldots + d_{1jn}(k) =$$

$$\sum_{i=1}^{H+f} g_{ijj} \; e_j(k-i+1) - \sum_{i=2}^{H+f} d_{ij1} \; u_1(k-i+1) -$$

$$\ldots\ldots\ldots\ldots\ldots - \sum_{i=2}^{H+f} d_{ijn} \; u_n(k-i+1) \ldots\ldots\ldots\ldots(16)$$

$j = 1 \rightarrow n$

Appendix 5.1

ie.:

$$\begin{bmatrix} d_1(11) & \cdots\cdots\cdots & d_1(1n) \\ \vdots & & \vdots \\ \vdots & & \vdots \\ \vdots & & \vdots \\ d_1(n1) & \cdots\cdots\cdots & d_1(nn) \end{bmatrix} \begin{bmatrix} u_1(k) \\ \vdots \\ \vdots \\ \vdots \\ u_n(k) \end{bmatrix} = \begin{bmatrix} x_1 \\ \vdots \\ x_j \\ \vdots \\ x_n \end{bmatrix}$$

where the numbers in brackets refer to the element of the matrix and x_j is given by the right-hand side of equation (16).

Finally, the controls may be computed from:

$$\begin{bmatrix} d_1(11) & \cdots\cdots\cdots & d_1(1n) \\ \vdots & & \vdots \\ \vdots & & \vdots \\ \vdots & & \vdots \\ d_1(n1) & \cdots\cdots\cdots & d_1(nn) \end{bmatrix}^{-1} \begin{bmatrix} x_1 \\ \vdots \\ \vdots \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} u_1(k) \\ \vdots \\ \vdots \\ \vdots \\ u_n(k) \end{bmatrix} \quad \ldots\ldots(17)$$

or:

$$\underline{u}(k) = \underline{D}^{-1}\ \underline{x} \quad \text{.......................................}(18)$$

It must be stated that the analysis, here, assumes a square, open-loop transfer matrix. If this is not the case, a Moore-Penrose generalized inverse may suffice, although the effects of this approximation on the stability of the overall system, has been shown to be undesirable [39 - 40][1].

An adhoc solution, to the problem of non-square plants, is discussed below.

The computation of $\underline{\underline{K}}_2(z)$, via equation (3), may be considered in an identical manner to the above with $\underline{\underline{R}}_1(z)$ as a diagonal matrix.

So that:

$$\underline{\underline{K}}_2(z) = \hat{\underline{\underline{G}}}_{OL}^{-1}(z)\,(\underline{\underline{I}} + \underline{\underline{S}}(z))\,\underline{\underline{T}}(z) - \hat{\underline{\underline{G}}}_L(z) \quad \text{..............}(19)$$

where:

$$\underline{\underline{S}}(z) = \underline{\underline{R}}_1(z)\,(\underline{\underline{I}} - \underline{\underline{R}}_1(z))^{-1}$$

and:

---

1 Appendix 5.1 and 5.2 -- see "References for Chapter Five".

Appendix 5.1

$$\underline{Y}(z) = \underline{R}_2(z)$$

Now, if:

$$\underline{u}_2 = \underline{K}_2\ \underline{\xi}$$

then:

$$\hat{\underline{G}}_{OL}\ \underline{u}_2 = [\ (\underline{I} + \underline{S})\ \underline{T} - \hat{\underline{G}}_{OL}\ \hat{\underline{G}}_{L}\ ]\ \underline{\xi} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

and thus a control law may be again written as a function of the parameter estimates.

Equation (20) defines an adaptive feedforward controller/compensator and, as such, makes use of the measurable disturbances in an active manner.

<u>The Problem of Non-Square Plants</u>

It has been implicitly assumed in the designs, outlined, that the open-loop plant transfer matrix estimate, $\hat{\underline{G}}_{OL}$, is square. This corresponds to a case in which the number of inputs is equal to the number of outputs. In a few situations, however, this is not the case and provision must be made for this.

Use is made of a fixed precompensator similar in concept to those designed, by Kuon, via the DNA method [52 - 53], although here, the major criterion of performance

is that the augmented plant is stable and square[1].

The scheme is then implemented, as before, using the augmented plant in place of the original plant. Figure A5-1.1 also includes an explicit identificaticn loop, since it is felt that this is a desirable information block.

---

1 The question of stability implies that an approximate knowledge of the open-loop poles is available.

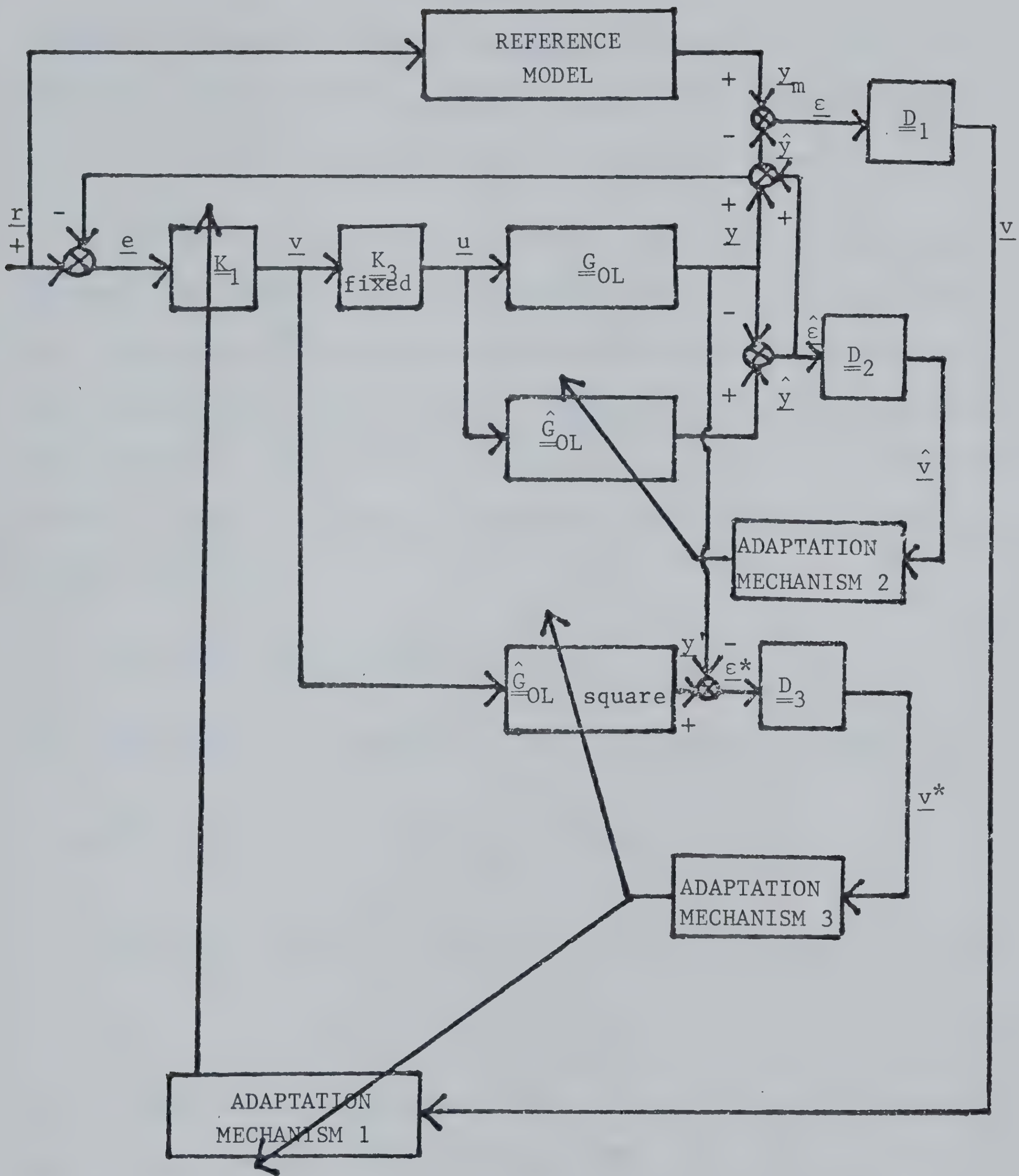


FIGURE A5-1.1 AUGMENTED OUTPUT MODEL REFERENCE ADAPTIVE CONTROL SCHEME FOR A NON-SQUARE PLANT

APPENDIX 5.2

Effect of Disturbances (Measurable or Not) and Bounded Nonlinearities on the Hyperstability of the Augmented Output Method for Model Reference Adaptive Control

All the derivations and proofs, of Chapter Five, have ignored the possibility of unmeasureable signals entering the system. The adaptation algorithms have no information concerning these disturbances, whatsoever. It is, therefore, of paramount importance to analyse the system and establish criteria which guarantee asymptotic hyperstability, under these conditions:

(i) The Identification System

The plant is assumed to obey an equation of the form:

$$\underline{y}(k) = \sum_{i=1}^{h} \underline{A}_i \, \underline{y}(k-i) + \sum_{j=1}^{f} \underline{B}_j \, \underline{u}(k-j) + \sum_{l=1}^{g} \underline{D}_l \, \underline{\xi}(k-l) + \sum_{l=1}^{g} \underline{D}'_l \, \underline{\xi}'(k-l) \quad \text{.....................(1)}$$

Here,

$\underline{y}$ is an nx1 output vector;

$\underline{u}$ is an mx1 control or input vector;

$\underline{\xi}$ is a px1 disturbance vector, which is known, $\forall k$;

and,

$\underline{\xi}'$ denotes an ex1 unmeasurable, disturbance vector, which is assumed bounded, $\forall k$.

$\underline{\underline{A}}_i$, $\underline{\underline{B}}_j$, $\underline{\underline{D}}_l$, and $\underline{\underline{D}}_l'$ are process parameter matrices of appropriate order[1].

An estimation model can be described by:

$$\hat{\underline{y}}(k) = \sum_{i=1}^{h} \hat{\underline{\underline{A}}}_i(k)\, \hat{\underline{y}}(k - i) + \sum_{j=1}^{f} \hat{\underline{\underline{B}}}_j(k)\, \underline{u}(k - j) + \sum_{l=1}^{g} \hat{\underline{\underline{D}}}_l\, \underline{\xi}(k - l) \quad \text{.......................................(2)}$$

where $\hat{\underline{y}}$ is an nx1 model output vector

$\hat{\underline{\underline{A}}}_i(k)$, $\hat{\underline{\underline{B}}}_j(k)$ and $\hat{\underline{\underline{D}}}_l(k)$ are time-varying model parameter matrices.

Defining an error vector, $\hat{\underline{\varepsilon}}(k)$:

$$\hat{\underline{\varepsilon}}(k) = \hat{\underline{y}}(k) - \underline{y}(k) \quad \text{....................................(3)}$$

and:

1

It would normally be assumed that n=m, ie. that the open-loop plant transfer matrix is square.

$$\hat{\underline{v}}(k) = \sum_{i=o}^{p_1} \underline{C}_i \, \hat{\underline{\varepsilon}}(k - i) \qquad \text{.......(4)}$$

$\underline{C}_0 = \underline{I}$

it is possible to write:

$$\hat{\underline{\varepsilon}}(k) = \sum_{i=1}^{h} \underline{A}_i \, \hat{\underline{\varepsilon}}(k - i) + \underline{I} \, \underline{w}(k) \qquad \text{.......(5)}$$

where:

$$\underline{w}(k) = - \sum_{i=1}^{h} (\underline{A}_i - \hat{\underline{A}}_i(k)) \, \hat{\underline{y}}(k - i) + \sum_{j=1}^{f} (\hat{\underline{B}}_j(k) - \underline{B}_j)$$

$$\underline{u}(k - j) + \sum_{l=1}^{g} (\hat{\underline{D}}_l(k) - \underline{D}_l) \, \underline{\xi}(k - l) - \sum_{l=1}^{g} \underline{D}'_l \, \underline{\xi}'(k - l) \qquad \text{.......(6)}$$

and:

$$\underline{w}_1(k) = -\underline{w}(k) \qquad \text{.......(7)}$$

Equations (3) - (7) describe an autonomous, discrete nonlinear, feedback system as depicted in Figure A5-2.1.

Further, if only those pairs $(\hat{\underline{v}}(k), \underline{w}_1(k))$ such that:

$$\eta(k_0, k_1) = \sum_{k=k_o}^{k_1} \hat{\underline{v}}^T(k) \, \underline{w}_1(k) \geq - \lambda_0^2 \qquad \forall k_1 \geq k_0 \qquad \text{.....(8)}$$

where λ_o is a finite constant only dependent on the initial

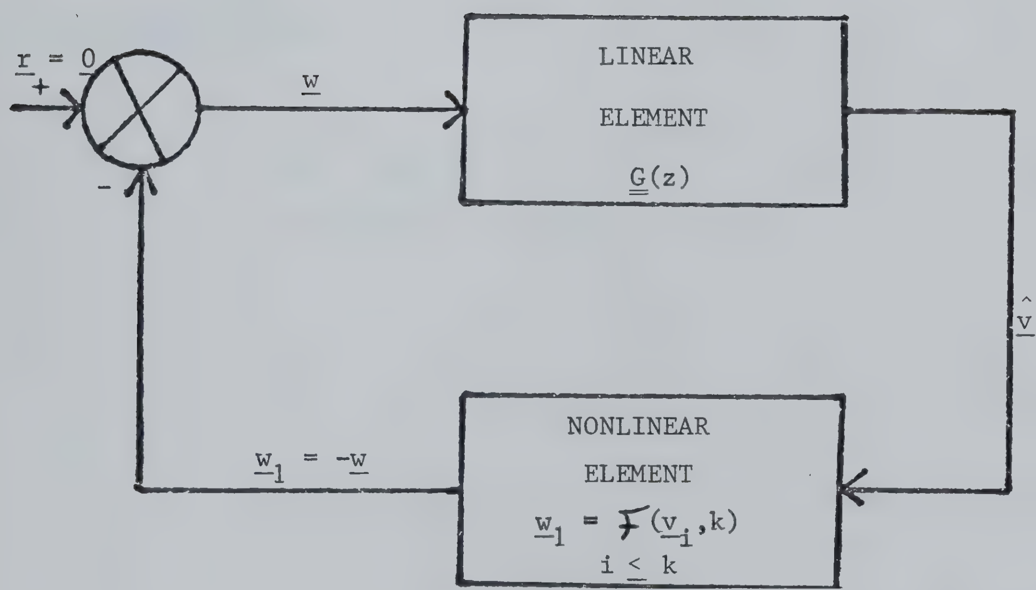


FIGURE A5-2.1 AUTONOMOUS NONLINEAR FEEDBACK IDENTIFICATION SYSTEM

system state, are considered, then it is possible to obtain sufficient conditions such that the system, described by equations (3) - (7) and inequality (8), is asymptotically hyperstable.

Theorem A5-2.1

Sufficient conditions such that the system, described by equations (3) - (7) and inequality (8), is asymptotic hyperstable, are:

(i) $\underline{\underline{G}}(z)=\underline{\underline{C}}(z)(\underline{\underline{I}} - \sum_{i=1}^{h} \underline{\underline{A}}_i z^{-i})^{-1} - 1/2\lambda\underline{\underline{I}}$ is a positive real discrete transfer matrix;

(ii)

$$(\hat{\underline{\underline{A}}}_i(k) - \underline{\underline{A}}_i)\,\hat{\underline{v}}(k - i) \qquad (i = 1 \rightarrow h)$$

$$(\hat{\underline{\underline{B}}}_j(k) - \underline{\underline{B}}_j)\,\underline{u}(k - j) \qquad (j = 1 \rightarrow f)$$

$$(\hat{\underline{\underline{D}}}_l(k) - \underline{\underline{D}}_l)\,\underline{\xi}(k - l) \qquad (l = 1 \rightarrow g)$$

$$\underline{\underline{D}}'_l\,\underline{\xi}'(k - l) \qquad (l = 1 \rightarrow g)$$

and $\hat{\underline{v}}$, are all of the same dimension;

(iii) The adaptation laws for $\hat{\underline{\underline{A}}}_i(k)$, $\hat{\underline{\underline{B}}}_j(k)$ and $\hat{\underline{\underline{D}}}_l(k)$ must admit the following nonlinear

relations:

$$\hat{\underline{\Phi}}_i(k) = [\ \hat{\phi}_{itq}(k)\ ] = [\ \alpha_{itq}(k-1)\,\hat{v}_t(k)\,\hat{y}_q(k-i)\ ]$$

$i = 1 \rightarrow h,\ t = 1 \rightarrow n,\ q = 1 \rightarrow n$

$$\hat{\underline{X}}_j(k) = [\ \hat{\eta}_{jtq}(k)\ ] = [\ \beta_{jtq}(k-1)\,\hat{v}_t(k)\,u_q(k-j)\ ]$$

$j = 1 \rightarrow f,\ t = 1 \rightarrow n,\ q = 1 \rightarrow m$

$$\hat{\underline{\Theta}}_l(k) = [\ \hat{\theta}_{ltq}(k)\ ] = [\ \delta_{ltq}(k-1)\,\hat{v}_t(k)\,\xi_q(k-l)\ ]$$

$l = 1 \rightarrow g,\ t = 1 \rightarrow n,\ q = 1 \rightarrow p$

where:

$$\alpha_{itq}(k) = \alpha_{itq}(k-1) + 1/\lambda[\ (\alpha_{itq}(k-1)\,\hat{y}_q(k-i))^2\ /$$

$$(1 - 1/\lambda\ \alpha_{itq}(k-1)\,\hat{y}^2_q(k-i))\ ]$$

$i = 1 \rightarrow h,\ t = 1 \rightarrow n,\ q = 1 \rightarrow n$

$$\beta_{jtq}(k) = \beta_{jtq}(k-1) + 1/\lambda[\ (\beta_{jtq}(k-1)\,u_q(k-j))^2\ /$$

$$(1 - 1/\lambda\ \beta_{jtq}(k-1)\,u^2_q(k-j))\ ]$$

$j = 1 \rightarrow f,\ t = 1 \rightarrow n,\ q = 1 \rightarrow m$

Appendix 5.2

$$\delta_{ltq}(k) = \delta_{ltq}(k-1) + 1/\lambda[\,(\delta_{ltq}(k-1)\,\xi_q(k-1))^2\,/$$

$$(1 - 1/\lambda \quad \delta_{ltq}\,\xi_q^2\,(k-1))\,]$$

$$l = 1 \rightarrow g,\ t = 1 \rightarrow n,\ q = 1 \rightarrow p$$

α_{itq}, β_{jtq} and δ_{ltq} are strictly negative coefficients and,

(iv) $\hat{\underline{v}}(k)$ is bounded.

The proof is analogous to that given for Theorem 5.3.

The inclusion of unmeasurable disturbances, thus, necessitates the inclusion of one more condition such that inequality (8) is satisfied.

The addition of this extra condition should come as no surprise as the guarantee of hyperstability, as noted by Thathachar and Gajendran [41], implies that $\hat{\underline{v}}(k)$ is bounded. In simplistic terms, this may be rationalized easily. Since hyperstability, in the configuration presently adopted, demands that the nonlinearity satisfies a boundedness condition (inequality (8)) and $\underline{\underline{G}}(z)$, the linear part, is positive real discrete, which implies absolute stability,

Appendix 5.2

then the output of the linear part, $\hat{\underline{v}}(k)$, must be bounded.

Unfortunately, at each sampling instant, $\hat{\underline{v}}(k)$ must be calculated. This is a function of the, as yet, unknown $\hat{\underline{\varepsilon}}(k)$, thus, equation (4) cannot be used. Another approach is, however, available:

From equations (2) - (4):

$$\hat{\underline{v}}(k) = -\underline{y}(k) + \sum_{i=1}^{h} \hat{\underline{A}}_i(k)\, \hat{\underline{y}}(k - i) + \sum_{j=1}^{f} \hat{\underline{B}}_j(k)\, \underline{u}(k - j) + \sum_{l=1}^{g} \hat{\underline{D}}_l(k)\, \underline{\xi}(k - l) + \sum_{i=1}^{p_1} \underline{C}_i\, \hat{\underline{\varepsilon}}(k - i) \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

or, in scalar form:

$$v_t(k) = -y_t(k) + \sum_{i=1}^{h}\sum_{q=1}^{n} \hat{a}_{itq}(k)\, \hat{y}_q(k - i) + \sum_{j=1}^{f}\sum_{q=1}^{m} \hat{b}_{jtq}(k)\, u_q(k - j) + \sum_{l=1}^{g}\sum_{q=1}^{p} \hat{d}_{ltq}\, \xi_q(k - l) + \sum_{i=1}^{p_1}\sum_{q=1}^{n} c_{itq}\, \hat{\varepsilon}_q(k - i) \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

$t = 1 \rightarrow n$

Introducing the parameter adaptation laws, of Theorem A5-2.1, in equation (10), one has:

$$v_t(k) = -y_t(k) + \sum_{i=1}^{h}\sum_{q=1}^{n} (\alpha_{itq}(k-1)\,\hat{v}_t(k)\,\hat{y}_q(k-i) +$$

$$\hat{a}_{itq}(k-1))\,\hat{y}_q(k-i) + \sum_{j=1}^{f}\sum_{q=1}^{m} (\beta_{jtq}(k-1)\,\hat{v}_t(k)$$

$$u_q(k-j) + \hat{b}_{jtq}(k-1))\,u_q(k-j) + \sum_{l=1}^{g}\sum_{q=1}^{p} (\delta_{ltq}(k-1)$$

$$\hat{v}_t(k)\,\xi_q(k-l) + \hat{d}_{ltq}(k-1))\,\xi_q(k-l) + \sum_{i=1}^{p_1}\sum_{q=1}^{n} c_{itq}$$

$$\hat{\varepsilon}_q(k-i) \quad \ldots\ldots (11)$$

So that with some algebraic manipulation:

$$\hat{v}_t(k) = X/Y \quad \ldots\ldots (12)$$

where:

$$X = -y_t(k) + \sum_{i=1}^{h}\sum_{q=1}^{n} \hat{a}_{itq}(k-1)\,\hat{y}_q(k-i) +$$

$$\sum_{j=1}^{f}\sum_{q=1}^{m} \hat{b}_{jtq}(k-1)\,u_q(k-j) + \sum_{l=1}^{g}\sum_{q=1}^{p} \hat{d}_{ltq}(k-1)$$

$$\xi_q(k-l) + \sum_{i=1}^{p_1}\sum_{q=1}^{n} c_{itq}\,\hat{\varepsilon}_q(k-i) \quad \ldots\ldots (13)$$

and:

$$Y = 1 - (\sum_{i=1}^{h}\sum_{q=1}^{n} \alpha_{itq}(k-1)\,\hat{y}_q^2(k-i) +$$

$$\sum_{j=1}^{f}\sum_{q=1}^{m} \beta_{jtq}(k-1)\, u_q^2(k-j) + \sum_{l=1}^{g}\sum_{q=1}^{p} \delta_{ltq}(k-1)$$

$$\xi_q^2(k-l)) \quad \ldots\ldots (14)$$

(ii) The Control Scheme

A very similar analysis, for the effect of unmeasurable disturbances on the overall control scheme of the proposed method, may be carried out.

Suppose that the closed-loop plant satisfies an input-output relation of the form:

$$\underline{y}(k) = \sum_{i=1}^{h} \underline{A}_{ip}(k)\, \underline{y}(k-i) + \sum_{j=1}^{f} \underline{B}_{jp}(k)\, \underline{r}(k-j) +$$

$$\sum_{l=1}^{g} \underline{D}_{lp}(k)\, \underline{\xi}(k-l) + \sum_{l=1}^{g} \underline{D}'_{lp}\, \underline{\xi}'(k-l) \quad \ldots\ldots (15)$$

The assumption is also made that $\underline{\xi}'(k-1)$, the unmeasurable disturbances, are bounded.

A reference model is then chosen such that:

$$\underline{y}_m(k) = \sum_{i=1}^{h_1} \underline{A}_{im}\, \underline{y}_m(k-i) + \sum_{j=1}^{f_1} \underline{B}_{jm}\, \underline{r}(k-j) +$$

$$\sum_{l=1}^{g_1} \underline{D}_{lm}\, \underline{\xi}(k-l) \quad \ldots\ldots (16)$$

where $h_1 \le h$, $f_1 \le f$, and $g_1 \le g$

Appendix 5.2

Using the definitions given by equations (3) and (4), one can write:

$$\underline{\varepsilon}(k) = \sum_{i=1}^{h_1} \mathbf{A}_{im}\ \underline{\varepsilon}(k-i) + \mathbf{I}\ \underline{w}(k) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(17)$$

where:

$$\underline{w}(k) = \sum_{i=1}^{h_1} (\mathbf{A}_{im} - \mathbf{A}_{ip}(k))\ \underline{y}(k-i) - \sum_{i=h_1+1}^{h} \mathbf{A}_{ip}(k)\ \underline{y}(k-i) +$$
$$\sum_{j=1}^{f_1} (\mathbf{B}_{jm} - \mathbf{B}_{jp}(k))\ \underline{r}(k-j) - \sum_{j=f_1+1}^{f} \mathbf{B}_{jp}(k)\ \underline{r}(k-j) +$$
$$\sum_{l=1}^{g_1} (\mathbf{D}_{lm} - \mathbf{D}_{lp}(k))\ \underline{\xi}(k-l) - \sum_{l=g_1+1}^{g} \mathbf{D}_{lp}(k)\ \underline{\xi}(k-l) -$$
$$\sum_{l=1}^{g} \mathbf{D}'_{lp}\ \underline{\xi}'(k-l) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

and:

$$\underline{w}_1(k) = -\underline{w}(k) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(19)$$

Equations (3), (4), (17) - (19) describe an autonomous discrete, nonlinear feedback system.

Once more, only those pairs $(\underline{v},\underline{w}_1)$ which satisfy the boundedness condition, given by inequality (8), are considered.

Thus we may write:

Theorem A5-2.2

Sufficient conditions such that the system, described by equations (3), (4), (17) - (19) and inequality (8), is an asymptotic hyperstable system, are:

(i) $\underline{\underline{G}}(z)=\underline{\underline{D}}_1(z)(\underline{\underline{I}}-\sum_{i=1}^{h_1}\underline{\underline{A}}_{im}z^{-i})^{-1}$ is positive real, discrete;

(ii)

$(\underline{\underline{A}}_{im} - \underline{\underline{A}}_{ip}(k))\,\underline{y}(k - i) \qquad (i = 1 \rightarrow h_1)$

$\underline{\underline{A}}_{ip}(k)\,\underline{y}(k - i) \qquad (i = h_1 + 1 \rightarrow h)$

$(\underline{\underline{B}}_{jm} - \underline{\underline{B}}_{jp}(k))\,\underline{r}(k - j) \qquad (j = 1 \rightarrow f_1)$

$\underline{\underline{B}}_{jp}(k)\,\underline{r}(k - j) \qquad (j = f_1 + 1 \rightarrow f)$

$(\underline{\underline{D}}_{lm} - \underline{\underline{D}}_{lp}(k))\,\underline{\xi}(k - l) \qquad (l = 1 \rightarrow g_1)$

$\underline{\underline{D}}_{lp}(k)\,\underline{\xi}(k - l) \qquad (l = g_1 + 1 \rightarrow g)$

$\underline{\underline{D}}'_{lp}\,\underline{\xi}'(k - l) \qquad (l = 1 \rightarrow g)$

and $\underline{v}$ are all of the same dimension;

(iii)

$$a_{itq}(k) = \alpha_{itq} \; v_t(k) \; y_q(k - i) + a_{itq}(k - 1)$$

$i = 1 \rightarrow h, \; t = 1 \rightarrow n, \; q = 1 \rightarrow n$

$$b_{jtq}(k) = \beta_{jtq} \; v_t(k) \; r_q(k - j) + b_{jtq}(k - 1)$$

$j = 1 \rightarrow f, \; t = 1 \rightarrow n, \; q = 1 \rightarrow n$

$$d_{ltq}(k) = \delta_{ltq} \; v_t(k) \; \xi_q(k - l) + d_{ltq}(k - 1)$$

$l = 1 \rightarrow g, \; t = 1 \rightarrow n, \; q = 1 \rightarrow p$

α_{itq}, β_{jtq} and δ_{ltq} are strictly positive constants and,

(iv) $\underline{v}(k)$ is bounded.

The proof is analogous to that given for Theorem 5.2.

Again $\underline{v}(k+1)$ must be calculated, and this may be done in an analogous fashion to the method given in Chapter Five (equations (18) - (20)).

Appendix 5.2

The conditions in the previous pages describe a system such as that depicted in Figure A5-2.2.

As with the scheme depicted in Figure 5.8 of Chapter Five, the closed-loop matrices of the plant are not directly accessible to adaptation. Therefore, computation methods for calculating the parameter matrices of the two compensator/controllers, $\underline{\underline{K}}_1$ and $\underline{\underline{K}}_2$ must be outlined.

With the configuration of Figure A5-2.2, it is possible to write:

$$\hat{\underline{y}} = (\underline{\underline{I}} + \hat{\underline{\underline{G}}}_{OL}\,\underline{\underline{K}}_1)^{-1}\,\hat{\underline{\underline{G}}}_{OL}\,\underline{\underline{K}}_1\,\underline{r} + (\underline{\underline{I}} + \hat{\underline{\underline{G}}}_{OL}\,\underline{\underline{K}}_1)^{-1}\,(\hat{\underline{\underline{G}}}_{L} + \hat{\underline{\underline{G}}}_{OL}\,\underline{\underline{K}}_2)\,\underline{\xi} \qquad \ldots\ldots(20)$$

or:

$$\hat{\underline{y}} = \underline{\underline{R}}_1\,\underline{r} + \underline{\underline{R}}_2\,\underline{\xi} \qquad \ldots\ldots(21)$$

where:

$\underline{\underline{R}}_1(z)$ is the closed-loop transfer matrix denoting the relationship between the outputs and setpoints, and

$\underline{\underline{R}}_2(z)$ is the closed-loop transfer matrix denoting the relationship between the outputs and the disturbances.

Using equations (20) and (21):

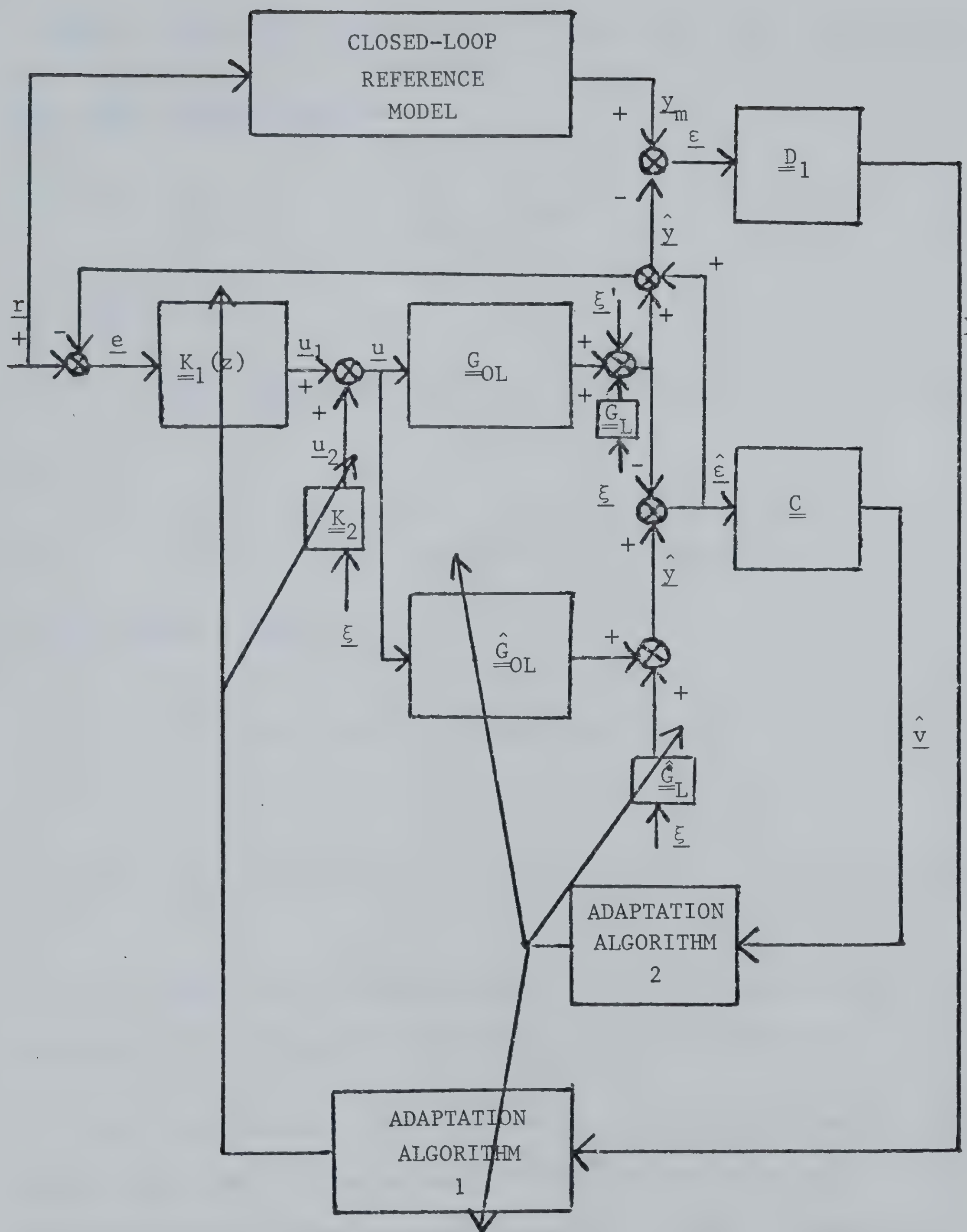


FIGURE A5-2.2 AUGMENTED OUTPUT MODEL REFERENCE ADAPTIVE SYSTEM WITH DISTURBANCES PRESENT

$$\underline{R}_1(z) = (\underline{I} + \hat{\underline{G}}_{OL}\ \underline{K}_1)^{-1}\ \hat{\underline{G}}_{OL}\ \underline{K}_1 \quad \ldots\ldots(22)$$

or, upon rearrangement:

$$\hat{\underline{G}}_{OL}\ \underline{K}_1 = (\underline{I} + \hat{\underline{G}}_{OL}\ \underline{K}_1)\ \underline{R}_1$$

So that:

$$\underline{K}_1(z) = \hat{\underline{G}}_{OL}^{-1}\ \underline{R}_1(z)\ (\underline{I} - \underline{R}_1(z))^{-1} \quad \ldots\ldots(23)$$

and:

$$\underline{R}_2(z) = (\underline{I} + \hat{\underline{G}}_{OL}\ \underline{K}_1)^{-1}\ (\hat{\underline{G}}_L + \hat{\underline{G}}_{OL}\ \underline{K}_2)$$

which implies that:

$$\hat{\underline{G}}_{OL}\ \underline{K}_2 = (\underline{I} + \hat{\underline{G}}_{OL}\ \underline{K}_1)\ \underline{R}_2 - \hat{\underline{G}}_L$$

and:

$$\underline{K}_2(z) = \hat{\underline{G}}_{OL}^{-1}\ (z)\ (\underline{I} + \hat{\underline{G}}_{OL}(z)\ \underline{K}_1(z))\ \underline{R}_2(z) - \hat{\underline{G}}_L(z) \quad \ldots\ldots(24)$$

In order that equations (23) and (24) be defined, it is necessary that $\hat{\underline{G}}_{OL}(z)$ and $(\underline{I} - \underline{R}_1(z))$ be invertible.

It would appear that the major problem involved in the implementation of the scheme, outlined, is that of dimensionality since calculations of $\underline{K}_1$ and $\underline{K}_2$ require

inversion of large matrix polynomials on-line. A practical alternative, to this approach, has been discussed in Appendix 5-1.

Nonlinear Plants

Since the stability approaches are based on an equivalent nonlinear autonomous feedback system [42 - 51,54], it is natural to expect that some some type of limited nonlinear plant behaviour could be tolerated.

A very obvious choice is a bounded nonlinearity. This enters the formulation in the same way as do unmeasurable disturbances. Once again the identification system will be affected, but as accurate parameter identification is only of secondary consideration, this is of little consequence.

Theorems A5.2-1 and A5.2-2 determine the stability of the control system when bounded nonlinearities are present and the development of the previous sections are directly applicable to this situation.

APPENDIX 6.1

Multivariable Simulation Run Data

System I

$$\begin{bmatrix} 2/(s+1) & 1/(s+1) \\ 1/(s+1) & 1/(s+1) \end{bmatrix}$$

System II

$$\begin{bmatrix} 2/(s+1) & 1/(s+1) \\ 1/(s+1) & 1/(s+10)^2 \end{bmatrix}$$

Appendix 6.1

RUNS 1, 2, 3, 4, 5.

Run Time = 100 minutes

Sampling Time = 60 seconds

Excitation Mode: setpoint change $\begin{pmatrix} 1.5 \\ 1.1 \end{pmatrix}$ @ 20 minutes

Initial Identification:

Excitation Mode: zero mean gaussian input sequence in both elements of the control vector:

element 1: s.d. = 0.2

element 2: s.d. = 0.3

Initial identification adaptive loop gain matrices:

$$\begin{bmatrix} -100 & -100 \\ -100 & -100 \end{bmatrix}$$

Initial identification model parameter matrices

Appendix 6.1

were all zero except for $\hat{\underline{\underline{B}}}_1$:

$$\hat{\underline{\underline{B}}}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

System III

$$\begin{bmatrix} 1/(s + 1)(5s + 1) & 1/(0.1s + 1) \\ 1/(0.1s + 1) & 1/(5s + 1)(0.2s + 1) \end{bmatrix}$$

RUNS 6, 7.

Run Time = 100 minutes

Sampling Time = 60 seconds

Excitation Mode: setpoint change $\begin{pmatrix} 6.0 \\ 8.0 \end{pmatrix}$ @ 20 minutes

Initial Identification:

Excitation Mode: step input change in both

elements of the control vector:

RUN 6

Step 1

element 1: step = 1.1

element 2: step = 1.2

time = 4 minutes

Step 2

element 1: step = 1.15

element 2: step = 1.1

time = 10 minutes

Step 3

element 1: step = 1.05

element 2: step = 1.15

time = 15 minutes

RUN 7

Step 1

element 1: step = 1.1

element 2: step = 1.15

time = 5 minutes

Step 2

element 1: step = 1.2

element 2: step = 1.1

time = 15 minutes

Appendix 6.1

Initial adaptive identification loop gain matrices:

$$\begin{bmatrix} -100 & -100 \\ -100 & -100 \end{bmatrix}$$

Initial identification model parameter matrices:

$$\hat{A}_1 = \begin{bmatrix} 1.2 & 0.0 \\ 0.0 & 1.0 \end{bmatrix} \quad \hat{A}_2 = \begin{bmatrix} -0.3 & 0.0 \\ 0.0 & 0.0 \end{bmatrix}$$

$$\hat{B}_1 = \begin{bmatrix} 0.0 & 10.0 \\ 10.0 & 0.0 \end{bmatrix} \quad \hat{B}_2 = \begin{bmatrix} 0.1 & -11.0 \\ -8.0 & 0.2 \end{bmatrix}$$

$$\hat{B}_3 = \begin{bmatrix} 0.0 & 3.0 \\ 0.0 & 0.0 \end{bmatrix}$$

Appendix 6.1

The initial desired adaptive model parameter matrices in all cases were set equal to zero. Further, in all runs the reference model was set equal to $\underline{\underline{I}}_2$.